INSTRUCTOR'S SOLUTIONS MANUAL

JAMES LAPP

Colorado Mesa University

ELEMENTARY STATISTICS

THIRTEENTH EDITION

Mario F. Triola

Pearson

CONTENTS

Chapter 1: Introduction To Statistics

Section 1-1: Statistical and Critical Thinking 1
Section 1-2: Types of Data 3
Section 1-3: Collecting Sample Data 3
Chapter Quick Quiz 5
Review Exercises 5
Cumulative Review Exercises 6

Chapter 2: Exploring Data with Tables and Graphs

Section 2-1: Frequency Distributions for Organizing and Summarizing Data 7
Section 2-2: Histograms 12
Section 2-3: Graphs That Enlighten and Graphs That Deceive 14
Section 2-4: Scatterplots, Correlation, and Regression 17
Chapter Quick Quiz 18
Review Exercises 19
Cumulative Review Exercises 20

Chapter 3: Describing, Exploring, and Comparing Data

Section 3-1: Measures of Center 21
Section 3-2: Measures of Variation 26
Section 3-3: Measures of Relative Standing and Boxplots 31
Chapter Quick Quiz 34
Review Exercises 35
Cumulative Review Exercises 36

Chapter 4: Probability

Section 4-1: Basic Concepts of Probability 37
Section 4-2: Addition Rule and Multiplication Rule 39
Section 4-3: Complements, Conditional Probability, and Bayes' Theorem 41
Section 4-4: Counting 43
Chapter Quick Quiz 45
Review Exercises 46
Cumulative Review Exercises 47

Chapter 5: Discrete Probability Distributions

Section 5-1: Probability Distributions 49
Section 5-2: Binomial Probability Distributions 51
Section 5-3: Poisson Probability Distributions 54
Chapter Quick Quiz 55
Review Exercises 56
Cumulative Review Exercises 57

Chapter 6: Normal Probability Distributions

Section 6-1: The Standard Normal Distribution 59
Section 6-2: Real Applications of Normal Distributions 60
Section 6-3: Sampling Distributions and Estimators 64
Section 6-4: The Central Limit Theorem 68
Section 6-5: Assessing Normality 71
Section 6-6: Normal as Approximation to Binomial 75
Chapter Quick Quiz 77
Review Exercises 78
Cumulative Review Exercises 79

Chapter 7: Estimating Parameters and Determining Sample Sizes

Section 7-1: Estimating a Population Proportion 81
Section 7-2: Estimating a Population Mean 85
Section 7-3: Estimating a Population Standard Deviation or Variance 89
Section 7-4: Bootstrapping: Using Technology for Estimates 92
Chapter Quick Quiz 94
Review Exercises 95
Cumulative Review Exercises 96

Chapter 8: Hypothesis Testing

Section 8-1: Basics of Hypothesis Testing 97
Section 8-2: Testing a Claim About a Proportion 99
Section 8-3: Testing a Claim About a Mean 105
Section 8-4: Testing a Claim About a Standard Deviation or Variance 109
Chapter Quick Quiz 112
Review Exercises 112
Cumulative Review Exercises 113

Chapter 9: Chapter 9: Inferences from Two Samples

Section 9-1: Two Proportions 115
Section 9-2: Two Means: Independent Samples 124
Section 9-3: Two Dependent Samples (Matched Pairs) 130
Section 9-4: Two Variances or Standard Deviations 134
Chapter Quick Quiz 136
Review Exercises 137
Cumulative Review Exercises 138

Chapter 10: Correlation and Regression

Section 10-1: Correlation 141
Section 10-2: Regression 148
Section 10-3: Prediction Intervals and Variation 157
Section 10-4: Multiple Regression 159
Section 10-5: Nonlinear Regression 161
Chapter Quick Quiz 165
Review Exercises 165
Cumulative Review Exercises 166

Chapter 11: Goodness-of-Fit and Contingency Tables

Section 11-1: Goodness-of-Fit 169
Section 11-2: Contingency Tables 173
Chapter Quick Quiz 177
Review Exercises 178
Cumulative Review Exercises 179

Chapter 12: Analysis of Variance

Section 12-1: One-Way ANOVA 181
Section 12-2: Two-Way ANOVA 183
Chapter Quick Quiz 184
Review Exercises 185
Cumulative Review Exercises 186

Chapter 13: Nonparametric Tests

Section 13-2: Sign Test 189
Section 13-3: Wilcoxon Signed-Ranks Test for Matched Pairs 190
Section 13-4: Wilcoxon Rank-Sum Test for Two Independent samples 192
Section 13-5: Kruskal-Wallis Test for Three or More Samples 195
Section 13-6: Rank Correlation 196
Section 13-7: Runs Test for Randomness 198
Chapter Quick Quiz 199
Review Exercises 200
Cumulative Review Exercises 201

Chapter 14: Statistical Process Control

Section 14-1: Control Charts for Variation and Mean 203
Section 14-2: Control Charts for Attributes 205
Chapter Quick Quiz 210
Review Exercises 210
Cumulative Review Exercises 212

Chapter 1: Introduction to Statistics

Section 1-1: Statistical and Critical Thinking

1. The respondents are a voluntary response sample or a self-selected sample. Because those with strong interests in the topic are more likely to respond, it is very possible that their responses do not reflect the opinions or behavior of the general population.
2. a. The sample consists of the 1046 adults who were surveyed. The population consists of all adults.
   b. When asked, respondents might be inclined to avoid the shame of the unhealthy habit of not washing their hands, so the reported rate of 70% might well be much higher than it is in reality. It is generally better to observe or measure human behavior than to ask subjects about it.
3. Statistical significance is indicated when methods of statistics are used to reach a conclusion that a treatment is effective, but common sense might suggest that the treatment does not make enough of a difference to justify its use or to be practical. Yes, it is possible for a study to have statistical significance, but not practical significance.
4. No. Correlation does not imply causation. The example illustrates a correlation that is clearly not the result of any interaction or cause effect relationship between deaths in swimming pools and power generated from nuclear power plants.
5. Yes, there does appear to be a potential to create a bias.
6. No, there does not appear to be a potential to create a bias.
7. No, there does not appear to be a potential to create a bias.
8. Yes, there does appear to be a potential to create a bias.
9. The sample is a voluntary response sample and has strong potential to be flawed.
10. The samples are voluntary response samples and have potential for being flawed, but this approach might be necessary due to ethical considerations involved in randomly selecting subjects and somehow imposing treatments on them.
11. The sampling method appears to be sound.
12. The sampling method appears to be sound.
13. With only a 1% chance of getting such results with a program that has no effect, the program appears to have statistical significance. Also, because the average loss of 22 pounds does seem substantial, the program appears to also have practical significance.
14. Because there is a 0.3% chance of getting such results by chance, the increase in scores does appear to have statistical significance. The typical increase of 5 points suggests that the course does have practical significance. The course does appear to be successful.
15. Because there is a 19% chance of getting that many girls by chance, the method appears to lack statistical significance. The result of 1020 girls in 2000 births (51% girls) is above the approximately 50% rate expected by chance, but it does not appear to be high enough to have practical significance. Not many couples would bother with a procedure that raises the likelihood of a girl from 50% to 51%.
16. Because there is a 25% chance of getting such results with a program that has no effect, the program does not appear to have statistical significance. Because the average increase is only 3 IQ points, the program does not appear to have practical significance.
17. Yes. Each column of 8 AM and 12 AM temperatures is recorded from the same subject, so each pair is matched.
18. No. The source is from university researchers who do not appear to gain from distorting the data.
19. The data can be used to address the issue of whether there is a correlation between body temperatures at 8 AM and at 12 AM. Also, the data can be used to determine whether there are differences between body temperatures at 8 AM and at 12 AM.
20. Because the differences could easily occur by chance (with a 64% chance), the differences do not appear to have statistical significance.
21. No. The white blood cell counts measure a different quantity than the red blood cell counts, so their differences are meaningless.

22. The issue that can be addressed is whether there is a correlation, or association, between white blood cell counts and red blood cell counts.
23. No. The National Center for Health Statistics has no reason to collect or present the data in a way that is biased.
24. No. Correlation does not imply causation, so a statistical correlation between white blood cell counts and red blood cell counts should not be used to conclude that higher white blood cell counts are the cause of higher red blood cell counts.
25. It is questionable that the sponsor is the Idaho Potato Commission and the favorite vegetable is potatoes.
26. The sample is a voluntary response sample, so there is a good chance that the results do not reflect the larger population of people who have a water preference.
27. The correlation, or association, between two variables does not mean that one of the variables is the cause of the other. Correlation does not imply causation. Clearly, sour cream consumption is not directly related in any way to motorcycle fatalities.
28. The sponsor of the poll is an electronic cigarette maker, so the sponsor does have an interest in the poll results. The source is questionable.
29. a. 700 adults
    b. 55%
30. a. 253.31 subjects
    b. No. Because the result is a count of people among the 347 who were surveyed, the result must be a whole number.
    c. 253 subjects
    d. 32%
31. a. 559.2 respondents
    b. No. Because the result is a count of respondents among the 1165 engaged or married women who were surveyed, the result must be a whole number.
    c. 559 respondents
    d. 8%
32. a. 293.17 women
    b. No. Because the result is a count of women among the 1543 who were surveyed, the result must be a whole number.
    c. 293 women
    d. 15%
    e. Interpretations of a "typical" week and what it means to "kick back and relax" might vary considerably by different survey respondents. The survey might be improved by asking about behavior within "the past seven days" instead of a "typical" week. Instead of "kick back and relax," respondents might be surveyed about specific behavior, such as reading, taking a nap, watching television, listening to music, or going for a walk.
33. Because a reduction of 100% would eliminate all of the size, it is not possible to reduce the size by 100% or more.
34. In an editorial criticizing the statement, the *New York Times* correctly interpreted the 100% improvement to mean that no baggage is being lost, which was not true.
35. Because a reduction of 100% would eliminate all plaque, it is not possible to reduce it by more than 100%.
36. If one subgroup receives a 4% raise and another subgroup receives a 4% raise, the combined group will receive a 4% raise, not an 8% raise. The percentages should not be added in this case.
37. The wording of the question is biased and tends to encourage negative responses. The sample size of 20 is too small. Survey respondents are self-selected instead of being randomly selected by the newspaper. If 20 readers respond, the percentages should be multiples of 5, so 87% and 13% are not possible results.
38. All percentages of success should be multiples of 5. The given percentages cannot be correct.

Section 1-2: Types of Data

1. The population consists of all adults in the United States, and the sample is the 2276 adults who were surveyed. Because the value of 33% refers to the sample, it is a statistic.
2. a. quantitative
   b. categorical
   c. categorical
   d. quantitative
3. Only part (a) describes discrete data.
4. a. The sample is the 1020 adults who were surveyed. The population is all adults in the United States.
   b. statistic
   c. ratio
   d. discrete
5. statistic
6. statistic
7. parameter
8. parameter
9. statistic
10. statistic
11. parameter
12. parameter
13. continuous
14. continuous
15. discrete
16. discrete
17. discrete
18. continuous
19. continuous
20. discrete
21. ordinal
22. nominal
23. nominal
24. ratio
25. interval
26. ordinal
27. ordinal
28. interval
29. The numbers are not counts or measures of anything. They are at the nominal level of measurement, and it makes no sense to compute the average (mean) of them.
30. The digits are not counts or measures of anything. They are at the nominal level of measurement and it makes no sense to calculate their average (mean).
31. The temperatures are at the interval level of measurement. Because there is no natural starting point with $0^\circ F$ representing "no heat," ratios such as "twice" make no sense, so it is wrong to say that it is twice as warm at the author's home as it is in Auckland, New Zealand.
32. The ranks are at the ordinal level of measurement. Differences between the universities cannot be determined, so there is no way to know whether the difference between Princeton and Harvard is the same as the difference between Yale and Columbia.
33. a. Continuous, because the number of possible values is infinite and not countable.
    b. Discrete, because the number of possible values is finite.
    c. Discrete, because the number of possible values is finite.
    d. Discrete, because the number of possible values is infinite and countable.

Section 1-3: Collecting Sample Data

1. The study is an experiment because subjects were given treatments.
2. The subjects in the study did not know whether they were taking a placebo or the paracetamol medication, and those who administered the pills also did not know.
3. The group sample sizes of 547, 550, and 546 are all large so that the researchers could see the effects of the paracetamol treatment.
4. The sample appears to be a convenience sample. Given that the subjects were randomly assigned to the three different treatment groups, it appears that the results of the study are good because they are not likely to be distorted from bias, but we should investigate the sample groups to ensure that they are not fundamentally different from the population.

5. The sample appears to be a convenience sample. By e-mailing the survey to a readily available group of Internet users, it was easy to obtain results. Although there is a real potential for getting a sample group that is not representative of the population, indications of which ear is used for cell phone calls and which hand is dominant do not appear to be factors that would be distorted much by a sample bias.
6. The study is an observational study because the subjects were not given any treatment.
7. With 717 responses, the response rate is 14%, which does appear to be quite low. In general, a very low response rate creates a serious potential for getting a biased sample that consists of those with a special interest in the topic.
8. Answers vary, but the following are good possibilities.
   a. Obtain a printed copy of the class roster, assign consecutive numbers (integers), then use a computer to randomly generate six of those numbers.
   b. Select every third student leaving class until six students are chosen.
   c. Randomly select three males and three females.
   d. Randomly select a row, and then select the students in that row. (Use only the first six to meet the requirement of a sample of size six.)
9. systematic
10. convenience
11. random
12. stratified
13. cluster
14. random
15. stratified
16. systematic
17. random
18. cluster
19. convenience
20. systematic
21. Observational study. The sample is a convenience sample consisting of subjects who decided themselves to respond. Such voluntary response samples have a high chance of not being representative of the larger population, so the sample may well be biased. The question was posted in an electronic edition of a newspaper, so the sample is biased from the beginning.
22. Experiment. The sample subjects consist of male physicians only. It would have been better to include females. Also, it would be better to include male and females who are not physicians.
23. Experiment. This experiment would create an *extremely* dangerous and illegal situation that has a real potential to result in injury or death. It's difficult enough to drive in New York City while being completely sober.
24. Observational study. The sample of four males and four females is too small.
25. Experiment. The biased sample created by using drivers from New York City cannot be fixed by using a larger sample. The larger sample will still be a biased sample that is not representative of drivers in the United States.
26. Experiment. Calling the subjects and asking them to report their weights has a high risk of getting results that do not reflect the actual weights. It would have been much better to somehow measure the weights instead of asking the subjects to report them.
27. Observational study. Respondents who have been convicted of felonies are not likely to respond honestly to the second question. The survey will suffer from a "social desirability bias" because subjects will tend to respond in ways that will be viewed favorably by those conducting the survey.
28. Observational study. The number of responses is very small, and the response rate of only 1.52% is far too small. With such a low response rate, there is a real possibility that the sample of respondents is biased and consists only of those with special interests in the survey topic.
29. prospective study
30. retrospective study
31. cross-sectional study
32. prospective study
33. matched pairs design
34. randomized block design
35. completely randomized design
36. matched pairs design

37. a. Not a simple random sample, but it is a random sample.
    b. Simple random sample and also a random sample.
    c. Not a simple random sample and not a random sample.

Quick Quiz

1. No. The numbers do not measure or count anything.
2. nominal
3. continuous
4. quantitative data
5. ratio
6. statistic
7. no
8. observational study
9. The subjects did not know whether they were getting aspirin or the placebo.
10. simple random sample

Review Exercises

1. The survey sponsor has the potential to gain from the results, which raises doubts about the objectivity of the results.
2. a. The sample is a voluntary response sample, so the results are questionable.
   b. statistic
   c. observational study
3. Randomized: Subjects were assigned to the different groups through a process of random selection, whereby they had the same chance of belonging to each group. Double-blind: The subjects did not know which of the three groups they were in, and the people who evaluated results did not know either.
4. No. Correlation does not imply causality.
5. Only part (c) is a simple random sample.
6. Yes. The two questions give the false impression that they are addressing very different issues. Most people would be in favor of defending marriage, so the first question is likely to receive a substantial number of "yes" responses. The second question better describes the issue and subjects are much more likely to have varied responses.
7. a. discrete
   b. ratio
   c. The mailed responses would be a voluntary response sample, so those with strong opinions or greater interest in the topics are more likely to respond. It is very possible that the results do not reflect the true opinions of the population of all full-time college students.
   d. stratified
   e. cluster
8. a. If they have no fat at all, they have 100% less than any other amount with fat, so the 125% figure cannot be correct.
   b. 686
   c. 28%
9. a. interval data; systematic sample
   b. nominal data; stratified sample
   c. ordinal data; convenience sample
10. Because there is a 15% chance of getting the results by chance, those results could easily occur by chance so the method does not appear to have statistical significance. The result of 236 girls in 450 births is a rate of 52.4%, so it is above the 50% rate expected by chance, but it does not appear to be high enough to have practical significance. The procedure does not appear to have either statistical significance or practical significance.

Cumulative Review Exercises

1. The mean is $\frac{3600+1700+4000+3900+3100+3800+2200+3000}{8}=3162.5$ grams. The weights all end with 00, suggesting that all of the weights are rounded to the hundreds place, so that the last two digits are always 00.

2. $0.5^6=0.015625$

3. $\frac{272-176}{6}=16,$ which is an unusually high value.

4. $\frac{98.2-98.6}{\frac{0.62}{\sqrt{106}}}=-6.64$

5. $\frac{1.96^2\cdot 0.25}{0.03^2}=1067$

6. $\frac{4000-1700}{4}=575$ grams

7. $\frac{(3600-3162.5)^2}{7}=27{,}343.75$ grams^2

8. $\sqrt{\frac{(98.4-98.6)^2+(98.6-98.6)^2+(98.8-98.6)^2}{3-1}}=\sqrt{0.04}=0.20$

9. $0.4^8=0.00065536$

10. $9^{11}=31{,}381{,}059{,}609$ (or about $31{,}381{,}060{,}000$)

11. $6^{14}=78{,}364{,}164{,}096$ (or about $78{,}364{,}164{,}000$)

12. $0.3^{12}=0.000000531441$

Chapter 2: Exploring Data with Tables and Graphs

Section 2-1: Frequency Distributions for Organizing and Summarizing Data

1. The table summarizes 50 service times. It is not possible to identify the exact values of all of the original times.
2. The classes of 60 – 120, 120 – 180, …, 300 – 360 overlap, so it is not always clear which class we should put a value in. For example, the value of 120 could go in the first class or the second class. The classes should be mutually exclusive.
3.

Time (sec)	Relative Frequency
60 – 119	14%
120 – 179	44%
180 – 239	28%
240 – 299	4%
300 – 359	10%

4. The sum of the relative frequencies is 125%, but it should be 100%, with a small round off error. All of the relative frequencies appear to be roughly the same, but if they are from a normal distribution, they should start low, reach a maximum, and then decrease.
5. Class width: 10
Class midpoints: 24.5, 34.5, 44.5, 54.5, 64.5, 74.5, 84.5
Class boundaries: 19.5, 29.5, 39.5, 49.5, 59.5, 69.5, 79.5, 89.5
Number: 87
6. Class width: 10
Class midpoints: 24.5, 34.5, 44.5, 54.5, 64.5, 74.5
Class boundaries: 19.5, 29.5, 39.5, 49.5, 59.5, 69.5, 79.5
Number: 87
7. Class width: 100
Class midpoints: 49.5, 149.5, 249.5, 349.5, 449.5, 549.5, 649.5
Class boundaries: –0.5, 99.5, 199.5, 299.5, 399.5, 499.5, 599.5, 699.5
Number: 153
8. Class width: 100
Class midpoints: 149.5, 249.5, 349.5, 449.5, 549.5
Class boundaries: 99.5, 199.5, 299.5, 399.5, 499.5, 599.5
Number: 147
9. No. The maximum frequency is in the second class instead of being near the middle, so the frequencies below the maximum do not mirror those above the maximum.
10. Yes. The frequencies start low, reach a maximum of 36, and then decrease. The values below the maximum are very roughly a mirror image of those above it.
11.

Duration (sec)	Frequency
125 – 149	1
150 – 174	0
175 – 199	0
200 – 224	3
225 – 249	34
250 – 274	12

12. The intensities do not appear to have a normal distribution.

Tornado F-Scale	Frequency
0	24
1	16
2	2
3	2
4	1

13.

Burger King Lunch Service Times (sec)	Frequency
70 – 109	11
110 – 149	23
150 – 189	7
190 – 229	6
230 – 269	3
230 – 269	6

14.

Burger King Dinner Service Times (sec)	Frequency
30 – 69	1
70 – 109	6
110 – 149	26
150 – 189	7
190 – 229	3
230 – 269	6
270 – 309	1

15. The distribution does not appear to be a normal distribution.

Wendy's Lunch Service Times (sec)	Frequency
70 – 149	25
150 – 229	15
230 – 309	6
310 – 389	3
390 – 469	1

16. The distribution does appear to be a normal distribution.

Wendy's Dinner Service Times (sec)	Frequency
30 – 69	4
70 – 109	11
110 – 149	15
150 – 189	10
190 – 229	4
230 – 269	6

17. Because there are disproportionately more 0s and 5s, it appears that the heights were reported instead of measured. Consequently, it is likely that the results are not very accurate.

x	Frequency
0	9
1	2
2	1
3	3
4	1
5	15
6	2
7	0
8	3
9	1

18. Because there are disproportionately more 0s and 5s, it appears that the weights were reported instead of measured. Consequently, it is likely that the results are not very accurate.

x	Frequency
0	26
1	1
2	1
3	2
4	2
5	12
6	1
7	0
8	4
9	1

19. The actresses appear to be younger than the actors.

Age When Oscar Was Won	Relative Frequency (Actresses)	Relative Frequency (Actors)
20 – 29	33.3%	1.1%
30 – 39	39.1%	32.2%
40 – 49	16.1%	41.4%
50 – 59	3.4%	17.2%
60 – 69	5.7%	6.9%
70 – 79	1.1%	1.1%
80 – 89	1.1%	

20. There do appear to be differences, but overall they are not very substantial differences.

Blood Platelet Count	Males	Females
0 – 99	0.7%	
100 – 199	33.3%	17.0%
200 – 299	58.8%	62.6%
300 – 399	6.5%	19.0%
400 – 499	0%	0%
500 – 599	0%	1.4%
600 – 699	0.7%	

21.

Age (years) of Best Actress When Oscar Was Won	Cumulative Frequency
Less than 30	29
Less than 40	63
Less than 50	77
Less than 60	80
Less than 70	85
Less than 80	86
Less than 90	87

22.

Age (years) of Best Actor When Oscar Was Won	Cumulative Frequency
Less than 30	1
Less than 40	29
Less than 50	65
Less than 60	80
Less than 70	86
Less than 80	87

23. No. The highest relative frequency of 24.8% is not much higher than the others.

Adverse Reaction	Relative Frequency
Headache	23.6%
Hypertension	8.7%
Upper Resp. Tract Infection	24.8%
Nasopharyngitis	21.1%
Diarrhea	21.9%

24. Yes, it appears that births occur on the days of the week with frequencies that are about the same.

Day	Relative Frequency
Monday	13.0%
Tuesday	16.5%
Wednesday	18.0%
Thursday	14.3%
Friday	14.3%
Saturday	10.8%
Sunday	13.3%

25. Yes, the frequency distribution appears to be a normal distribution.

Systolic Blood Pressure (mm Hg)	Frequency
80 – 99	11
100 – 119	116
120 – 139	131
140 – 159	34
160 – 179	7
180 – 199	1

26. Yes, the frequency distribution appears to be a normal distribution.

Diastolic Blood Pressure (mm Hg)	Frequency
40 – 54	27
55 – 69	107
70 – 84	133
85 – 99	31
100 – 114	2

27. Yes, the frequency distribution appears to be a normal distribution.

Magnitude	Frequency
1.00 – 1.49	19
1.50 – 1.99	97
2.00 – 2.49	187
2.50 – 2.99	147
3.00 – 3.49	100
3.50 – 3.99	38
4.00 – 4.49	8
4.50 – 4.99	4

28. No, the frequency distribution does not appear to be a normal distribution.

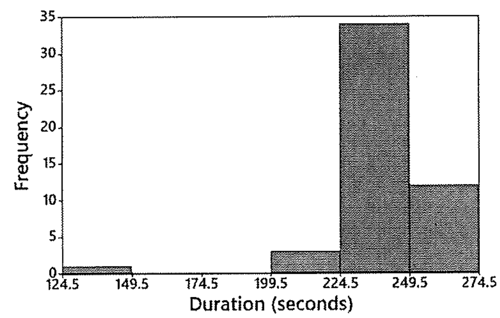
Depth (km)	Frequency
0.0 – 9.9	539
10.0 – 19.9	49
20.0 – 29.9	10
30.0 – 39.9	1
40.0 – 49.9	1

29. An outlier can dramatically increase the number of classes.

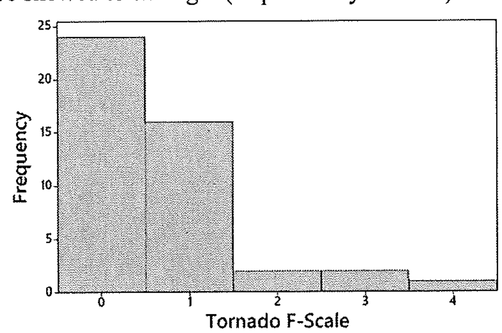
Weight (lb)	With Outlier	Without Outlier
200 – 219	6	6
220 – 239	5	5
240 – 259	12	12
260 – 279	36	36
280 – 299	87	87
300 – 319	28	28
320 – 339	0	
340 – 359	0	
360 – 379	0	
380 – 399	0	
400 – 419	0	
420 – 439	0	
440 – 459	0	
460 – 479	0	
480 – 499	0	
500 – 519	1	

Section 2-2: Histograms

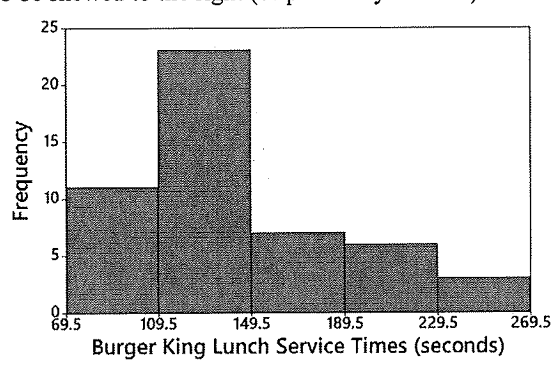
1. The histogram should be bell-shaped.
2. Not necessarily. Because the sample subjects themselves chose to be included, the voluntary response sample might not be representative of the population.
3. With a data set that is so small, the true nature of the distribution cannot be seen with a histogram.
4. The outlier will result in a single bar that is far away from all of the other bars in the histogram, and the height of that bar will correspond to a frequency of 1.
5. 40
6. Approximate values: Class width: 0.1 gram, lower limit of first class: 5.5 grams, upper limit of first class: 5.6 grams
7. The shape of the graph would not change. The vertical scale would be different, but the relative heights of the bars would be the same.
8. 40 of the quarters are "pre-1964" made with 90% silver and 10% copper, and the other 40 quarters are "post-1964" made with a copper-nickel alloy. The histogram depicts weights from two different populations of quarters.
9. Because it is far from being bell-shaped, the histogram does not appear to depict data from a population with a normal distribution.



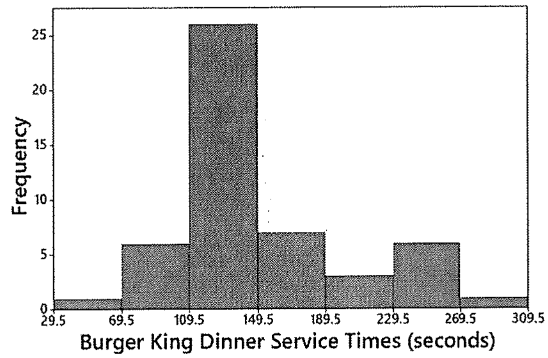
10. The histogram appears to be skewed to the right (or positively skewed).



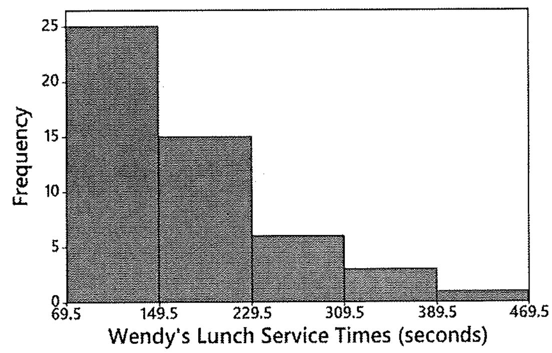
11. The histogram appears to be skewed to the right (or positively skewed).



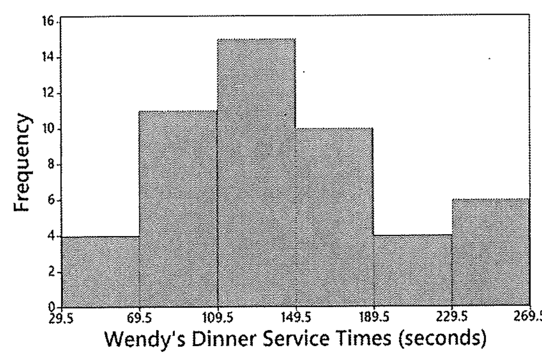
12. Because the histogram isn't close enough to being bell-shaped, it does not appear to depict data from a population with a normal distribution.



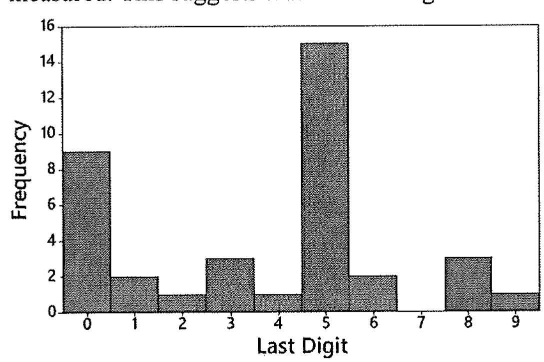
13. The histogram appears to be skewed to the right (or positively skewed).



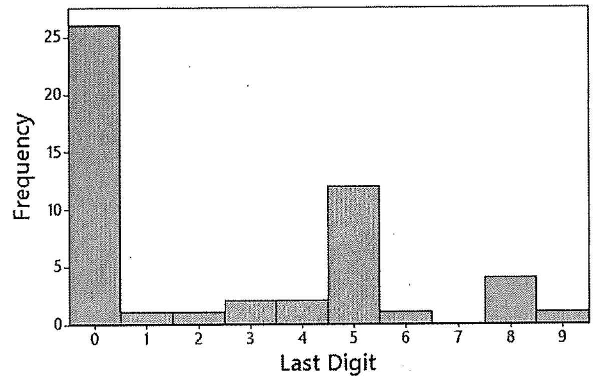
14. Because the histogram is not close enough to being bell-shaped, it does not appear to depict data from a population with a normal distribution.



15. The digits 0 and 5 appear to occur more often than the other digits, so it appears that the heights were reported and not actually measured. This suggests that the data might not be very useful.



16. The digits 0 and 5 appear to occur more often than the other digits, so it appears that the weights were reported and not actually measured. This suggests that the data might not be very useful.



17. The ages of actresses are lower than the ages of actors.

18. Only part (c) appears to represent data from a normal distribution. Part (a) has a systematic pattern that is not that of a straight line, part (b) has points that are not close to a straight-line pattern, and part (d) is really bad because it shows a systematic pattern and points that are not close to a straight-line pattern.

Section 2-3: Graphs That Enlighten and Graphs That Deceive

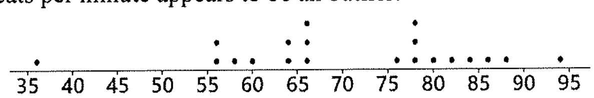
1. The data set is too small for a graph to reveal important characteristics of the data. With such a small data set, it would be better to simply list the data or place them in a table.

2. No. If the sample is a bad sample, such as one obtained from voluntary responses, there are no graphs or other techniques that can be used to salvage the data.

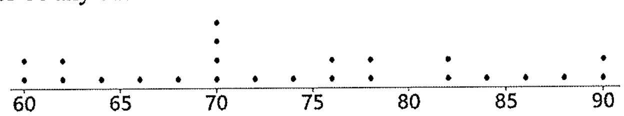
3. No. Graphs should be constructed in a way that is fair and objective. The readers should be allowed to make their own judgments, instead of being manipulated by misleading graphs.

4. Center, variation, distribution, outliers, change in the characteristics of data over time. The time-series graph does the best job of giving us insight into the change in the characteristics of data over time.

5. The pulse rate of 36 beats per minute appears to be an outlier.



6. There do not appear to be any outliers.



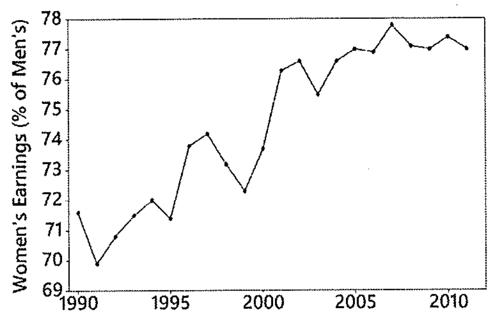
7. The data are arranged in order from lowest to highest, as 36, 56, 56, and so on.

3 | 6
4 |
5 | 668
6 | 044666
7 | 6888
8 | 02468
9 | 4

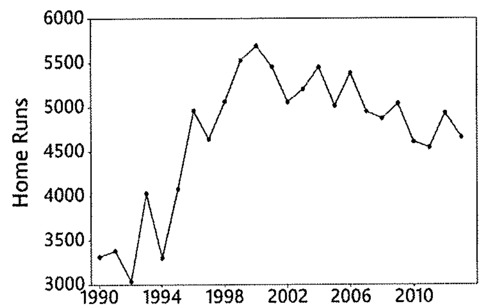
8. The two values closes to the middle are 72 mm Hg and 74 mm Hg.

6 | 0022468
7 | 0000246688
8 | 22468
9 | 00

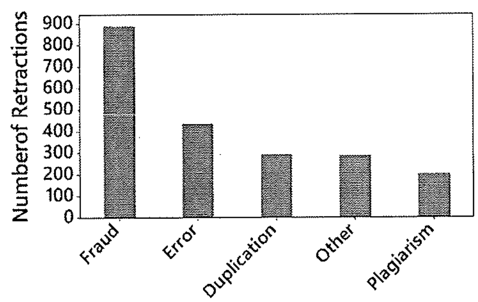
9. There is a gradual upward trend that appears to be leveling off in recent years. An upward trend would be helpful to women so that their earnings become equal to those of men.



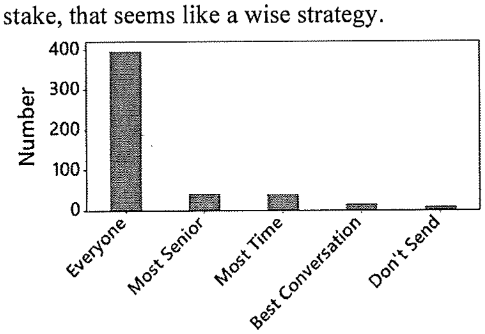
10. The numbers of home runs rose from 1990 to 2000, but after 2000 there has been a gradual decline.



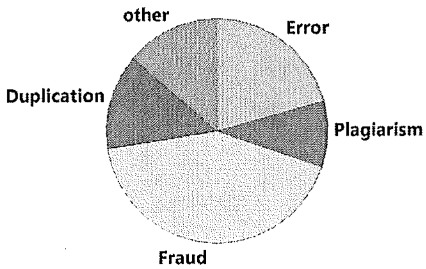
11. Misconduct includes fraud, duplication, and plagiarism, so it does appear to be a major factor.



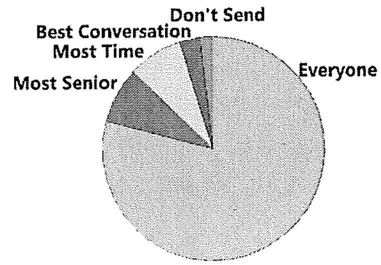
12. The overwhelming response was that thank-you notes should be sent to everyone who is met during a job interview. Given what is at stake, that seems like a wise strategy.



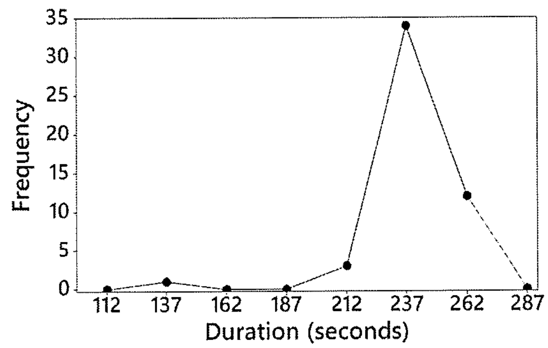
13.



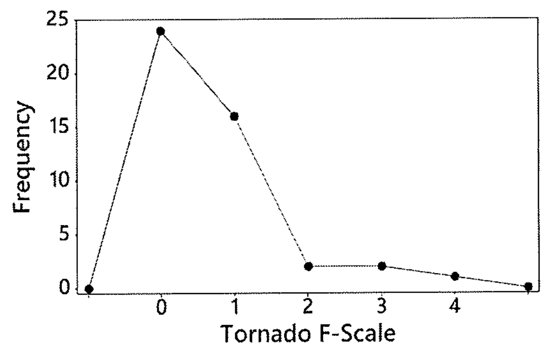
14.



15. The distribution appears to be skewed to the left (or negatively skewed).



16. The distribution appears to be skewed to the right (or positively skewed).



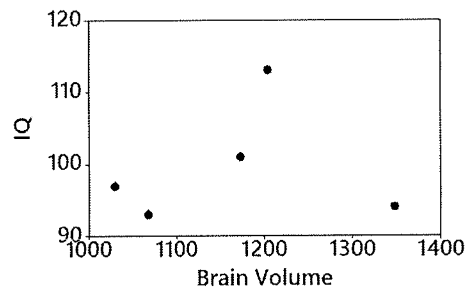
17. Because the vertical scale starts with a frequency of 200 instead of 0, the difference between the "no" and "yes" responses is greatly exaggerated. The graph makes it appear that about five times as many respondents said "no," when the ratio is actually a little less than 2.5 to 1.
18. The fare increased from \$1 to \$2.50, so it increased by a factor of 2.5. But when the larger bill is drawn so that the width is 2.5 times that of the smaller bill and the height is 2.5 times that of the smaller bill, the larger bill has an area that is 6.25 times that of the smaller bill (instead of being 2.5 times its size, as it should be). The illustration greatly exaggerates the increase in the fare.
19. The two costs are one-dimensional in nature, but the baby bottles are three-dimensional objects. The \$4500 cost isn't even twice the \$2600 cost, but the baby bottles make it appear that the larger cost is about five times the smaller cost.
20. The graph is misleading because it depicts one-dimensional data with three-dimensional boxes. See the first and last boxes in the graph. Workers with advanced degrees have annual incomes that are roughly 3 times the incomes of those with no high school diplomas, but the graph exaggerates this difference by making it appear that workers with advanced degrees have incomes that are roughly 27 times the amounts for workers with no high school diploma.

21.

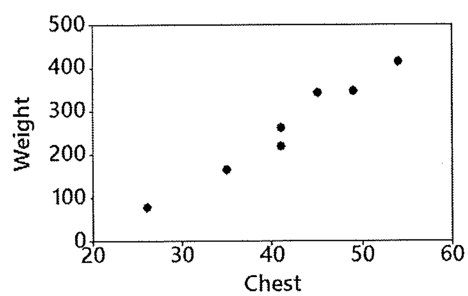
96 |
96 | 59
97 | 0001112333444
97 | 55666666788888999
98 | 5555666666666666666677777788888889
96 | 001244
96 | 56

Section 2-4: Scatterplots, Correlation, and Regression

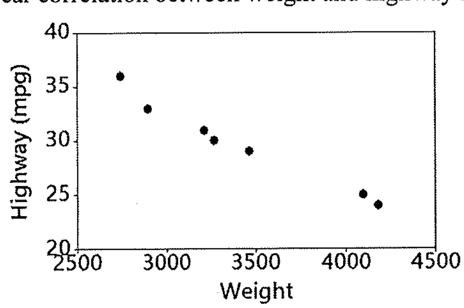
1. The term linear refers to a straight line, and r measures how well a scatterplot of the sample paired data fits a straight-line pattern.
2. No. Finding the presence of a statistical correlation between two variables does not justify any conclusion that one of the variables is a cause of the other.
3. A scatterplot is a graph of paired (x, y) quantitative data. It helps us by providing a visual image of the data plotted as points, and such an image is helpful in enabling us to see patterns in the data and to recognize that there may be a correlation between the two variables.
4. a. 1
   b. 0
   c. 0
   d. −1
5. There does not appear to be a linear correlation between brain volume and IQ score.



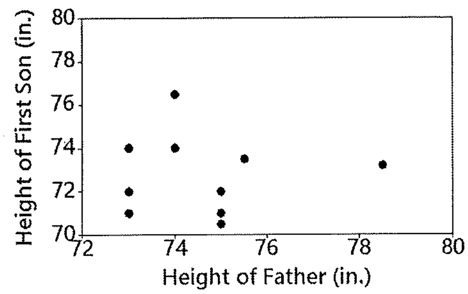
6. There does appear to be a linear correlation between chest sizes and weights of bears.



7. There does appear to be a linear correlation between weight and highway fuel consumption.



8. There does not appear to be a linear correlation between heights of fathers and the heights of their first sons.



9. With $n = 5$ pairs of data, the critical values are ± 0.878. Because $r = 0.127$ is between -0.878 and 0.878, evidence is not sufficient to conclude that there is a linear correlation.
10. With $n = 7$ pairs of data, the critical values are ± 0.754. Because $r = 0.980$ is in the right tail region beyond 0.754, there are sufficient data to conclude that there is a linear correlation.
11. With $n = 7$ pairs of data, the critical values are ± 0.754. Because $r = 0.987$ is in the left tail region below -0.754, there are sufficient data to conclude that there is a linear correlation.
12. With $n = 10$ pairs of data, the critical values are ± 0.632. Because $r = 0.017$ is between -0.632 and 0.632, evidence is not sufficient to conclude that there is a linear correlation.
13. Because the *P*-value is not small (such as 0.05 or less), there is a high chance (83.9% chance) of getting the sample results when there is no correlation, so evidence is not sufficient to conclude that there is a linear correlation.
14. Because the *P*-value is small (such as 0.05 or less), there is a small chance of getting the sample results when there is no correlation, so there is sufficient evidence to conclude that there is a linear correlation.
15. Because the *P*-value is small (such as 0.05 or less), there is a small chance of getting the sample results when there is no correlation, so there is sufficient evidence to conclude that there is a linear correlation.
16. Because the *P*-value is not small (such as 0.05 or less), there is a high chance (96.3% chance) of getting the sample results when there is no correlation, so the evidence is not sufficient to conclude that there is a linear correlation.

Quick Quiz

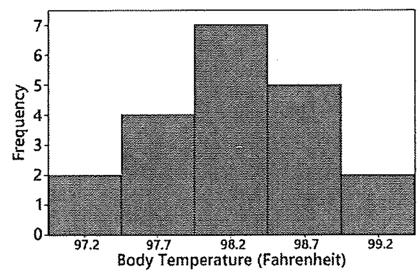
1. Class width: 3. It is not possible to identify the original data values.
2. Class boundaries: 17.5 and 20.5
   Class limits: 18 and 20.
3. 40
4. 19 and 19
5. pareto chart
6. histogram
7. scatterplot
8. No, the term "normal distribution" has a different meaning than the term "normal" that is used in ordinary speech. A normal distribution has a bell shape, but the randomly selected lottery digits will have a uniform or flat shape.
9. variation
10. The bars of the histogram start relatively low, increase to some maximum, and then decrease. Also, the histogram is symmetric, with the left half being roughly a mirror image of the right half.

Review Exercises

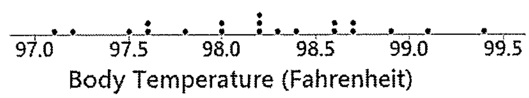
1.

Temperature (°F)	Frequency
97.0–97.4	2
97.5–97.9	4
98.0–98.4	7
98.5–98.9	5
99.0–99.4	2

2. Yes, the data appear to be from a population with a normal distribution because the bars start low and reach a maximum, then decrease, and the left half of the histogram is approximately a mirror image of the right half.



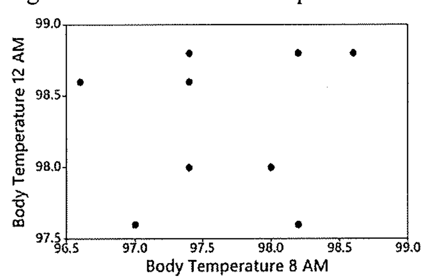
3. The distribution is closer to being a normal distribution than the others.



4. There are no outliers.

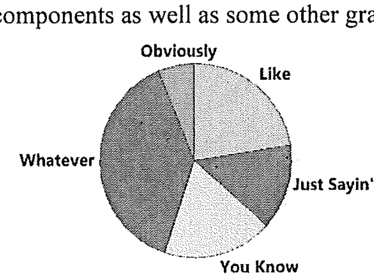
```
97. |125668
98. |002223466779
99. |14
```

5. No. There is no pattern suggesting that there is a relationship.

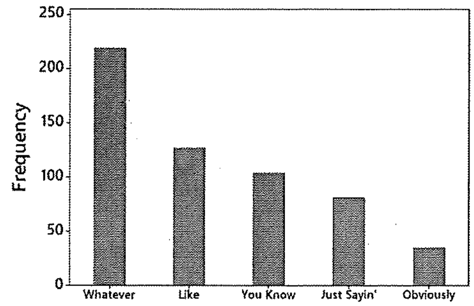


6. a. time-series graph
   b. scatterplot
   c. pareto chart

7. A pie chart wastes ink on components that are not data; pie charts lack an appropriate scale; pie charts don't show relative sizes of different components as well as some other graphs, such as a Pareto chart.



8. The Pareto chart does a better job. It draws attention to the most annoying words or phrases and shows the relative sizes of the different categories.

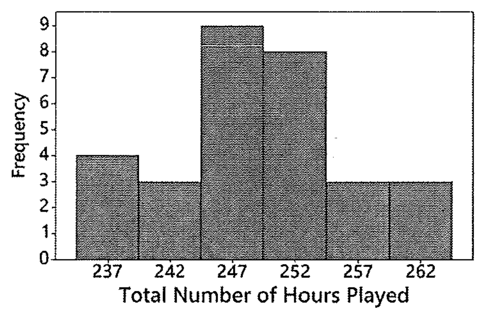


Cumulative Review Exercises

1.

Total Hours	Frequency
235–239	4
240–244	3
245–249	9
250–254	8
255–259	3
260–264	3

2. a. 235 hours and 239 hours
   b. 234.5 hours and 239.5 hours
   c. 237 hours
3. The distribution is closer to being a normal distribution than the others.



4. Start the vertical scale at a frequency of 2 instead of the frequency of 0.
5. Looking at the stemplot sideways, we can see that the distribution approximates a normal distribution.

```
23 | 6789
24 | 112555677889
25 | 00012233888
26 | 024
```

6. a. continuous
   b. quantitative
   c. ratio
   d. convenience sample
   e. sample

Chapter 3: Describing, Exploring, and Comparing Data

Section 3-1: Measures of Center

1. The term *average* is not used in statistics. The term *mean* should be used for the result obtained by adding all of the sample values and dividing the total by the number of sample values.
2. No. The 50 amounts are all weighed equally in the calculation that yields the mean of 27.3 cents per gallon of gas, but some states consume many more gallons of gas than others, so the mean amount of state sales tax should be calculated using a weighted mean that takes into account the numbers of gallons sold in the different states.
3. They use different approaches for providing a value (or values) of the center or middle of the sorted list of data.
4. Using the four values, $\bar{x} = \frac{13.5+10.2+21.1+15.1}{4} = 14.98$ Mbps and the median is $\frac{13.5+15.1}{2} = 14.30$ Mbps.

   Using the five values, including the outlier of 142, $\bar{x} = \frac{13.5+10.2+21.1+15.1+142}{5} = 40.38$ Mbps and the median is 15.10 Mbps.
   The outlier caused the mean to change by a substantial amount, but the median changed very little. The median is resistant to the effect of the outlier, but the mean is not resistant.
5. The mean is $\bar{x} = \frac{7+19+20+25+55+60+81+82+89+91+99}{11} = 57.1$.

   The median is 60.
   There is no mode.
   The midrange is $\frac{7+99}{11} = 53$.
   The jersey numbers are nominal data that are just replacements for names, and they do not measure or count anything, so the resulting statistics are meaningless.
6. The mean is $\bar{x} = \frac{189+190+190+195+202+225+235+252+254+305+305}{11} = 231.1$ lb.

   The median is 225 lb.
   The modes are 190 lb and 305 lb.
   The midrange is $\frac{189+305}{2} = 247.0$ lb.
   The jersey numbers are nominal data that are just replacements for names, and they do not measure or count anything, so the resulting statistics are meaningless.
7. The mean is $\bar{x} = \frac{150+150+150+150+160+160+165+185+200+250}{10} = 172$, or \$172.0 million.

   The median is $\frac{160+160}{2} = 160$, or \$160 million.
   The mode is \$150 million
   The midrange is $\frac{150+250}{2} = 200$, or \$200 million.
   Apart from the fact that all other celebrities have amounts of net worth lower than those given, nothing meaningful can be known about the population of net worth of all celebrities. The numbers all end in 0 or 5, and they appear to be rounded estimates.

8. The mean is $\bar{x}=\frac{77+104+121+153+195+212+244+264}{8}=171.3$, or \$173.30.

The median is $\frac{153+195}{2}=174.0$, or \$174.00.

There is no mode.

The midrange is $\frac{77+264}{2}=170.5$, or \$170.50.

The lowest price is an important statistic that is not one of the measures of center. Another factor to consider is the quality of the hotel, which can be judged by using ratings of others who stayed at the hotels. Other factors might be the convenience of the specific location, hotel restaurants, hotel nightclubs, and so on.

9. The mean is $\bar{x}=\frac{2+4+5+6+7+7+8+8+8+8+9+9+12+15}{14}=7.7$ hurricanes.

The median is $\frac{8+8}{2}=8.0$ hurricanes.

The mode is 8 hurricanes.

The midrange is $\frac{2+15}{2}=6.5$ hurricanes.

The data are time-series data, but the measures of center do not reveal anything about a trend consisting of a pattern of change over time.

10. The mean is $\bar{x}=\frac{1+1+1+1+1+1+1+1+1+1+1+\cdots+2+2+2+3+3+3+3+3+3+4}{25}=1.9$.

The median is 2.
The mode is 1.

The midrange is $\frac{1+4}{2}=2.5$.

The mode of 1 correctly indicates that the smooth-yellow peas occur more than any other phenotype, but the other measures of center do not make sense with these data at the nominal level of measurement.

11. The mean is $\bar{x}=\frac{950+1150+1200+1400+\cdots+1600+1600+1750+1800}{12}=1454.2$, or \$1454.20.

The median is $\frac{1500+1500}{2}=1500.0$, or \$1500.00.

The mode is \$1500.

The midrange is $\frac{950+1800}{2}=1375.0$, or \$1375.00.

The sample consists of "best buy" TVs, so it is not a random sample and is not likely to be representative of the population. The lowest price is a relevant statistic for someone planning to buy one of the TVs.

12. The mean is $\bar{x}=\frac{0.51+0.74+0.89+1.04+1.18+1.38+1.41+1.42+1.45+1.45+1.49}{11}=1.178$ W/kg.

The median is 1.380 W/kg.
The mode is 1.45 W/kg.

The midrange is $\frac{0.51+1.49}{2}=1.000$ W/kg.

If concerned about radiation absorption, you might purchase the cell phone with the *lowest* absorption rate. All of the cell phones in the sample have absorption levels below the FCC maximum of 1.6 W/kg.

13. The mean is $\bar{x} = \dfrac{0+0+0+\cdots+34+36+38+41+41+41+\cdots+53+54+55}{20} = 32.0$ mg.

The median is $\dfrac{38+41}{2} = 39.5$ mg.

The mode is 0 mg.

The midrange is $\dfrac{0+55}{2} = 27.5$ mg.

Americans consume some brands much more often than others, but the 20 brands are all weighted equally in the calculations, so the statistics are not necessarily representative of the population of all cans of the same 20 brands consumed by Americans.

14. The mean is $\bar{x} = \dfrac{8+9+11+12+15+15+18+20+20+23+24+25+30+34}{14} = 18.9$ firefighters.

The median is $\dfrac{18+20}{2} = 19.0$ firefighters.

The mode is 15 firefighters.

The midrange is $\dfrac{8+34}{2} = 21.0$ firefighters.

The data are time series data, but the measures of center do not reveal anything about a trend consisting of a pattern of change over time.

15. The mean is $\bar{x} = \dfrac{8.6+9.1+9.1+9.1+9.3+9.3+9.4+9.8+9.9+10+10.4}{11} = 9.45$ in.

The median is 9.30 in.
The mode is 9.1 in.

The midrange is $\dfrac{8.6+10.4}{2} = 9.50$ in.

Because the measurements were made in 1988, they are not necessarily representative of the current population of all Army women.

16. The mean is $\bar{x} = \dfrac{46,785+\cdots+46,930+46,944+46,962+47,343+\cdots+49,138}{10} = 47,327.4$, or \$47,327.40.

The median is $\dfrac{46,944+46,962}{2} = 46,953.0$, or \$46,953.00.

There is no mode.

The midrange is $\dfrac{46,785+49,138}{2} = 47,961.5$, or \$47,961.50.

Apart from the fact that all other colleges have tuition and fee amounts less than those listed, nothing meaningful can be known about the population.

17. The mean is $\bar{x} = \dfrac{39+50+50+50+\cdots+175+200+209+\cdots+500+500+1500+2500}{25} = 365.3$, or \$365.30.

The median is 200.0, or \$200.00.
The median is 500, or \$500.00.

The midrange is $\dfrac{39+2500}{2} = 1269.5$, or \$1269.5.

The amounts of \$1500 and \$2500 appear to be outliers.

18. The mean is $\bar{x} = \dfrac{0.3+0.6+0.8+\cdots+1.2+1.2+1.3+1.4+1.4++\cdots+3.9+4.6+6.1}{21} = 1.80$ million albums.

The median is 1.3 million albums.
The mode is 1.4 million albums.

The midrange is $\dfrac{0.3+6.1}{2} = 3.20$ million albums.

None of the measures of center give us any information about the changing trend over time

19. The mean is $\bar{x} = \dfrac{0+0+0+0+0+0+\cdots+9+10+10+20+40+50}{50} = 2.8$ cigarettes.

The median is 0 cigarettes.
The mode is 0 cigarettes.

The midrange is $\dfrac{0+50}{2} = 25.0$ cigarettes.

Because the selected subjects report the number of cigarettes smoked, it is very possible that the data are not at all accurate. And what about that person who smokes 50 cigarettes (or 2.5 packs) a day? What are they thinking?

20. The mean is $\bar{x} = \dfrac{3+3+3+\cdots+6+6+6+7+7+7+7+8+8+8+\cdots+8+9+10}{26} = 6.6.$

The median is $\dfrac{7+7}{2} = 7.0.$

The mode is 8.

The midrange is $\dfrac{3+10}{2} = 6.5.$

Because the male subjects volunteered to participate in speed dating, they are not likely to be typical of adult males, so the results should not be used to describe the attractiveness of the population of adult males.

21. Systolic:

The mean is $\bar{x} = \dfrac{96+116+118+120+122+126+128+136+156+158}{10} = 127.6$ mm Hg,

The median is $\dfrac{122+126}{2} = 124.0$ mm Hg.

Diastolic:

The mean is $\bar{x} = \dfrac{52+58+64+72+74+76+80+82+88+90}{10} = 73.6$ mm Hg.

The median is $\dfrac{74+76}{2} = 75.0$ mm Hg.

Given that systolic and diastolic blood pressures measure different characteristics, a comparison of the measures of center doesn't make sense. Because the data are matched, it would make more sense to investigate whether there is an association or *correlation* between systolic blood pressure measurements and diastolic blood pressure measurements.

22. Brinks:

The mean is $\bar{x} = \dfrac{1.3+1.3+1.4+1.5+1.5+1.6+1.7+1.7+1.8}{10} = 1.55$, or \$1.55 million.

The median is $\dfrac{1.5+1.6}{2} = 1.55$, or \$1.55 million.

22. (continued)

Not Brinks:

The mean is $\bar{x}=\dfrac{1.5+1.5+1.6+1.6+1.6+1.7+1.8+1.9+1.9+2.2}{10}=1.73$, or \$1.73 million.

The median is $\dfrac{1.6+1.7}{2}=1.65$, or \$1.65 million.

The data do suggest that collections were considerably lower when Brinks was the collection contractor.

23. Males:

The mean is $\bar{x}=\dfrac{54+56+58+\cdots+66+66+72+\cdots+80+86+96}{15}=69.5$, beats per minute.

The median is 66.0 beats per minute.

Females:

The mean is $\bar{x}=\dfrac{64+68+70+\cdots+82+84+86+\cdots+90+90+94}{15}=82.1$, beats per minute.

The median is 84.0 beats per minute.

The pulse rates of males appear to be lower than those of females.

24. Single Line:

The mean is $\bar{x}=\dfrac{390+396+402+408+426+438+444+462+462+462}{10}=429.0$ seconds.

The median is $\dfrac{426+438}{2}=432.0$ seconds.

Individual Lines:

The mean is $\bar{x}=\dfrac{252+324+348+372+402+462+462+510+558+600}{10}=429.0$ seconds.

The median is $\dfrac{402+462}{2}=432.0$ seconds.

Individual lines: same results as with a single line. Although the mean and median are the same, the times with individual lines are much more *varied* than those with a single line.

25. The mean is $\bar{x}=0.8$ and the median is 1.0. Ten of the tornadoes have missing F-scale measurements.

26. The mean is $\bar{x}=2.572$ and the median is 2.490. Yes, the magnitude of that World Series earthquake is an outlier because a magnitude of 7.0 is substantially greater than all of the other magnitudes listed in Data Set 21.

27. The mean is $\bar{x}=98.20°\text{F}$ and the median is 98.40°F. These results suggest that the mean is less than 98.6°F.

28. The mean is $\bar{x}=3152.0$ g and the median is 3300 g. All of the weights end in 00, so they are all rounded to the nearest 100 grams. This suggests that the results should be rounded as follows: $\bar{x}=3150.0$ g and the median is 3300 g.

29. The mean is $\bar{x}=\dfrac{29(24.5)+34(34.5)+14(44.5)+3(54.5)+5(64.5)+1(74.5)+1(84.5)}{29+34+14+3+5+1+1}=36.2$ years. This result is the same as the mean of 36.2 years found by using the original list of data values.

30. The mean is $\bar{x}=\dfrac{1(24.5)+28(34.5)+36(44.5)+15(54.5)+6(64.5)+1(74.5)}{1+28+36+15+6+1}=44.5$ years. The mean from the frequency distribution is close to the mean of 44.1 years obtained by using the original list of values.

31. The mean is $\bar{x}=\dfrac{1(49.5)+51(149.5)+90(249.5)+10(349.5)+0(449.5)+0(549.5)+1(649.5)}{1+51+90+10+0+0+1}$

$=224.0\,(1000\text{ cells}/\mu\text{L})$. The mean from the frequency distribution is quite close to the mean of

$=224.3\,(1000\text{ cells}/\mu\text{L})$. obtained by using the original list of values.

32. The mean is $\bar{x}=\dfrac{25(149.5)+92(249.5)+28(349.5)+0(449.5)+2(549.5)}{25+92+28+0+2}=255.6\,(1000\text{ cells}/\mu\text{L})$. The mean from the frequency distribution is quite close to the mean of $=225.1\,(1000\text{ cells}/\mu\text{L})$. obtained by using the original list of values.

33. The mean is $\bar{x}=\dfrac{3(4)+3(2)+3(3)+4(4)+1(1)}{3+3+3+4+1}=3.14$, so the student made the dean's list.

34. The mean is $\bar{x}=0.60\left(\dfrac{63+91+88+84+79}{5}\right)+0.10(86)+0.15(90)+0.15(70)=81.2$, so the student earned a B.

35. a. The missing value is $5(78.0)-82-78-56-84=90$ beats per minute.

    b. $n-1$

36. The mean ignoring the presidents who are still alive is 15.0 years. The mean including the presidents who are still alive is at least 15. years. The results do differ by a considerable amount.

37. 504 lb is an outlier. For the original data, the median is 285.5 lb and the mean is 294.4 lb. The 10% trimmed mean is 285.4 lb and the 20% trimmed mean is 285.8 lb. The median, 10% trimmed mean, and 20% trimmed mean are all quite close, but the untrimmed mean of 294.4 lb differs from them because it is strongly affected by the inclusion of the outlier.

38. The harmonic mean is $2/\left(\frac{1}{38}+\frac{1}{56}\right)=45.3$ mi/h.

39. The geometric mean is $\sqrt[6]{(1.05154)(1.02730)(1.00488)(1.00319)(1.00313)(1.00268)}=1.015289767$, or 1.015290 when rounded, so the growth rate is 1.5290%.

40. The root mean square (RMS) is $\sqrt{\left(0^2+60^2+110^2+(-110)^2+(-60)^2+0^2\right)/6}=72.3$ volts, which is very different from the mean of 0 volts.

41. The median is $125+(50)\left(\left(\dfrac{50+1}{2}-(11+1)\right)/2\right)=153.125$ seconds, which is rounded to 153.1 seconds.

    This value differs by 2.6 seconds from the median of 150.5 seconds found by using the original list of service times. The value of 150.5 seconds is better because it is based on the original data and does not involve interpolation.

Section 3-2: Measures of Variation

1. $s\approx\dfrac{1439-963}{4}=119.0\text{ cm}^3$, which is quite close to the exact value of the standard deviation of 124.9 cm^3.

2. Significantly low values are less than or equal to $1126.0-2(124.9)=876.2\text{ cm}^3$ and significantly high values are greater than or equal to $1126.0+2(124.9)=1375.8\text{ cm}^3$. A brain volume of 1440 cm^3 is significantly high.

3. $(20.0414\text{ kg})^2=401.6577\text{ kg}^2$

4. $s,\ \sigma,\ s^2,\ \sigma^2$

5. The range is $99-7=92.0$.

   The variance is $s^2=\dfrac{(7-57.1)^2+(19-57.1)^2+\cdots+(91-57.1)^2+(99-57.1)^2}{11-1}=1149.5$.

   The standard deviation is $s=\sqrt{1149.5}=33.9$.
   The jersey numbers are nominal data that are just replacements for names, and they do not measure or count anything, so the resulting statistics are meaningless.

6. The range is $305-189=116.0$ lb.

The variance is $s^2=\frac{(189-231.1)^2+(190-231.1)^2+\cdots+(305-231.1)^2+(305-231.1)^2}{11-1}=1923.7\ \text{lb}^2$.

The standard deviation is $s=\sqrt{1923.7}=43.9$ lb.

All of the weights are from players on only 1 of the 32 teams, but it isn't very likely that any of the teams has weights that are substantially different from those of any other team, so it is reasonable to conclude that the measures of variation are typical of NFL players.

7. The range is $250-150=100$ million dollars.

The variance is $s^2=\frac{(150-172)^2+(150-172)^2+\cdots+(200-172)^2+(250-172)^2}{10-1}=1034.4$ (million dollars)^2.

The standard deviation is $s=\sqrt{1034.4}=32.0$ million dollars.

Because the data are from celebrities with the highest net worth, the measures of variation are not at all typical for all celebrities.

8. The range is $264-77=\$187.0$.

The variance is $s^2=\frac{(77-171.3)^2+(104-171.3)^2+\cdots+(244-171.3)^2+(264-171.3)^2}{8}=4626.2$ (dollars)^2.

The standard deviation is $s=\sqrt{4626.2}=\$68.0$.

The measures of variation are not very helpful for someone searching for a room. Such a person is likely looking for a low price, a good location, and quality accommodations.

9. The range is $15-2=13$ hurricanes.

The variance is $s^2=\frac{14(966)-(108)^2}{14(14-1)}=10.2$ hurricanes^2.

The standard deviation is $s=\sqrt{10.2}=3.2$ hurricanes.

Data are time-series data, but the measures of variation do not reveal anything about a trend consisting of a pattern of change over time.

10. The range is $4-1=3$.

The variance is $s^2=\frac{25(109)-(47)^2}{25(25-1)}=0.9$.

The standard deviation is $s=\sqrt{0.9}=0.9$.

The measures of variation can be found, but they make no sense because the data don't measure or count anything. They are nominal data.

11. The range is $1800-950=\$850$.

The variance is $s^2=\frac{12(26{,}047{,}500)-(17{,}450)^2}{12(12-1)}=61{,}117.4$ (dollars)^2.

The standard deviation is $s=\sqrt{61{,}117.4}=\$247.2$.

The sample consists of "best buy" TVs, so it is not a random sample and is not likely to be representative of the population. The measures of variation are not likely to be typical of all TVs that are 60 inches or larger.

12. The range is $1.49-0.51=0.980$ W/kg.

The variance is $s^2=\frac{11(16.4078)-(12.96)^2}{11(12-1)}=0.114\ (\text{W/kg})^2$.

The standard deviation is $s=\sqrt{0.114}=0.337$ W/kg.

No. Some models of cell phones have a much larger market share than others, so the measures from the different models should be weighted according to their size in the population.

13. The range is $55-0=55.0$ mg.

    The variance is $s^2=\dfrac{20(29{,}045)-(651)^2}{20(20-1)}=413.4\text{ mg}^2$.

    The standard deviation is $s^2=\sqrt{413.4}=20.3$ mg.
    Americans consume some brands much more often than others, but the 20 brands are all weighted equally in the calculations, so the statistics are not necessarily representative of the population of all cans of the same 20 brands consumed by Americans.
14. The range is $34-8=26.0$ firefighters.

    The variance is $s^2=\dfrac{14(5770)-(264)^2}{14(14-1)}=60.9\text{ firefighters}^2$.

    The standard deviation is $s=\sqrt{60.9}=7.8$ firefighters.
    The data are time-series data, but the measures of variation do not reveal anything about a trend consisting of a pattern of change over time.
15. The range is $10.4-8.6=1.80$ in.

    The variance is $s^2=\dfrac{11(985.94)-(107)^2}{11(11-1)}=0.27\text{ in.}^2$

    The standard deviation is $s=\sqrt{0.27}=0.52$ in.
    Because the measurements were made in 1988, they are not necessarily representative of the current population of all Army women.
16. The range is $49{,}138-46{,}785=\$2353$.

    The variance is $s^2=\dfrac{10(22{,}403{,}579{,}102)-(47{,}374)^2}{10(10-1)}=527{,}910.5\text{ (dollars)}^2$.

    The standard deviation is $s=\sqrt{527{,}910.5}=\$726.6$.
    Because the data include only the 10 highest costs, the measures of variation don't tell us anything about the variation among costs for the population of all U.S. college tuitions.
17. The range is $2500-39=\$2461.0$.

    The variance is $s^2=\dfrac{25(10{,}306{,}077)-(9133)^2}{25(25-1)}=290{,}400.4\text{ (dollars)}^2$.

    The standard deviation is $s=\sqrt{290{,}400.4}=\$538.9$.
    The amounts of \$1500 and \$2500 appear to be outliers, and it is likely that they have a large effect on the measures of variation.
18. The range is $6.1-0.3=5.80$ million albums.

    The variance is $s^2=\dfrac{21(110.11)-(37.9)^2}{21(21-1)}=2.09\text{ (million albums)}^2$.

    The standard deviation is $s=\sqrt{2.09}=1.44$ million albums.
    None of the measures of variation give us any information about the changing trend over time.
19. The range is $50-0=50.0$ cigarettes.

    The variance is $s^2=\dfrac{50(4781)-(139)^2}{50(50-1)}=89.7\text{ (cigarettes)}^2$.

    The standard deviation is $s=\sqrt{89.7}=9.5$ cigarettes.
    Because the selected subjects report the number of cigarettes smoked, it is very possible that the data are not at all accurate, so the results might not reflect the actual smoking behavior of California adults.

20. The range is $10-3=7.0$.

The variance is $s^2=\dfrac{26(1213)-(171)^2}{26(26-1)}=3.5$.

The standard deviation is $s=\sqrt{3.5}=1.9$.
Because the male subjects volunteered to participate in speed dating, they are not likely to be typical of adult males, so the results are not likely to reflect the variation among attractiveness ratings for the population of adult males.

21. Systolic: $\bar{x}=127.6$, $s=18.6$; The coefficient of variation is $\dfrac{18.6}{127.6}\cdot 100\%=14.6\%$.

Diastolic: $\bar{x}=73.6$, $s=12.5$; The coefficient of variation is $\dfrac{12.5}{73.6}\cdot 100\%=16.9\%$.

The variation is roughly about the same.

22. Brinks: $\bar{x}=1.55$, $s=0.18$; The coefficient of variation is $\dfrac{0.18}{1.55}\cdot 100\%=11.5\%$.

Not Brinks: $\bar{x}=1.73$, $s=0.22$; The coefficient of variation is $\dfrac{0.22}{1.73}\cdot 100\%=12.8\%$.

The results are not very different, so the two samples do not appear to have different amounts of variation.

23. Male: $\bar{x}=69.5$, $s=11.3$; The coefficient of variation is $\dfrac{11.3}{69.5}\cdot 100\%=16.2\%$.

Female: $\bar{x}=82.1$, $s=9.2$; The coefficient of variation is $\dfrac{9.2}{82.1}\cdot 100\%=11.2\%$.

Pulse rates of males appear to vary more than pulse rates of females.

24. Single Line: $\bar{x}=429.0$, $s=28.6$; The coefficient of variation is $\dfrac{28.6}{429.0}\cdot 100\%=6.7\%$.

Individual Lines: $\bar{x}=429.0$, $s=109.3$; The coefficient of variation is $\dfrac{109.3}{429.0}\cdot 100\%=25.5\%$.

The single line has much less variation than with individual lines.

25. Range $=4.0$, $s^2=0.9$, $s=0.9$

26. Range $=3.600$, $s^2=0.423$, $s=0.651$; Adding 7.0: Range $=5.910$, $s^2=0.455$, $s=0.675$;
The range changes by a considerable amount, but the variance and standard deviation do not change very much.

27. Range $=3.10°\text{F}$, $s^2=0.39\,(°\text{F})^2$, $s=0.62°\text{F}$

28. Range $=4600.0$ g, $s^2=480{,}848.1\text{ g}^2$, $s=693.4$ g
All of the weights end in 00, so they are all rounded to the nearest 100 grams. This suggests that the results should be rounded as follows: Range $=4600.0$ g, $s^2=480{,}850\text{ g}^2$, $s=690$ g.

29. The rule of thumb standard deviation is $s\approx\dfrac{4-0}{4}=1.0$, which is very close to $s=0.9$ found by using all of the data.

30. The rule of thumb standard deviation is $s\approx\dfrac{4.69-1.09}{4}=0.900$, which is not substantially different from $s=0.651$ found by using all of the data.

31. The rule of thumb standard deviation is $s\approx\dfrac{99.6-96.5}{4}=0.78°\text{F}$, which is not substantially different from $s=0.62°\text{F}$ found by using all of the data.

32. The rule of thumb standard deviation is $s \approx \dfrac{4900-300}{4} = 1150.0$ g, which differs from $s = 693.4$ g found by using all of the data by a considerable amount. Several of the lowest weights correspond to premature births, and they cause the range to be larger, with the resulting estimate being larger.
33. Significantly low values are less than or equal to $74.0-2(12.5)=49.0$ beats per minute, and significantly high values are greater than or equal to $74.0+2(12.5)=99.0$ beats per minute. A pulse rate of 44 beats per minute is significantly low.
34. Significantly low values are less than or equal to $69.6-2(11.3)=42.0$ beats per minute, and significantly high values are greater than or equal to $69.6+2(11.3)=92.2$ beats per minute. A pulse rate of 50 beats per minute is neither significantly low or high.
35. Significantly low values are less than or equal to $77.32-2(1.29)=24.74$ cm, and significantly high values are greater than or equal to $77.32+2(1.29)=29.90$ cm. A foot length of 30 cm is significantly high.
36. Significantly low values are less than or equal to $98.20-2(0.62)=96.96°\text{F}$, and significantly high values are greater than or equal to $98.20+2(0.62)=99.44°\text{F}$. A body temperature of $100°\text{F}$ is significantly high.
37. $s=\sqrt{\dfrac{87\left(29\cdot 24.5^2+\cdots+1\cdot 84.5^2\right)-\left(29\cdot 24.5+\cdots+1\cdot 84.5\right)^2}{87(87-1)}}=12.7$ years, which differs from the exact value of 11.5 years by a somewhat large amount.
38. $s=\sqrt{\dfrac{87\left(1\cdot 24.5^2+\cdots+1\cdot 74.5^2\right)-\left(1\cdot 24.5+\cdots+1\cdot 74.5\right)^2}{87(87-1)}}=9.6$ years, which differs from the exact value of 8.9 years by a somewhat large amount.
39. $s=\sqrt{\dfrac{103\left(1\cdot 49.5^2+\cdots+1\cdot 649.5^2\right)-\left(1\cdot 49.5+\cdots+1\cdot 649.5\right)^2}{103(103-1)}}=68.4$, which is somewhat far from the exact value of 59.5.
40. $s=\sqrt{\dfrac{147\left(25\cdot 149.5^2+\cdots+2\cdot 549.5^2\right)-\left(25\cdot 149.5+\cdots+2\cdot 549.5\right)^2}{147(147-1)}}=69.5$, which is not substantially far from the exact value of 65.5.
41. a. The empirical rule states that approximately 95% of women should fall between two standard deviations of the mean.
    b. Since $\dfrac{189.7-255.1}{65.4}=-1$ and $\dfrac{320.5-255.1}{65.4}=1$, the empirical rule states that approximately 68% of women should fall between one standard deviation of the mean.
42. a. The empirical rule states that approximately 68% of healthy adults should fall between one standard deviation of the mean.
    b. Since $\dfrac{96.34-98.20}{0.62}=-3$ and $\dfrac{98.82-98.20}{0.62}=3$, the empirical rule states that approximately 99.7% healthy adults should fall between three standard deviations of the mean.
43. At least $1-1/3^2=89\%$ of women have platelet counts within 3 standard deviations of the mean. The minimum count is $255.1-3(65.4)=58.9$ and the maximum count is $255.1+3(65.4)=451.3$.

44. At least $1-1/2^2=75\%$ of healthy adults have body temperatures within 2 standard deviations of the mean. The minimum temperature is $98.20-2(0.62)=96.96°\text{F}$ and the maximum temperature is $98.20+2(0.62)=96.96°\text{F}$.

45. a. $\mu=\dfrac{9+10+20}{3}=13$ cigarettes and $\sigma^2=\dfrac{(9-13)^2+(10-13)^2+(20-13)^2}{3}=24.7$ cigarettes^2

   b. The nine possible samples of two values are the following: {(9, 9), (9, 10), (9, 20), (10, 9), (10, 10), (10, 20), (20, 9), (20, 10), (20,20)} which have the following corresponding sample variances: {0, 0.5, 60.5, 0.5, 0, 50, 60.5, 50, 0}, that have a mean of $s^2=2.47$ cigarettes^2.

   c. The population variances of the nine samples above are {0, 0.25, 30.25, 0.25, 0, 25, 30.25, 25, 0} that have a mean of $s^2=12.3$ cigarettes^2.

   d. Part (b), because repeated samples result in variances that target the same value (24.7 cigarettes^2) as the population variance. Use division by $n-1$.

   e. No. The mean of the sample variances (24.7 cigarettes^2) equals the population variance (24.7 cigarettes^2), but the mean of the sample standard deviations (3.5 cigarettes) does not equal the population standard deviation (5.0 cigarettes).

46. The mean absolute deviation of the population is 4.7 cigarettes. With repeated samplings of size 2, the nine different possible samples have mean absolute deviations of 0, 0, 0, 0.5, 0.5, 5, 5, 5.5, 5.5. With many such samples, the mean of those nine results is 2.4 cigarettes, showing that the sample mean absolute deviations tend to center about the value of 2.4 cigarettes instead of the mean absolute deviation of the population, which is 4.7 cigarettes. The sample mean deviations do not target the mean deviation of the population. This is not good. This indicates that a sample mean absolute deviation is not a good estimator of the mean absolute deviation of a population.

Section 3-3: Measures of Relative Standing and Boxplots

1. James' height is 4.07 standard deviations above the mean.
2. The minimum height is 155 cm, the first quartile, Q_1, is 169.1 cm, the second quartile, Q_2, (or the median) is 173.8 cm, the third quartile, Q_3, is 179.4 cm, and the maximum height is 193.3 cm.
3. The bottom boxplot represents weights of women, because it depicts weights that are generally lower.
4. 2.00 should be preferred, because it is 2.00 standard deviations above the mean and would correspond to the highest of the five different possible scores.
5. a. The difference is $77.8-17.60=60.20$ Mbps.

   b. $\dfrac{60.20}{16.02}=3.76$ standard deviations

   c. $z=3.76$

   d. The data speed of 77.8 Mbps is significantly high.
6. a. The difference is $0.8-17.60=-16.8$ Mbps.

   b. $\dfrac{16.8}{16.02}=1.05$ standard deviations

   c. $z=-1.05$

   d. The data speed of 0.8 Mbps is not significant. It is neither significantly low nor significantly high.
7. a. The difference is $36-74.0=-38$ beats per minute.

   b. $\dfrac{38}{12.5}=3.04$ standard deviation

   c. $z=-3.04$

   d. The pulse rate of 36 beats per minute is significantly low.

8. a. The difference is $5.28-1.911=3.369$ lb.
   b. $\frac{3.369}{1.065}=3.16$ standard deviations
   c. $z=3.16$
   d. The weight of 5.28 lb is significantly high.
9. Significantly low scores are less than or equal to $21.1-2(5.1)=10.9$, and significantly high scores are greater than or equal to $21.1+2(5.1)=31.3$.
10. Significantly low scores are less than or equal to $25.2-2(6.4)=12.4$, and significantly high scores are greater than or equal to $25.2+2(6.4)=38.0$.
11. Significantly low weights are less than or equal to $5.63930-2(0.06194)=5.51542$ g, and significantly high weights are greater than or equal to $5.63930+2(0.06194)=5.76318$ g.
12. Significantly low hip breadths are less than or equal to $36.6-2(2.5)=31.6$ cm, and significantly high hip breadths are greater than or equal to $36.6+2(2.5)=41.6$ cm.
13. The tallest man's z score is $z=\frac{251-174.12}{7.10}=10.83$ and the shortest man's z score is $z=\frac{54.6-174.12}{7.10}$ $=-16.83$. Chandra Bahadur Dangi has the more extreme height because his z score of -16.83 is farther from the mean than the z score of 10.83 for Sultan Kosen
14. The female has a higher red blood cell count because her z score is $z=\frac{5.23-4.439}{0.402}=1.97$, which is a higher number than the z score of $z=\frac{5.58-4.719}{0.490}=1.76$ for the male.
15. The male has a more extreme birth weight because his z score is $z=\frac{1500-3272.8}{660.2}=-2.19$, which is a lower number than the z score of $z=\frac{1500-3037.1}{706.3}=-2.18$ for the female.
16. Julianne Moore had the more extreme age since her z score is $z=\frac{54-36.2}{11.5}=1.55$, which is farther from the mean than the z score of $z=\frac{33-44.1}{8.9}=-1.25$ for Eddie Redmayne.
17. For 2.4 Mbps, $\frac{29}{50}\cdot 100=58$, so it is the 58th percentile.
18. For 13.0 Mbps, $\frac{45}{50}\cdot 100=90$, so it is the 90th percentile.
19. For 0.7 Mbps, $\frac{17}{50}\cdot 100=34$, so it is the 34th percentile.
20. For 9.6 Mbps, $\frac{43}{50}\cdot 100=86$, so it is the 86th percentile.
21. $L=\frac{60\cdot 50}{100}=30$, so $P_{60}=\frac{2.4+2.5}{2}=2.45$ Mbps (Tech: Excel: 2.44 Mbps)
22. $L=\frac{25\cdot 50}{100}=12.5$, so $Q_1=P_{25}=0.5$ Mbps

23. $L=\frac{75\cdot 50}{100}=37.5$, so $Q_3=P_{75}=3.8$ Mbps (Tech: Minitab: 3.85 Mbps; Excel: 3.75 Mbps)

24. $L=\frac{40\cdot 50}{100}=20$, so $P_{10}=\frac{1.0+1.1}{2}=1.05$ Mbps (Tech: Excel: 2.44 Mbps)

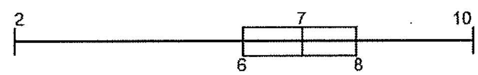
25. $L=\frac{50\cdot 50}{100}=25$, so $P_{10}=\frac{1.6+1.6}{2}=1.6$ Mbps

26. $L=\frac{75\cdot 50}{100}=37.5$, so $P_{75}=3.8$ Mbps (Tech: Minitab: 3.85 Mbps; Excel: 3.75 Mbps)

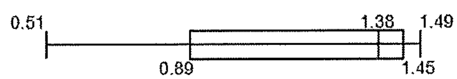
27. $L=\frac{25\cdot 50}{100}=12.5$, so $P_{25}=0.5$ Mbps

28. $L=\frac{85\cdot 50}{100}=42.5$, so $P_{85}=8.2$ Mbps (Tech: Excel: 7.29 Mbps)

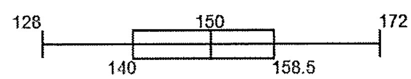
29. The five number summary is 2, 6.1, 7.0, 8.0, 10.



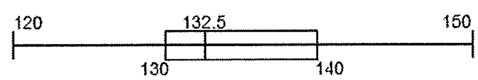
30. The five number summary is 0.51 W/kg, 0.890 W/kg, 1.380 W/kg, 1.450 W/kg, 1.49 W/kg. (Tech: Excel yields $Q_1=0.965$ W/kg, $Q_3=1.435$ W/kg.)



31. The five number summary is 128 mBq, 140.0 mBq, 150.0 mBq, 158.5 mBq, 172 mBq (Tech: Minitab yields $Q_1=139.0$ mBq and $Q_3=159.75$ mBq. Excel yields $Q_1=141.0$ mBq and $Q_3=157.25$ mBq.)

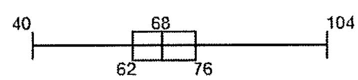


32. The five number summary is 120 mm Hg, 130.0 mm Hg, 132.5 mm Hg, 140.0 mm Hg, 150 mm Hg. (Tech: Minitab yields $Q_1=128.75$ mm Hg and $Q_3=140.75$ mm Hg.)

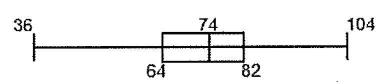


33. The top boxplot represents males. Males appear to have slightly lower pulse rates than females.

Male Pulse

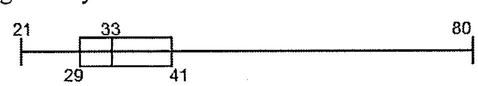


Female Pulse

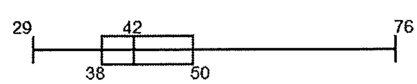


34. The top boxplot represents actresses. Although actresses include the oldest age of 80 years, their boxplot shows that they have ages that are generally lower than those of actors.

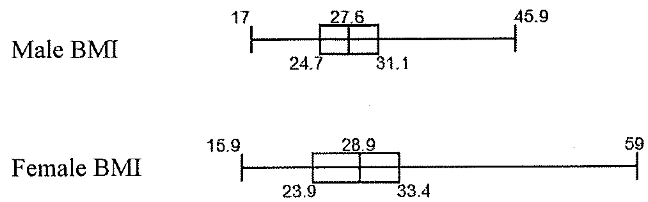
Actresses



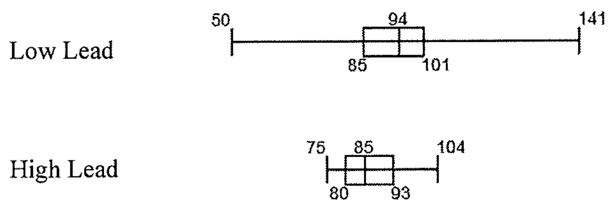
Actors



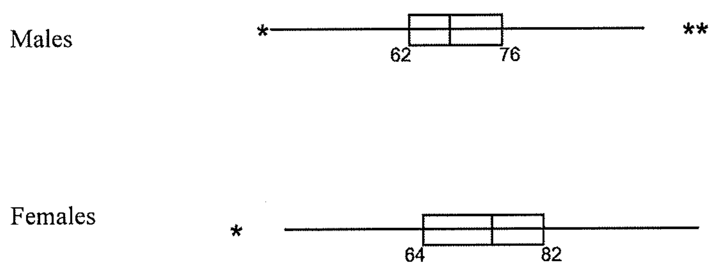
35. The top boxplot represents BMI values for males. The two boxplots do not appear to be very different, so BMI values of males and females appear to be about the same, except for a few very high BMI values for females that caused the boxplot to extend farther to the right.



36. The low lead level group represented in the top boxplot has much more variation and the IQ scores tend to be higher than the IQ scores from the high lead level group.



37. Top boxplot represents males. Males appear to have slightly lower pulse rates than females. The outliers for males are 40 beats per minute, 102 beats per minute, and 104 beats per minute. The outlier for females is 36 beats per minute.



Quick Quiz

1. The sample mean is $\bar{x} = \dfrac{4+5+6+6+7+7+7+7+8+8+8+8}{12} = 6.8$ hours.
2. The median is $\dfrac{7+7}{2} = 7.0$ hours.
3. The modes are 7 hours and 8 hours.
4. The variance is $(1.3\text{ hour})^2 = 1.7\text{ hour}^2$
5. Yes, because 0 hours is substantially less than all of the other data values.
6. $z = \dfrac{5-6.3}{1.4} = -0.93$
7. 75% or 60 sleep times
8. minimum, first quartile Q_1, second quartile Q_2 (or median), third quartile Q_3, maximum
9. $s \approx \dfrac{10-4}{4} = 1.5$
10. $\bar{x}, \mu, s, \sigma, s^2, \sigma^2$

Review Exercises

1. a. The mean is $\bar{x} = \frac{(-7)+(-7)+(-5)+(-4)+(-1)+0+1+1+1+4+7+22}{12} = 1.0$ min.

   b. The median is $\frac{0+1}{2} = 0.5$ min.

   c. The mode is 1 min.

   d. The midrange is $\frac{(-7)+22}{2} = 7.5$ min.

   e. The range is $22-(-7) = 29.0$ min.

   f. $s = \sqrt{\frac{(-7-1.0)^2+(-7-1.0)^2+(-5-1.0)^2+\cdots+(4-1.0)^2+(7-1.0)^2+(7-1.0)^2}{12-1}} = 7.9$ min

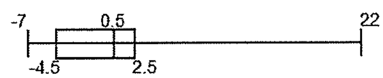
   g. $s^2 = 7.9^2 = 61.8 \text{ min}^2$

   h. $L = \frac{25\cdot 12}{100} = 3$, so $Q_1 = \frac{(-5)+(-4)}{2} = -4.5$ min

   i. $L = \frac{75\cdot 12}{100} = 9$, so $Q_1 = \frac{1+4}{2} = 2.5$ min (Tech: Minitab yields $Q_1 = -4.75$ min and $Q_3 = 3.25$ min. Excel yields $Q_1 = -4.25$ min and $Q_3 = 1.75$ min.)

2. $z = \frac{0-1}{7.9} = -0.13$; The prediction error of 0 minutes is not significant because its z score is between –2 and 2, so it is within two standard deviations of the mean.

3. The five number summary is –7 min, –4.5 min, 1.5 min, 2.5 min, 22 min. (Tech: Minitab yields $Q_1 = -4.75$ min and $Q_3 = 3.25$ min. Excel yields $Q_1 = -4.25$ min and $Q_3 = 1.75$ min.)



4. The mean is $\bar{x} = \frac{12+14+22+27+40}{5} = 23.0$. The numbers don't measure or count anything. They are used as replacements for the names of the categories, so the numbers are at the nominal level of measurement. In this case the mean is a meaningless statistic.

5. The male z score is $z = \frac{3400-3272.8}{660.2} = 0.19$. The female z score is $z = \frac{3200-3037.1}{706.3} = 0.23$. The female has the larger relative birth because the female has the larger z score.

6. The outlier is 646. The mean and standard deviation with the outlier included are $\bar{x} = 267.8$ and $s = 131.6$. With the outlier excluded, the values are $\bar{x} = 230.0$ and $s = 42.0$. Both statistics changed by a substantial amount, so here the outlier has a very strong effect on the mean and standard deviation.

7. The minimum value is 119 mm, the first quartile is 128 mm, the second quartile (or median) is 131 mm, the third quartile is 135 mm, and the maximum value is 141 mm.

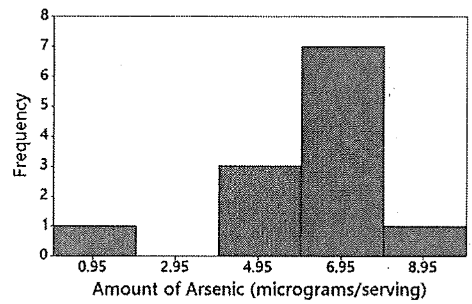
8. Based on a minimum of 117 seconds and a maximum of 256 seconds, an estimate of the standard deviation of duration times would be $s \approx \frac{256-117}{4} = 34.5$ seconds.

Cumulative Review Exercises

1.

Arsenic (μg)	Frequency
0.0 – 1.9	1
2.0 – 3.9	0
4.0 – 5.9	3
6.0 – 7.9	7
8.0 – 9.9	1

2.



3.

```
1 | 5
2 |
3 |
4 | 9
5 | 44
6 | 13679
7 | 38
8 | 2
```

4. a. The mean is $\bar{x} = \dfrac{1.5+4.9+5.4+5.4+6.1+6.3+6.6+6.7+6.9+7.3+7.8+8.2}{12} = 6.09\ \mu g.$

   b. The median is $\dfrac{6.3+6.6}{2} = 6.45\ \mu g.$

   c. $s = \sqrt{\dfrac{(1.5-6.09)^2+(4.9-6.09)^2+\cdots+(7.8-6.09)^2+(8.2-6.09)^2}{12-1}} = 1.75\ \mu g$

   d. $s^2 = 1.75^2 = 3.06\ (\mu g)^2$

   e. The range is $8.2-1.5 = 6.70\ \mu g.$

5. The vertical scale does not begin at 0, so the differences among different outcomes are exaggerated.
6. No. A normal distribution would appear in a histogram as being bell-shaped, but the histogram is not bell-shaped.

Chapter 4: Probability

Section 4-1: Basic Concepts of Probability

1. $P(A)=\frac{1}{1000}$, or 0.001, $P(\overline{A})=1-\frac{1}{1000}=\frac{999}{1000}$, or 0.999
2. The probability of rain today is 0.25, or $\frac{1}{4}$.
3. part (c)
4. The answers vary, but a high answer in the neighborhood of 0.999 is reasonable.
5. 0, 3/5, 1, 0.135
6. 1/5, or 0.2
7. 1/9, or 0.111
8. {bb, bg, gb, gg}
9. 47 girls is significantly high.
10. 26 girls is neither significantly low nor significantly high.
11. 23 girls is neither significantly low nor significantly high.
12. 5 girls is significantly low.
13. 1/2, or 0.5
14. 1/5, or 0.2
15. 0.43
16. 0.292
17. 1/10, or 0.1
18. 1/2, or 0.5
19. 0
20. 1
21. $\frac{5}{555}=\frac{1}{111}$, or 0.00901; The employer would suffer because it would be at a risk by hiring someone who uses drugs.
22. $\frac{25}{555}=\frac{5}{111}$, or 0.0450; The person tested would suffer because he or she would be suspected of using drugs when in reality he or she does not use drugs.
23. $\frac{50}{555}=\frac{10}{111,}$ or 0.0901; This result does appear to be a reasonable estimate of the prevalence rate.
24. $\frac{25+480}{555}=\frac{505}{555}=\frac{101}{111}$, or 0.910; The result does appear to be reasonable as an estimate of the proportion of the adult population that does not use drugs.
25. $\frac{879}{945}$, or 0.93; Yes, the technique appears to be effective.
26. $\frac{239}{291}$, or 0.821; Yes, the technique appears to be effective.
27. $\frac{428}{580}$ = 0.738; Yes, it is reasonable.
28. a. 1/365
    b. yes
    c. He already knew.
    d. 0
29. $\frac{1380}{1380+3732}=\frac{115}{426}$, or 0.270; No, it is not unlikely for someone to not use social networking sites.
30. $\frac{83,600}{83,600+5,127,400}=\frac{418}{26,055}$, or 0.016; Yes, in a passenger car crash, a rollover is unlikely.

31. a. brown /brown, brown/blue, blue/brown, blue/blue
    b. 1/4
    c. 3/4
32. In the following, the first letter represents the chromosome contributed by the father and the second letter represents the chromosome contributed by the mother. Let X_1 and X_2 represent the possible X chromosomes contributed by the mother.
    a. 0; The possible outcomes are $\{xX_1, xX_2, YX_1, YX_2\}$, neither son will have the disease.
    b. 0; The possible outcomes are $\{xX_1, xX_2, YX_1, YX_2\}$, neither daughter will have the disease.
    c. 1/2, or 0.5; The possible outcomes are $\{Xx_1, XX_2, Yx_1, YX_2\}$, one of the two sons will have the disease.
    d. 0; The possible outcomes are $\{Xx_1, XX_2, Yx_1, YX_2\}$, neither daughter will have the disease.
33. 3/8, or 0.375
34. 3/8, or 0.375
35. {bbbb, bbbg, bbgb, bbgg, bgbb, bgbg, bggb, bggg, gbbg, gbbb, gbgb, gbgg, ggbb, ggbg, gggb, gggg}; 4/16, or 0.25
36. 2/16, or 0.125
37. The high probability of 0.327 shows that the sample results could have easily occurred by chance. It appears that there is not sufficient evidence to conclude that pregnant women can correctly predict the gender of their baby.
38. The low probability of less than 0.0001 shows that the sample results are not likely to occur by chance. It appears that seatbelts do have an effect on survival rate. So buckle up!
39. a. In comparing the 200 mg treatment group to the placebo group, the low probability of less than 0.049 shows that the sample results could not have easily occurred by chance. It appears that 200 mg of caffeine does have an effect on memory.
    b. In comparing the 300 mg group to the 200 mg group, the high probability of 0.75 indicates the sample results could have easily occurred by chance. There is not sufficient evidence to conclude that there are different effects from the 300 mg treatment group and the 200 mg treatment group.
40. The high probability of 0.512 shows that the sample results could have easily occurred by chance. It appears that there is not sufficient evidence to conclude that cell phones have an effect on cancer of the brain or nervous system.
41. a. 9999 : 1
    b. 4999 : 1
    c. The description is not accurate. The odds against winning are 9999 : 1 and the odds in favor are 1 : 9999, not 1 : 10,000.
42. a. 18/38, or 0.474
    b. 10 : 9
    c. $18
    d. $20
43. a. $7-2=\$5$
    b. 5 : 2
    c. 772 : 228 or 193 : 57 or about 3.39 : 1 (roughly 17 : 5)
    d. The worth of the $2 bet would be approximately $3.39 \cdot 2+2=\$8.80$ (instead of the actual payoff of $7.00).

44. Relative risk: $\dfrac{\frac{26}{2103}}{\frac{22}{1671}}=0.939$ Odds ratio: $\dfrac{\dfrac{\frac{26}{2103}}{1-\frac{26}{2103}}}{\dfrac{\frac{22}{1671}}{1-\frac{22}{1671}}}=0.938$

The probability of a headache with Nasonex (0.0124) is slightly less than the probability of a headache with the placebo (0.0132), so Nasonex does not appear to pose a risk of headache.

Section 4-2: Addition Rule and Multiplication Rule

1. $P(A)$ represents the probability of selecting an adult with blue eyes, and $P(\overline{A})$ represents the probability of selecting an adult who does not have blue eyes.
2. $P(M \mid B)$ represents the probability of getting a male, given that someone with blue eyes has been selected. $P(M \mid B)$ is not the same as $P(B \mid M)$.
3. Because the selections are made without replacement, the events are dependent. Because the sample size of 1068 is less than 5% of the population size of 15,524,971, the selections can be treated as being independent (based on the 5% guideline for cumbersome calculations).
4. It is certain that the selected adult has type B blood or does not have type B blood.
5. $1-0.26=0.74$
6. $1-0.803=0.197$
7. $P(\overline{N})=1-0.330=0.670,$ where $P(\overline{N})$ is the probability of randomly selecting someone with a response different from "never."
8. $P(\overline{I})$ denotes the probability of screening a driver and finding that he or she is not intoxicated, and $P(\overline{I})=0.99112,$ or 0.991 when rounded.

Use the following table for Exercises 9–20

	McDonald's	Burger King	Wendy's	Taco Bell	**Total**
Order Accurate	329	264	249	145	**987**
Order Not Accurate	33	54	31	13	**131**
Total	**362**	**318**	**280**	**158**	**1118**

9. $\dfrac{1118-362}{1118}=\dfrac{756}{1118}=\dfrac{387}{559},$ or 0.676

10. $\dfrac{131}{1118},$ or 0.117

11. $\dfrac{362}{1118}+\dfrac{987}{1118}-\dfrac{329}{1118}=\dfrac{1020}{1118}=\dfrac{510}{559},$ or 0.912; The two events are not disjoint.

12. $\dfrac{280}{1118}+\dfrac{131}{1118}-\dfrac{31}{1118}=\dfrac{380}{1118}=\dfrac{190}{559},$ or 0.340; The two events are not disjoint.

13. a. $\dfrac{158}{1118}\cdot\dfrac{158}{1118}=0.0200;$ Yes, the events are independent.

b. $\dfrac{158}{1118}\cdot\dfrac{157}{1117}=0.0199;$ The events are dependent, not independent.

14. a. $\frac{131}{1118} \cdot \frac{131}{1118} = 0.0137$; Yes, the events are independent.

    b. $\frac{131}{1118} \cdot \frac{130}{1117} = 0.0136$; The events are dependent, not independent.

15. a. $\frac{987}{1118} \cdot \frac{987}{1118} = 0.779$; Yes, the events are independent.

    b. $\frac{987}{1118} \cdot \frac{986}{1117} = 0.779$; The events are dependent, not independent.

16. a. $\frac{318}{1118} \cdot \frac{318}{1118} = 0.0809$; Yes, the events are independent.

    b. $\frac{318}{1118} \cdot \frac{317}{1117} = 0.0807$; The events are dependent, not independent.

17. $\frac{362+280}{1118} + \frac{131}{1118} - \frac{33+31}{1118} = \frac{709}{1118}$, or 0.634

18. $\frac{318+158}{1118} + \frac{987}{1118} - \frac{264+145}{1118} = \frac{1054}{1118} = \frac{527}{559}$, or 0.943

19. $\frac{280}{1118} \cdot \frac{279}{1117} \cdot \frac{278}{1116} = 0.0156$

20. $\frac{131}{1118} \cdot \frac{130}{1117} \cdot \frac{129}{1116} = 0.00158$

21. Use the following table for parts (a) and (b).

	Positive Test Result	Negative Test Result	**Total**
Subject Used Marijuana	True Positive 119	False Negative 3	**122**
Subject Did Not Use Marijuana	False Positive 24	True Negative 154	**178**
Total	**143**	**157**	**300**

   a. There were a total of 300 subjects in the study.

   b. 154 subjects had a true negative result.

   c. $\frac{154}{300} = \frac{77}{150} = 0.513$

22. $\frac{119+3+154}{300} = \frac{23}{25} = 0.92$

23. $\frac{119+24+154}{300} = \frac{99}{100} = 0.990$

24. $\frac{178}{300} = \frac{89}{150} = 0.593$; No, in the general population, the rate of subjects who do not use marijuana is probably much greater than 0.593 or 59.3%.

25. a. 0.03

    b. $0.03 \cdot 0.03 = 0.0009$

    c. $0.03 \cdot 0.03 \cdot 0.03 = 0.000027$

    d. By using one drive without a backup, the probability of total failure is 0.03, and with three independent disk drives, the probability drops to 0.000027. By changing from one drive to three, the probability of total failure drops from 0.03 to 0.000027, and that is a very substantial improvement in reliability. Back up your data!

26. a. $0.22 \cdot 0.22 = 0.0484$

    b. $1 - 0.0484 = 0.9516$, or 0.952; This probability seems high, but both generators fail about 5% of the time that they are needed. Given the importance of the hospital's needs, the reliability should be improved.

27. $8834-504=8330$, $\frac{8330}{8834}\cdot\frac{8329}{8833}\cdot\frac{8328}{8832}=0.838$; The probability of 0.838 is high, so it is likely that the entire batch will be accepted, even though it includes many firmware defects.

28. $2875-288=2587$, $\frac{2587}{2875}\cdot\frac{2586}{2874}\cdot\frac{2585}{2873}=0.728$; It is likely that the entire lot would be accepted. With about 10% of the containers not meeting requirements, the probability of 0.728 seems too high.

29. a. $\frac{47,637}{47,637+111,874}=\frac{47,637}{159,511}=0.299$

    b. Using the 5% guideline for cumbersome calculations, $(0.299)^5=0.00239$. Using exact probabilities, $\frac{47,637}{159,511}\cdot\frac{47,636}{159,510}\cdot\frac{47,635}{159,509}\cdot\frac{47,634}{159,508}\cdot\frac{47,633}{159,507}=0.00238$.

30. Using the 5% guideline for cumbersome calculations, $\left(\frac{47,637-188}{47,637}\right)^{40}=0.854$.

31. a. $0.985\cdot0.985+0.985\cdot0.015+0.015\cdot0.985=0.999775$

    b. $0.985\cdot0.985=0.970225$

    c. The series arrangement provides better protection.

32. $\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdot\ldots\cdot\frac{341}{365}=0.431$

33. $P(A \text{ or } B)=P(A)+P(B)-2P(A \text{ and } B)$

34. $P(\overline{A \text{ or } B})=\frac{249+31+145+13}{1118}=\frac{438}{1118}$, or 0.392, $P(\bar{A} \text{ or } \bar{B})=\frac{1118}{1118}$, or 1; The results are different. In general, $P(\overline{A \text{ or } B})$ is not the same as $P(\bar{A} \text{ or } \bar{B})$.

Section 4-3: Complements, Conditional Probability, and Bayes' Theorem

1. $\bar{A}$ is the event of not getting at least 1 defect among the 3 iPhones, which means that all 3 iPhones are good.
2. Parts (a) and (b) are correct.
3. The probability that the selected person is a high school classmate given that the selected person is female.
4. Confusion of the inverse is to think that $P(F \mid H)=P(H \mid F)$ or to switch one of those values for the other.

   That is, confusion of the inverse is to think that the following two probabilities are equal or to incorrectly use one of them for the other: (1) the probability of selecting a female given that the selected person is a high school classmate; (2) the probability of selecting a high school classmate given that the selected person is female.
5. $1-\left(\frac{1}{2}\right)^3=\frac{7}{8}$, or 0.875
6. 1/2, or 0.5
7. $1-(0.512)^6=0.982$
8. $1-(0.545)^6=0.974$; The system cannot continue indefinitely because eventually there would be no women to give birth.
9. $1-(1-0.10)^4=0.344$
10. $1-(1-0.20)^{10}=0.893$; There is a good chance of continuing.
11. $1-(1-0.20)^{15}=0.965$; The probability is high enough so that she can be reasonably sure of getting a defect for her work.

12. $1-(1-0.67)^4 = 0.988$; It is very possible that the result is not valid because it is based on data from a voluntary response survey.

Use the following table for Exercises 13–16.

	Purchased Gum	Kept the Money	**Total**
Students Given Four Quarters	27	16	**43**
Students Given a $1 Bill	12	34	**46**
Total	**39**	**50**	**89**

13. a. $P(\text{spent money} \mid \text{given quarters}) = \frac{27}{43}$, or 0.628

    b. $P(\text{kept money} \mid \text{given quarters}) = \frac{16}{43}$, or 0.372

    c. It appears that when students are given four quarters, they are more likely to spend the money than keep it.

14. a. $P(\text{spent money} \mid \text{given dollar bill}) = \frac{12}{46} = \frac{6}{23}$, or 0.261

    b. $P(\text{kept money} \mid \text{given dollar bill}) = \frac{34}{46} = \frac{17}{23}$, or 0.739

    c. It appears that when students are given a $1 bill, they are more likely to keep the money than spend it.

15. a. $P(\text{spent money} \mid \text{given quarters}) = \frac{27}{43}$, or 0.628

    b. $P(\text{spent money} \mid \text{given dollar bill}) = \frac{12}{46} = \frac{6}{23}$, or 0.261

    c. It appears that students are more likely to spend the money when given four quarters than when given a $1 bill.

16. a. $P(\text{kept money} \mid \text{given quarters}) = \frac{16}{43}$, or 0.373

    b. $P(\text{kept money} \mid \text{given dollar bill}) = \frac{34}{46} = \frac{17}{23}$, or 0.739

    c. It appears that students are more likely to spend the money when given four quarters than when given a $1 bill.

Use the following table for Exercises 17–20.

	Positive Test Result	Negative Test Result	**Total**
Hepatitis C	335	10	**345**
No Hepatitis C	2	1153	**1155**
Total	**337**	**1163**	**1500**

17. $P(\text{positive result} \mid \text{no hepatitis C}) = \frac{2}{1155}$, or 0.00173; This is the probability of the test making it appear that the subject has hepatitis C when the subject does not have it, so the subject is likely to experience needless stress and additional testing.

18. $P(\text{negative result} \mid \text{hepatitis C}) = \frac{10}{345}$, or 0.0290; The subject gets a test result showing that hepatitis C is not present, but it actually is present, so the subject might delay or forego helpful treatment.

19. $P(\text{hepatitis C} \mid \text{positive result}) = \frac{335}{337}$, or 0.994; The very high result makes the test appear to be effective in identifying hepatitis C.

20. $P(\text{no hepatitis C} \mid \text{negative result}) = \frac{1153}{1163}$, or 0.991; The very high result makes the test appear to be effective in identifying subjects not having hepatitis C.

21. a. $1-(0.03)^2 = 0.9991$

    b. $1-(0.03)^3 = 0.999973$; The usual round-off rule for probabilities would result in a probability of 1.00, which would incorrectly indicate that we are certain to have at least one working hard drive.

22. $1-(0.22)^3 = 0.989$; The result is quite high, but there is roughly a 1% chance that in the event of a power failure, none of the backup generators will work. With the possibility of an outage during a major event with thousands of spectators and millions of televisions viewers (as happened in the Superdome—Super Bowl XLVII), it would be wise for manufacturers to improve the reliability of the backup generators so that the 22% failure rate is lowered and the probability of 0.989 is increased.

23. $1-(1-0.126)^5 = 0.490$; The probability is not low, so further testing of the individual samples will be necessary for about 49% of the combined samples.

24. $1-(1-0.005)^{10} = 0.0489$; The probability is quite low, indicating that further testing of the individual samples will be necessary for about 5% of the combined samples.

25. $1-\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdot\ldots\cdot\frac{341}{365} = 1-0.431 = 0.569$

Section 4-4: Counting

1. The symbol ! is the factorial symbol that represents the product of decreasing whole numbers, as in $6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$. Six people can stand in line 720 different ways.

2. Combinations, because order does not count and six numbers are selected (from 1 to 49) without replacement.

3. Because repetition is allowed, numbers are selected *with replacement*, so the combinations rule and the two permutation rules do not apply. The multiplication counting rule can be used to show that the number of possible outcomes is $10\cdot10\cdot10\cdot10 = 10{,}000$, so the probability of winning is $1/10{,}000$.

4. No, because order counts. The "combination" of 8 – 21 – 8 will not work if it is entered as 8 – 8 – 21. (Also, because repetition of numbers is allowed, the numbers can be selected with replacement, so the combinations rule does not apply.)

5. $\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10} = \frac{1}{10{,}000}$

6. $\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10} = \frac{1}{100{,}000}$

7. There are ${}_{19}C_2 = \frac{19!}{(19-2)!2!} = 171$ ways to choose a quinela. The probability is $1/171$, or 0.00585.

8. ${}_{11}C_3 = \frac{11!}{(11-3)!3!} = 165$, $3! = 6$

9. $8! = 40{,}320$; The probability is $1/40{,}320$.

10. For three additional letters, there are $2\cdot26\cdot26\cdot26 = 35{,}152$ possibilities, for two additional letters, there are $2\cdot26\cdot26 = 1352$ possibilities, for a total of $35{,}152+1352 = 36{,}504$ possibilities.

11. There are ${}_{50}P_5 = \frac{50!}{(50-5)!} = 254{,}251{,}200$ possible routes. The probability is $1/254{,}251{,}200$.

12. $\frac{12!}{3!4!} = 3,326,400$

13. $\frac{1}{100 \cdot 100 \cdot 100 \cdot 100} = \frac{1}{100,000,000}$; No, there are too many different possibilities.

14. ${}_{10}C_2 = \frac{10!}{(10-2)!2!} = 10$

15. ${}_{16}C_4 = \frac{16!}{(16-4)!4!} = 1820$; The probability is $1/1820$, or 0.000549.

16. $5! = 120$; The probability is $1/120$.

17. $\frac{1}{{}_{69}C_5 \cdot 26} = \frac{1}{292,201,338}$

18. $4! = 24$; The probability is $1/24$.

19. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100,000$; The probability is $1/100,000$, or 0.00001.

20. ${}_9P_6 = \frac{9!}{(9-6)!} = 60,480$

21. There are $8 \cdot 10 \cdot 10 = 800$ possible area codes. There are $8 \cdot 10 \cdot 10 \cdot 8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 6,400,000,000$ possible area codes. Yes. (With a total population of about 400,000,000, there would be about 16 phone numbers for every adult and child.)

22. $\frac{11!}{4!4!2!} = 34,650$

23. a. ${}_{10}P_4 = \frac{10!}{(10-4)!} = 5040$

    b. ${}_{10}C_4 = \frac{10!}{(10-4)!4!} = 210$

    c. The probability is $1/210$.

24. a. $1/4$, or 0.25

    b. $\frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16}$, or 0.188

    c. Trick question. There is no finite number of attempts, because you could continue to get the wrong position every time.

25. $8! = 40,320$; The probability is $1/40,320$.

26. a. $\frac{1}{10^{16}} = \frac{1}{10,000,000,000,000,000}$

    b. $\frac{1}{10^{12}} = \frac{1}{1,000,000,000,000}$

    c. $\frac{1}{10^8} = \frac{1}{100,000,000}$; The number of possibilities (100,000,000) is still quite large, so there is no reason to worry(unless the information is taken from your credit card or is hacked from the Internet).

27. $\frac{16!}{2!2!2!2!2!} = 653,837,184,000$

28. a. ${}_{16}P_{14}\dfrac{16!}{(16-14)!}=10,461,394,944,000$; (Most calculators will give a result in scientific notation, so an answer such as 10,461,395,000,000 is OK.)

b. ${}_{16}C_{14}\dfrac{16!}{(16-14)!14!}=120$

c. The probability is $1/120$.

29. $\dfrac{1}{{}_{75}C_5\cdot 15}=\dfrac{1}{258,890,850}$; There is a *much* better chance of being struck by lightning.

30. $\dfrac{{}_2C_1}{{}_{12}C_6}=\dfrac{2}{924}=\dfrac{1}{462}$; Yes, if everyone treated is of one gender while everyone in the placebo group is of the opposite gender, you would not know if different reactions are due to the treatment or gender.

31. There are $2+2\cdot2+2\cdot2\cdot2+2\cdot2\cdot2\cdot2+2\cdot2\cdot2\cdot2\cdot2=2+4+8+16+32=62$ different possible characters. The alphabet requires 26 characters and there are 10 digits, so the Morse code system is more than adequate.

32. $\dfrac{{}_2C_1}{{}_8C_4}=\dfrac{2}{70}=\dfrac{1}{35}$

33. $\dfrac{4\cdot16+16\cdot4}{52\cdot51}=\dfrac{128}{2652}=\dfrac{32}{663}$, or about 0.0483; 4.83%, or about 5%, of hands are blackjack hands.

34. You can touch five fingers, four fingers, three fingers or two fingers in $\dfrac{5!}{5!0!}+\dfrac{5!}{4!1!}+\dfrac{5!}{3!2!}+\dfrac{5!}{2!3!}$ $=1+5+10+10=26$ different ways.

35. 12 ways: {25p, 1n 20p, 2n 15p, 3n 10p, 4n 5p, 5n, 1d 15p, 1d 1n 10p, 1d 2n 5p, 1d 3n, 2d 5p,2d 1n} (Note: 25p represents 25 pennies, etc.)

36. a. $32+16+8+4+2+1=63$

b. $\left(\dfrac{1}{2}\right)^{63}=\dfrac{1}{9,223,372,036,854,775,808}=0.000000000000000000108$, or extremely unlikely!

37. $26+26\cdot36+26\cdot36^2+26\cdot36^3+26\cdot36^4+26\cdot36^5+26\cdot36^6+26\cdot36^7=2,095,681,645,538$ or about 2 trillion.

38. a. ${}_5C_2=10$

b. ${}_nC_2=\dfrac{n(n-1)}{2}$

c. $4!=24$

d. $(n-1)!$

Quick Quiz

1. $\dfrac{4}{5}$, or 0.8

2. $1-0.20=0.80$

3. $\dfrac{4}{12}=\dfrac{1}{3}$

4. $0.74^2=0.5476$, or 0.548

5. Answer varies, but the probability should be low, such as 0.001.

Use the following table for Exercises 6–10.

	Developed Flu	Did Not Develop Flu	**Total**
Vaccine Treatment	14	1056	**1070**
Placebo	95	437	**532**
Total	**109**	**1493**	**1602**

6. $\frac{109}{1602}$, or 0.0680

7. $\frac{14+1056+95}{1602}=\frac{1165}{1602}$, or $\frac{109}{1602}+\frac{1070}{1602}-\frac{14}{1602}=\frac{1165}{1602}$, or 0.727

8. $\frac{14}{1602}=\frac{7}{801}=0.00847$

9. $\frac{109}{1602}\cdot\frac{108}{1601}=0.00459$

10. $P(\text{developed flu} \mid \text{given vaccine})=\frac{14}{1070}=\frac{7}{535}$, or 0.0131

Review Exercises

Use the following table for Exercises 1–10.

	Driver Killed	Driver Not Killed	**Total**
Seatbelt Used	3655	7005	**10,660**
Seatbelt Not Used	4402	3040	**7442**
Total	**8057**	**10,045**	**18,102**

1. $\frac{10{,}660}{18{,}102}=\frac{5330}{9051}$, or 0.589

2. $P(\text{not killed} \mid \text{seatbelt used})=\frac{7005}{10{,}660}=\frac{1401}{2132}$, or 0.657

3. $P(\text{killed} \mid \text{seatbelt not used})=\frac{4402}{7442}=\frac{2201}{3721}$, or 0.592

4. $\frac{10{,}660}{18{,}102}+\frac{8057}{18{,}102}-\frac{3655}{18{,}102}=\frac{7531}{9051}$, or 0.832

5. $\frac{7442}{18{,}102}+\frac{10{,}045}{18{,}102}-\frac{3040}{18{,}102}=\frac{14{,}447}{18{,}102}$, or 0.798

6. $\frac{10{,}660}{18{,}102}\cdot\frac{10{,}659}{18{,}101}=0.347$

7. $\frac{8057}{18{,}102}\cdot\frac{8057}{18{,}102}=0.198$

8. A is the event of selecting a driver and getting someone who was not using a seatbelt. $P(\bar{A})=1-0.589$ $=0.411$

9. A is the event of selecting a driver and getting someone who was killed.
   $P(\bar{A})=1-\frac{10{,}045}{18{,}102}=\frac{8057}{18{,}102}=\frac{1151}{2586}=0.445$

10. $\frac{10{,}045}{18{,}102}\cdot\frac{10{,}044}{18{,}101}\cdot\frac{10{,}043}{18{,}100}=0.171$

11. Answer varies, but Forbes reports that about 19% of cars are black, so any estimate between 0.10 and 0.30 would be good.

12. a. $1-0.75=0.25$, or 25%
    b. $0.75\cdot0.75\cdot0.75\cdot0.75=0.316$
    c. No, it is not unlikely because the probability of 0.316 shows that the event occurs quite often.

13. a. 1/365
    b. 31/365
    c. Answers will vary, but it is probably quite small, such as 0.01 or less.
    d. yes

14. $1-\left(1-\frac{34}{10,000}\right)^{10}=0.0335$; No, it is not likely.

15. a. $\frac{1}{{}_{33}C_5}=\frac{1}{237,336}$

b. 1/4

c. 1/4

d. $\frac{1}{237,336}\cdot\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{3,797,376}$

16. $\frac{1}{{}_{43}C_5}=\frac{1}{962,598}$

17. a. $1-\frac{1}{1000}=\frac{999}{1000}$, or 0.999

b. $1-\left(\frac{1}{1000}\right)^2=\frac{999,999}{1,000,000}$, or 0.999999

18. ${}_{19}P_2=342$; The probability is $1/342$.

Cumulative Review Exercises

1. a. The mean is $\bar{x}=\frac{0.09+0.11+\cdots+0.15+0.17+\cdots+0.23+0.35}{12}=0.165\ \text{g/dL}$.

b. The median is $\frac{0.15+0.17}{2}=0.160\ \text{g/dL}$.

c. The midrange is $\frac{0.09+0.35}{2}=0.220\ \text{g/dL}$.

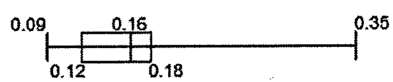
d. The range is $0.35-0.09=0.260\ \text{g/dL}$.

e. $s=\sqrt{\frac{(0.09-0.165)^2+(0.11-0.165)^2+\ldots+(0.23-0.165)^2+(0.35-0.165)^2}{12-1}}=0.069\ \text{g/dL}$

f. $s^2=(0.069)^2=0.005\ (\text{g/dL})^2$

2. a. The five number summary is 0.090 g/dL, 0.120 g/dL, 0.160 g/dL, 0.180 g/dL, 0.350 g/dL. The value of 0.350 g/dL is an outlier.

b.



c.

0. | 9
1. | 113457788
2. | 3
3. | 5

3. a. $\frac{2346}{5100}=0.46=46\%$

b. 0.460

c. stratified sample

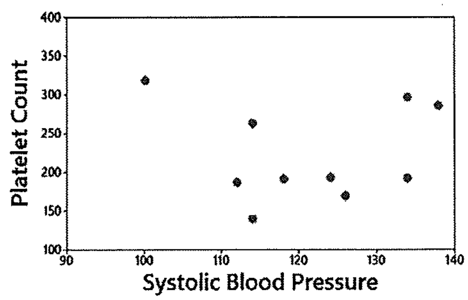
4. a. a convenience sample

b. If the students at the college are mostly from a surrounding region that includes a large proportion of one ethnic group, the results will not reflect the general population of the United States.

c. $0.35+0.4=0.75$

d. $1-(0.6)^2=0.64$

5. The lack of any pattern of the points in the scatterplot suggests that there does not appear to be an association between systolic blood pressure and blood platelet count.



6. a. $\dfrac{1}{_{52}C_5} = \dfrac{1}{2{,}598{,}960}$

   b. $\dfrac{1}{28}$

   c. $\dfrac{1}{2{,}298{,}960} \cdot \dfrac{1}{28} = \dfrac{1}{72{,}770{,}880}$

Chapter 5: Discrete Probability Distributions

Section 5-1: Probability Distributions

1. The random variable is x, which is the number of girls in four births. The possible values of x are 0, 1, 2, 3, and 4. The values of the random variable x are numerical.
2. The random variable is discrete because the number of possible values is 5, and 5 is a finite number. The random variable is discrete if it has a finite number of values or a countable number of values.
3. Table 5-7 does describe a probability distribution because the three requirements are satisfied. First, the variable x is a numerical random variable and its values are associated with probabilities. Second, $\Sigma P(x) = 0.063 + 0.250 + 0.375 + 0.250 + 0.063 = 1.001$, which is not exactly 1 due to round-off error, but is close enough to satisfy the requirement. Third, each of the probabilities is between 0 and 1 inclusive, as required.
4. The probability of 0.136 is relevant; 56 is not significantly high because the probability of 56 or more girls is 0.136, which is not small, such as 0.05 or less. With random chance, it is likely that the outcome could be 56 or more girls.
5. a. continuous random variable
   b. not a random variable
   c. discrete random variable
   d. continuous random variable
   e. discrete random variable
6. a. not a random variable
   b. continuous random variable
   c. discrete random variable
   d. not a random variable
   e. discrete random variable
7. Probability distribution with
   $\mu = 0 \cdot 0.031 + 1 \cdot 156 + 2 \cdot 0.313 + 3 \cdot 313 + 4 \cdot 0.156 + 5 \cdot 0.031 = 2.5$
   $\sigma = \sqrt{(0-2.5)^2 \cdot 0.031 + (1-2.5)^2 \cdot 0.156 + \cdots + (4-2.5)^2 \cdot 0.156 + (5-2.5)^2 \cdot 0.031} = 1.1$
8. Probability distribution (The sum of the probabilities is 1.001, but that is due to rounding errors.) with
   $\mu = 0 \cdot 0.659 + 1 \cdot 0.287 + 2 \cdot 0.050 + 3 \cdot 004 + 4 \cdot 0.001 + 5 \cdot 0 = 0.4$
   $\sigma = \sqrt{(0-0.4)^2 \cdot 0.659 + (1-0.4)^2 \cdot 0.287 + \cdots + (4-0.4)^2 \cdot 0.001 + (5-0.4)^2 \cdot 0} = 0.6$
9. Not a probability distribution because the sum of the probabilities is 0.1, which is not 1 as required. Also, Ted clearly needs a new approach.
10. Not a probability distribution because the responses are not values of a numerical random variable.
11. Probability distribution with
    $\mu = 0 \cdot 0.091 + 1 \cdot 0.334 + 2 \cdot 0.408 + 3 \cdot 0.166 = 1.6$
    $\sigma = \sqrt{(0-1.6)^2 \cdot 0.091 + (1-1.6)^2 \cdot 0.334 + (2-1.6)^2 \cdot 0.408 + (3-1.6)^2 \cdot 0.166} = 0.9$
    (The sum of the probabilities is 0.999, but that is due to rounding errors.)
12. Probability distribution with
    $\mu = 0 \cdot 0.358 + 1 \cdot 0.439 + 2 \cdot 0.179 + 3 \cdot 0.024 = 0.9$
    $\sigma = \sqrt{(0-0.9)^2 \cdot 0.358 + (1-0.9)^2 \cdot 0.439 + (2-0.9)^2 \cdot 0.179 + (3-0.9)^2 \cdot 0.024} = 0.8$
13. This is not a probability distribution because the responses are not values of a numerical random variable.
14. This is not a probability distribution because the sum of the probabilities is 0.934 instead of 1 as required. The discrepancy between 0.934 and 1 is too large to attribute to rounding errors.
15. $\mu = 0 \cdot 0.004 + 1 \cdot 0.031 + 2 \cdot 0.109 + \cdots + 6 \cdot 0.109 + 7 \cdot 0.031 + 8 \cdot 0.004 = 4.0$ girls
    $\sigma = \sqrt{(0-4.0)^2 \cdot 0.004 + (1-4.0)^2 \cdot 0.031 + \cdots + (7-4.0)^2 \cdot 0.031 + (8-4.0)^2 \cdot 0.004} = 1.4$ girls
16. The lower limit is $\mu - 2\sigma = 4.0 - 2(1.4) = 1.2$ girls. Because 1 girl is less than or equal to 1.2 girls, it is a significantly low number of girls.
17. The upper limit is $\mu + 2\sigma = 4.0 + 2(1.4) = 6.8$ girls. Because 6 girls is not greater than or equal to 6.8 girls, it is not a significantly high number of girls.

18. a. $P(X=7)=0.031$
    b. $P(X \ge 7)=0.031+0.004=0.035$
    c. The probability from part (b), since it is the probability of the given or more extreme result.
    d. Yes, because the probability of 7 or more girls is 0.035, which is low (less than or equal to 0.05).
19. a. $P(X=6)=0.109$
    b. $P(X \ge 6)=0.109+0.031+0.004=0.144$
    c. The result from part (b), since it is the probability of the given or more extreme result.
    d. No, because the probability of six or more girls is 0.144, which is not very low (less than or equal to 0.05).
20. a. $P(X=1)=0.031$
    b. $P(X \le 1)=0.004+0.031=0.035$
    c. The result from part (b), since it is the probability of the given or more extreme result.
    d. Yes, because the probability of one or fewer girls is 0.035, which is low (less than or equal to 0.05).
21. $\mu = 0\cdot 0.172+1\cdot 0.363+2\cdot 0.306+3\cdot 0.129+4\cdot 0.027+5\cdot 0.002=1.5$ sleepwalkers

    $\sigma=\sqrt{(0-1.5)^2\cdot 0.172+(1-1.5)^2\cdot 0.363+...+(4-1.5)^2\cdot 0.027+(5-1.5)^2\cdot 0.002}=1.0$ sleepwalkers
22. Significantly high numbers of sleepwalkers are greater than or equal to $\mu+2\sigma=1.5+2(1.0)=3.5$ sleepwalkers. Because 4 sleepwalkers is greater than or equal to 3.5 sleepwalkers, 4 sleepwalkers is a significantly high number.
23. Significantly high numbers of sleepwalkers are greater than or equal to $\mu+2\sigma=1.5+2(1.0)=3.5$ sleepwalkers. Because 3 sleepwalkers is not greater than or equal to 3.5 sleepwalkers, 3 sleepwalkers is not a significantly high number.
24. a. $P(X=4)=0.027$
    b. $P(X \ge 4)=0.027+0.002=0.029$
    c. The probability from part (b), since it is the probability of the given or more extreme result.
    d. Yes, because the probability of four or more sleepwalkers is 0.029, which is very low (less than or equal to 0.05)
25. a. $P(X=1)=0.363$
    b. $P(X \le 1)=0.172+0.363=0.535$
    c. The probability from part (b), since it is the probability of the given or more extreme result.
    d. No, because the probability of one or fewer sleepwalkers is 0.535, which is not low (less than or equal to 0.05)
26. a. $10\cdot 10\cdot 10\cdot 10=10{,}000$
    b. 1/10,000
    c. $\$5000-\$1=\$4999$
    d. $-\$1\cdot 1+\$5000\cdot\frac{1}{10{,}000}=-\$0.50=-50\,¢$
    e. The \$1 bet on the pass line in craps is better because its expected value of −1.4¢ is much greater than the expected value of −50¢ for the Ohio Pick 4 lottery.
27. a. $10\cdot 10\cdot 10=1000$
    b. 1/1000
    c. $\$500-\$1=\$499$
    d. $-\$1\cdot 1+\$500\cdot\frac{1}{1000}=-\$0.50=-50\,¢$
    e. Because both bets have the same expected value of −50¢, neither bet is better than the other.

28. a. $-\$0.26+\$30\cdot\frac{5}{38}-\$5\cdot\frac{33}{38}=-\$0.39$, or $-39¢$

    b. The bet on the number 27 is better because its expected value of $-26¢$ is greater than the expected value of $-39¢$ for the other bet.

29. a. Surviving the year: –\$226; Not surviving the year: \$50,000 – \$226 = \$49,774

    b. $-\$161\cdot 0.9986+\$99{,}839\cdot(1-0.9986)=-\$21$

    c. Yes; the expected value for the insurance company is \$21, which indicates that the company can expect to make an average of \$21 for each such policy.

30. a. –\$226 and \$49,774

    b. $-\$226\cdot 0.9968+\$46{,}774\cdot(1-0.9968)=-\$66$

    c. Yes; the expected value for the insurance company is \$66, which indicates that the company can expect to make an average of \$66 for each such policy.

Section 5-2: Binomial Probability Distributions

1. The given calculation assumes that the first two consumers are comfortable with the drones and the last three consumers are not comfortable with drones, but there are other arrangements consisting of two consumers who are comfortable and three who are not. The probabilities corresponding to those other arrangements should also be included in the result.
2. $n=5, x=2, p=0.42, q=0.58$
3. Because the 30 selections are made without replacement, they are dependent, not independent. Based on the 5% guideline for cumbersome calculations, the 30 selections can be treated as being independent. (The 30 selections constitute 3% of the population of 1009 responses, and 3% is not more than 5% of the population.) The probability can be found by using the binomial probability formula, but it would require application of that formula 21 times (or 10 times if we are clever), so it would be better to use technology.
4. The 0+ indicates that the probability is a very small positive value. (The actual value is 0.0000205.) The notation of 0+ does not indicate that the event is impossible; it indicates that the event is possible, but very unlikely.
5. Not binomial; each of the weights has more than two possible outcomes.
6. binomial
7. binomial
8. Not binomial; each of the responses has more than two possible outcomes.
9. Not binomial; because the senators are selected without replacement, the selections are not independent. (The 5% guideline for cumbersome calculations cannot be applied because the 40 selected senators constitute 40% of the population of 100 senators, and that exceeds 5%.)
10. Not binomial; because the senators are selected without replacement, they are not independent. (The 5% guideline for cumbersome calculations cannot be applied because the 10 selected senators constitute 10% of the population of 100 senators, and that exceeds 5%.). Also, the numbers of terms have more than two possible outcomes.
11. Binomial; although the events are not independent, they can be treated as being independent by applying the 5% guideline. The sample size of 1019 is not more than 5% of the population of all adults.
12. Binomial; although the events are not independent, they can be treated as being independent by applying the 5% guideline. The sample size of 1000 is not more than 5% of the population of all adults.

13. a. $\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{1}{5}=0.128$

    b. {WWC, WCW, CWW}; Each has a probability of 0.128.

    c. $0.128\cdot 3=0.384$

14. a. $P(\text{OOOD})=0.4\cdot 0.4\cdot 0.4\cdot 0.6=0.0384$

    b. {OOOD, OODO, ODOO, DOOO }; Each has a probability of 0.0384.

    c. $0.0384\cdot 4=0.154$

15. ${}_8C_7 \cdot 0.2^7 \cdot 0.8^1 = 0.0000819$ (Table: 0+)

16. ${}_8C_4 \cdot 0.2^4 \cdot 0.8^4 + {}_8C_5 \cdot 0.2^5 \cdot 0.8^3 + {}_8C_6 \cdot 0.2^6 \cdot 0.8^2 + {}_8C_7 \cdot 0.2^7 \cdot 0.8^1 + {}_8C_8 \cdot 0.2^8 \cdot 0.8^0 = 0.0563$ (Table: 0.056)

17. ${}_8C_0 \cdot 0.2^0 \cdot 0.8^8 + {}_8C_1 \cdot 0.2^1 \cdot 0.8^7 + {}_8C_2 \cdot 0.2^2 \cdot 0.8^6 = 0.797$ (Table: 0.798)

18. ${}_8C_0 \cdot 0.2^0 \cdot 0.8^8 + {}_8C_1 \cdot 0.2^1 \cdot 0.8^7 + {}_8C_2 \cdot 0.2^2 \cdot 0.8^6 = 0.797$ (Table: 0.798)

19. ${}_8C_0 \cdot 0.2^0 \cdot 0.8^8 = 0.168$

20. $1 - {}_8C_0 \cdot 0.2^0 \cdot 0.8^8 = 0.832$ (Table: 0.833)

21. ${}_8C_6 \cdot 0.54^6 \cdot 0.46^2 = 0.147$

22. ${}_{20}C_{15} \cdot 0.54^{15} \cdot 0.46^5 = 0.0309$

23. ${}_{10}C_8 \cdot 0.54^8 \cdot 0.46^2 + {}_{10}C_9 \cdot 0.54^9 \cdot 0.46^1 + {}_{10}C_{10} \cdot 0.54^{10} \cdot 0.46^0 = 0.0889$

24. ${}_{10}C_0 \cdot 0.54^0 \cdot 0.46^{10} + {}_{10}C_1 \cdot 0.54^1 \cdot 0.46^9 + {}_{10}C_2 \cdot 0.54^2 \cdot 0.46^8 = 0.00952$

25. ${}_{90}C_0 \cdot 0.27^0 \cdot 0.73^{90} + {}_{90}C_1 \cdot 0.27^1 \cdot 0.73^{89} + \cdots + {}_{90}C_6 \cdot 0.27^6 \cdot 0.73^{84} + {}_{90}C_7 \cdot 0.27^7 \cdot 0.73^{83} = 0.00000451$; The result of 7 minorities is significantly low. The probability shows that it is very highly unlikely that a process of random selection would result in 7 or fewer minorities. (The Supreme Court rejected the claim that the process was random.)

26. ${}_{20}C_{19} \cdot 0.79^{19} \cdot 0.21^1 + {}_{20}C_{20} \cdot 0.79^{20} \cdot 0.21^0 = 0.0566$; The probability of 19 or more adults requiring eyesight correction is not low (less than or equal to 0.05), so the result of 19 is not significantly high.

27. a. ${}_6C_5 \cdot 0.20^5 \cdot 0.80^1 = 0.002$ (Tech: 0.00154)
    b. ${}_6C_6 \cdot 0.20^6 \cdot 0.80^0 = 0+$ (Tech: 0.000064)
    c. $0.002 + 0 = 0.002$ (Tech: 0.00160)
    d. Yes, the small probability from part (c) suggests that 5 is an unusually high number.

28. a. ${}_5C_0 \cdot 0.20^0 \cdot 0.80^5 = 0.328$
    b. ${}_5C_1 \cdot 0.20^1 \cdot 0.80^4 = 0.410$
    c. $0.328 + 0.410 = 0.738$ (Tech: 0.737)
    d. No, the probability from part (c) is not small, so 1 is not an unusually low number.

29. a. $\mu = np = 36 \cdot 0.5 = 18.0$ girls, $\sigma = \sqrt{np(1-p)} = \sqrt{36 \cdot 0.5 \cdot 0.5} = 3.0$ girls
    b. Values of $18.0 - 2(3.0) = 12.0$ girls or fewer are significantly low, values of $18.0 + 2(3.0) = 24.0$ girls or more are significantly high, and values between 12.0 girls and 24.0 girls are not significant.
    c. The result is significantly high because the result of 26 girls is greater than or equal to 24.0 girls. A result of 26 girls would suggest that the XSORT method is effective.

30. a. $\mu = np = 16 \cdot 0.5 = 8.0$ girls, $\sigma = \sqrt{np(1-p)} = \sqrt{16 \cdot 0.5 \cdot 0.5} = 2.0$ girls
    b. Values of $8.0 - 2(2.0) = 4.0$ girls or fewer are significantly low, values of $8.0 + 2(2.0) = 12.0$ girls or more are significantly high, and values between 4.0 girls and 12.0 girls are not significant.
    c. The result is not significant because the result of 11 girls is not greater than or equal to 12.0 girls. A result of 11 girls would not suggest that the XSORT method is effective.

31. a. $\mu = np = 10 \cdot 0.75 = 7.5$ peas, $\sigma = \sqrt{np(1-p)} = \sqrt{10 \cdot 0.75 \cdot 0.25} = 1.4$ peas
    b. Values of $7.5 - 2(1.4) = 4.7$ peas or fewer are significantly low, values of $7.5 + 2(1.4) = 10.3$ peas or more are significantly high, and values between 4.7 peas and 10.3 peas are not significant.
    c. The result is not significant because the result of 9 peas is not greater than or equal to 10.3 peas.

32. a. $\mu = np = 16 \cdot 0.75 = 12.0$ peas, $\sigma = \sqrt{np(1-p)} = \sqrt{16 \cdot 0.75 \cdot 0.25} = 1.7$ peas
b. Values of $12.0 - 2(1.7) = 8.6$ peas or fewer are significantly low, values of $12.0 + 2(1.7) = 15.4$ peas or more are significantly high, and values between 4.7 peas and 10.3 peas are not significant.
c. The result is significant because the result of 7 peas is less than or equal to 8.6 peas.

33. $1 - {}_{36}C_0 \cdot 0.01^0 \cdot 0.99^{36} = 0.304$; It is not unlikely for such a combined sample to test positive.

34. $1 - {}_{8}C_0 \cdot 0.1^0 \cdot 0.9^8 = 0.570$; It is likely for such a combined sample to test positive.

35. ${}_{40}C_1 \cdot 0.03^1 \cdot 0.97^{39} + {}_{40}C_0 \cdot 0.03^0 \cdot 0.97^{40} = 0.662$; The probability shows that about 2/3 of all shipments will be accepted. With about 1/3 of the shipments rejected, the supplier would be wise to improve quality.

36. ${}_{50}C_2 \cdot 0.02^2 \cdot 0.98^{48} + {}_{50}C_1 \cdot 0.02^1 \cdot 0.98^{49} + {}_{50}C_0 \cdot 0.02^0 \cdot 0.98^{50} = 0.922$; About 92% of all shipments will be accepted. Almost all shipments will be accepted, and only 8% of the shipments will be rejected.

37. a. $\mu = np = 100 \cdot 0.16 = 16.0$ M&Ms, $\sigma = \sqrt{np(1-p)} = \sqrt{100 \cdot 0.16 \cdot 0.84} = 3.7$ M&Ms; Values between $16.0 - 2(3.7) = 8.8$ M&Ms and $16.0 + 2(3.7) = 23.4$ M&Ms are not significant (8.7 and 23.3 if using unrounded values). 19 M&Ms lies between these limits, so it is not significant.
b. The probability of exactly 19 green M&Ms is ${}_{100}C_{19} \cdot 0.16^{19} \cdot 0.84^{81} = 0.0736$.
c. The probability of 19 or more green M&Ms is ${}_{100}C_{19} \cdot 0.16^{19} \cdot 0.84^{81} + \cdots + {}_{100}C_{100} \cdot 0.16^{100} \cdot 0.84^0$ $= 0.242$.
d. The probability from part (c) is relevant. The result of 19 green M&Ms is not significantly high.
e. The results do not provide strong evidence against the claim of 16% for green M&Ms.

38. a. $\mu = np = 41 \cdot 0.5 = 20.5$, $\sigma = \sqrt{np(1-p)} = \sqrt{41 \cdot 0.5 \cdot 0.5} = 3.2$; Values between $20.5 - 2(3.2) = 14.1$ and $20.5 + 2(3.2) = 26.9$ are not significant. The value of 40 is greater than or equal to 26.9, so it is significant.
b. The probability of exactly 40 top lines for Democrats is ${}_{41}C_{40} \cdot 0.5^{40} \cdot 0.5^1$, which is approximately zero when rounded.
c. The probability of 19 or more green M&Ms is ${}_{41}C_{40} \cdot 0.5^{40} \cdot 0.5^1 + {}_{41}C_{41} \cdot 0.5^{41} \cdot 0.5^0$, which is approximately zero when rounded.
d. The probability from part (c) is relevant. The result of 40 top lines for Democrats is significantly high.
e. The results suggest that the clerk did not assign ballot lines randomly.

39. a. $\mu = np = 611 \cdot 0.43 = 262.7$ votes, $\sigma = \sqrt{np(1-p)} = \sqrt{611 \cdot 0.43 \cdot 0.57} = 12.2$ votes; Values between $262.7 - 2(12.2) = 238.3$ votes and $262.7 + 2(12.2) = 287.1$ votes are not significant. The value of 308 votes is greater than or equal to 128.1, so it is significant.
b. The probability of exactly 308 voters is ${}_{611}C_{308} \cdot 0.43^{308} \cdot 0.57^{303} = 0.0000369$.
c. The probability of 308 or more voters is ${}_{611}C_{308} \cdot 0.43^{308} \cdot 0.57^{303} + \cdots + {}_{611}C_{611} \cdot 0.43^0 \cdot 0.57^{611}$ $= 0.000136$.
d. The probability from part (c) is relevant. The value of 308 votes is significantly high.
e. The results suggest that the surveyed voters either lied or had defective memory of how they voted.

40. a. $\mu = np = 580 \cdot 0.25 = 145.0$ yellow peas, $\sigma = \sqrt{np(1-p)} = \sqrt{580 \cdot 0.25 \cdot 0.75} = 10.4$ yellow peas; Values between $145.0 - 2(10.4) = 124.2$ yellow peas and $145.0 + 2(10.4) = 165.8$ yellow peas are not significant (124.1 and 165.9 if using unrounded values). 152 yellow peas lies between these limits, so it is not significant.
b. The probability of exactly 152 yellow peas is ${}_{580}C_{152} \cdot 0.25^{152} \cdot 0.75^{428} = 0.0301$.
c. The probability of 152 or more yellow peas is ${}_{580}C_{152} \cdot 0.25^{152} \cdot 0.75^{428} + \cdots + {}_{580}C_{580} \cdot 0.25^{580} \cdot 0.75^0$ $= 0.265$.

40. (continued)
   d. The probability from part (c) is relevant. The value of 152 yellow peas is not significantly high.
   e. The results do not provide strong evidence against Mendel's claim of 25% for yellow peas.

41. $P(5)=0.06(1-0.06)^4=0.0468$

42. $\dfrac{15!}{7!6!2!}\cdot\left(\dfrac{18}{38}\right)^7\cdot\left(\dfrac{18}{38}\right)^6\cdot\left(\dfrac{2}{38}\right)^2=0.0302$

43. $P(4)=\dfrac{6!}{(6-2)!2!}\cdot\dfrac{43!}{(43-6+2)!(6-2)!}\div\dfrac{(6+43)!}{(6+43-6)!6!}=0.1324$

Section 5-3: Poisson Probability Distributions

1. $\mu=535/576=0.929$, which is the mean number of hits per region. $x=2$, because we want the probability that a randomly selected region had exactly 2 hits, and $e\approx 2.71828$ which is a constant used in all applications of Formula 5-9.

2. The mean is $\mu=153/64=2.4$ tornadoes, the standard deviation is $\sigma=\sqrt{2.4}=1.5$ tornadoes, and the variance is $\sigma^2=2.4$ tornadoes^2.

3. The possible values of x are 0, 1, 2, ... (with no upper bound), so x is a discrete random variable. It is not possible to have $x=2.3$ calls in a day

4. $P(0)$ represents the probability of no occurrences of an event during the relevant interval. If $x=0$, $P(0)=e^{-\mu}$.

5. a. $P(5)=\dfrac{6.1^5\cdot e^{-6.1}}{5!}=0.158$
   b. In 55 years, the expected number of years with 5 hurricanes is $55\cdot 0.158=8.7$.
   c. The expected value of 8.7 years is close to the actual value of 8 years, so the Poisson distribution works well here.

6. a. $P(0)=\dfrac{6.1^0\cdot e^{-6.1}}{0!}=0.00224$
   b. In 55 years, the expected number of years with 5 hurricanes is $55\cdot 0.00224=0.1$.
   c. The expected value of 0.1 years is close to the actual value of 0 years, so the Poisson distribution works well here.

7. a. $P(7)=\dfrac{6.1^7\cdot e^{-6.1}}{7!}=0.140$
   b. In 55 years, the expected number of years with 5 hurricanes is $55\cdot 0.140=7.7$.
   c. The expected value of 7.7 years is close to the actual value of 7 years, so the Poisson distribution works well here.

8. a. $P(4)=\dfrac{6.1^4\cdot e^{-6.1}}{4!}=0.129$
   b. In 55 years, the expected number of years with 5 hurricanes is $55\cdot 0.129=7.1$.
   c. The expected value of 7.1 years is not very close to the actual value of 10 years, so the Poisson distribution does not work so well here.

9. $\mu=\dfrac{4221}{365}=11.6$ births, $P(15)=\dfrac{11.6^{15}\cdot e^{-11.6}}{15!}=0.0649$ (0.0643 if using the unrounded mean.) There is less than a 7% chance of getting exactly 15 births on any given day.

10. $\mu = \dfrac{33}{365} = 0.9$ murders, $P(0) = \dfrac{0.9^0 \cdot e^{-0.9}}{0!} = 0.407$ (0.402 if using the unrounded mean.) There should be many days (roughly 40%) with no murders.

11. a. $\mu = \dfrac{22713}{365} = 62.2$

    b. $P(50) = \dfrac{62.2^{50} \cdot e^{62.2}}{50!} = 0.0155$

12. $\mu = \dfrac{196}{20} = 9.8$

    a. $P(0) = \dfrac{9.8^0 \cdot e^{-9.8}}{0!} = 0.497$

    b. $P(1) = \dfrac{9.8^1 \cdot e^{-9.8}}{1!} = 0.348$

    c. $P(2) = \dfrac{9.8^2 \cdot e^{-9.8}}{2!} = 0.122$

    d. $P(3) = \dfrac{9.8^3 \cdot e^{-9.8}}{3!} = 0.0284$

    e. $P(4) = \dfrac{9.8^4 \cdot e^{-9.8}}{4!} = 0.00497$; The expected frequencies of 139, 97, 34, 8 , and 1.4 compare reasonably well to the actual frequencies, so the Poisson distribution does provide good results.

13. a. $P(2) = \dfrac{0.929^2 \cdot e^{-0.929}}{2!} = 0.17$

    b. The expected number of regions with exactly 2 hits is 98.2.

    c. The expected number of regions with 2 hits is close to 93, which is the actual number of regions with 2 hits.

14. a. $\mu = 12429 \cdot 0.000011 = 0.1367$

    b. $P(0) = \dfrac{0.137^0 \cdot e^{-0.137}}{0!} = 0.872$ and $P(1) = \dfrac{0.137^1 \cdot e^{-0.137}}{1!} = 0.119$; So the probability of 0 or 1 is $0.872 + 0.119 = 0.991$.

    c. $1 - 0.991 = 0.009$

    d. No, the probability of more than one case is extremely small, so the probability of getting as many as four cases is even smaller.

15. $\mu = \dfrac{33,561}{2969} = 11.3$ fatalities, $1 - P(0) = 1 - \dfrac{11.3^0 \cdot e^{-11.3}}{0!} = 0.9999876$ or 0.9999877 if using the unrounded mean. There is a very high chance ("almost certain") that at least one fatality will occur.

16. $\mu = \dfrac{181}{365} = 0.5$ fatalities, $1 - P(0) = 1 - \dfrac{0.5^0 \cdot e^{-0.5}}{0!} = 0.393$ or 0.391 if using the unrounded mean.

17. The Poisson distribution approximation is valid since $n = 5200 \geq 100$ and $\mu = np = \dfrac{5200}{292,201,338}$ $= 0.0000178 \leq 10$. The probability of winning at least one time is $1 - P(0) = 1 - \dfrac{0.0000178^0 \cdot e^{-0.0000178}}{0!}$ $= 0.0000178$, so it is highly unlikely that at least one jackpot win will occur in 50 years.

Quick Quiz

1. No, the sum of the probabilities is $4/3$, or 1.333, which is greater than 1.
2. $\mu = 80 \cdot 0.2 = 16.0$; $\sigma = \sqrt{80 \cdot 0.2 \cdot 0.8} = 3.6$
3. The values are parameters because they represent the mean and standard deviation for the population of all who make random guesses for the 80 questions, not a sample of actual results.
4. No, (Using the range rule of thumb, the limit separating significantly high values is $\mu + 2\sigma = 16.0 + 2(3.6)$ $= 23.2$, but 20 is not greater than or equal to 23.2. Using probabilities, the probability of 20 or more correct answers is 0.163, which is not low.)

5. Yes, (Using the range rule of thumb, the limit separating significantly low values is $\mu - 2\sigma = 16.0 - 2(3.6) = 8.8$, and 8 is less than 8.8. Using probabilities, the probability of 8 or fewer correct answers is 0.0131, which is low.)
6. This is probability distribution because the three requirements are satisfied. First, the variable x is a numerical random variable and its values are associated with probabilities. Second, $\Sigma P(x) = 0 + 0.006 + 0.051 + 0.205 + 0.409 + 0.328 = 0.999$, which is not exactly 1 due to round-off error, but is close enough to satisfy the requirement. Third, each of the probabilities is between 0 and 1 inclusive, as required.
7. $\mu = 0 \cdot 0 + 1 \cdot 0.006 + 2 \cdot 0.051 + 3 \cdot 0.205 + 4 \cdot 0.409 + 5 \cdot 0.328 = 4.0$ flights
8. $\sigma^2 = (0.9 \text{ flight})^2 = 0.8 \text{ flight}^2$
9. 0+ indicates that the probability is a very small positive number. It does not indicate that it is impossible for none of the five flights to arrive on time.
10. $P(X < 3) = P(X \leq 2) = 0 + 0.006 + 0.051 = 0.057$

Review Exercises

1. $P(X = 3) = {}_5C_3 \cdot 0.74^3 \cdot 0.26^2 = 0.247$
2. $P(X \geq 1) = 1 - {}_5C_0 \cdot 0.74^0 \cdot 0.26^5 = 0.999$; No; the five friends are not randomly selected from the population of adults. Also, the fact that they are vacationing together suggests that their financial situations are more likely to include credit cards.
3. $\mu = 5 \cdot 0.74 = 3.7$, $\sigma = \sqrt{5 \cdot 0.74 \cdot 0.26} = 1.0$.
4. No, the limit separating significantly high values is $\mu + 2\sigma = 3.7 + 2(1.0) = 5.7$, but 5 is not greater than or equal to 5.7. Also, the probability that all five adults have credit cards is ${}_5C_5 \cdot 0.74^5 \cdot 0.26^0 = 0.222$, which is not low (less than or equal to 0.05).
5. Yes, the limit separating significantly low values is $\mu - 2\sigma = 3.7 - 2(1.0) = 1.7$, and 1 is less than or equal to 1.7. Also, the probability of one or fewer adults having a credit card is ${}_5C_0 \cdot 0.74^0 \cdot 0.26^5 + {}_5C_1 \cdot 0.74^1 \cdot 0.26^4 = 0.0181$, which is low (less than or equal to 0.05).
6. This is not a probability distribution because the responses are not values of a numerical random variable.
7. This is not a probability distribution because $\Sigma P(x) = 0.0016 + 0.0250 + 0.1432 + 0.3892 + 0.4096 = 0.9686$, instead of 1 as required. The discrepancy between 0.9686 and 1 is too large to attribute to rounding errors.
8. This is a probability distribution (The sum of the probabilities is 0.999, but that is due to rounding errors.) with $\mu = 0 \cdot 0 + 1 \cdot 0.003 + 2 \cdot 0.025 + 3 \cdot 0.111 + 4 \cdot 0.279 + 5 \cdot 0.373 + 6 \cdot 0.208 = 4.6$ people and
   $\sigma = \sqrt{(0-4.6)^2 \cdot 0.0 + (1-4.6)^2 \cdot 0.003 + \cdots + (5-4.6)^2 \cdot 0.373 + (6-4.6)^2 \cdot 0.208} = 1.0$ people.
9. a. $784 \cdot 0.301 = 236.0$ checks
   b. $\mu = 784 \cdot 0.301 = 236.0$, $\sigma = \sqrt{784 \cdot 0.301 \cdot 0.699} = 12.8$
   c. The limit separating significantly low values is $\mu - 2\sigma = 236.0 - 2(12.8) = 210.3$ (210.4 if using unrounded values.)
   d. Yes, because 0 is less than or equal to 210.3 (or 210.4) checks.

10. a. $\mu = 7/365 = 0.0192$

b. $P(X=0) = \dfrac{0.0192^0 \cdot e^{-0.0192}}{0!} = 0.981$

c. $P(X>1) = P(X \geq 2) = 1 - P(X \leq 1) = 1 - \dfrac{0.0192^0 \cdot e^{-0.0192}}{0!} - \dfrac{0.0192^1 \cdot e^{-0.0192}}{1!} = 0.000182$

d. No, because the event is so rare. (But it is possible that more than one death occurs in a car crash or some other such event, so it might be wise to consider a contingency plan.)

Cumulative Review Exercises

1. a. The mean is $\bar{x} = \dfrac{0+0+1+2+8+17+21+28}{8} = 9.6$ moons.

b. The median is $\dfrac{2+8}{2} = 5.0$ moons.

c. The mode is 0 moons.

d. The range is $28-0 = 28.0$ moons.

e. The standard deviation is $s = \sqrt{\dfrac{(0-9.6)^2+(0-9.6)^2+\cdots+(21-9.6)^2+(28-9.6)^2}{8-1}} = 11.0$ moons.

f. The variance is $s^2 = \dfrac{(0-9.6)^2+(0-9.6)^2+\cdots+(21-9.6)^2+(28-9.6)^2}{8-1} = 120.3$ moons^2.

g. The minimum is $9.6 - 2\cdot 11.0 = -12.4$ moons and the maximum is $9.6+2\cdot 11.0 = 31.6$ moons.

h. No, because none of the planets have a number of moons less than or equal to $\mu - 2\sigma = 9.6 - 2(11.0) = -12.4$ moons (which is impossible, anyway) and none of the planets have a number of moons equal to or greater than $\mu - 2\sigma = 9.6 + 2(11.0) = 31.6$ moons.

i. ratio

j. discrete

2. a. $\dfrac{1}{10}\cdot\dfrac{1}{10}\cdot\dfrac{1}{10} = \dfrac{1}{1000} = 0.001$

b. $365 \cdot 0.001 = 0.365$

c. $P(1) = \dfrac{0.365^1 \cdot e^{-0.365}}{1!} = 0.254$

d. $-1\cdot 0.999 + 499 \cdot 0.001 = -0.50$ or $-50¢$.

3. Refer to the following table.

	Challenge Upheld with Overturned Call	Challenge Rejected with Overturned Call	**Total**
Challenges by Men	152	412	**564**
Challenges by women	79	236	**315**
Total	**231**	**648**	**879**

a. $\dfrac{231}{879} = \dfrac{77}{293} = 0.263$

b. $\dfrac{79}{231} = 0.342$

c. $\dfrac{231}{879}\cdot\dfrac{230}{878} = 0.0688$

d. $\dfrac{564}{879} + \dfrac{231}{879} - \dfrac{152}{879} = \dfrac{643}{879} = 0.732$

e. $\dfrac{152}{231} = 0.658$

4. a. $0.73 \cdot 347 = 253$

b. The sample is the 347 human resource professionals who were surveyed. The population is all human resource professionals.

c. 73% is a statistic because it is a measure based on a sample, not the entire population.

5. No vertical scale is shown, but a comparison of the numbers shows that 7,066,000 is roughly 1.2 times the number 6,000,000. However, the graph makes it appear that the goal of 7,066,000 people is roughly 3 times the number of people enrolled. The graph is misleading in the sense that it creates the false impression that actual enrollments are far below the goal, which is not the case. Fox News apologized for their graph and provided a corrected graph.

6. a. $P(X=5) = {}_8C_5 \cdot 0.7^5 \cdot 0.3^3 = 0.254$

   b. $P(X \geq 7) = {}_8C_7 \cdot 0.7^7 \cdot 0.3^1 + {}_8C_8 \cdot 0.7^8 \cdot 0.3^0 = 0.255$

   c. $\mu = 8 \cdot 0.7 = 5.6$ adults, $\sigma = \sqrt{8 \cdot 0.7 \cdot 0.3} = 1.3$ adults

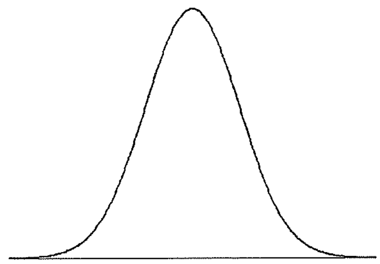
   d. Yes. (Using the range rule of thumb, the limit separating significantly low values is $\mu - 2\sigma = 5.6 - 2(1.3) = 3$, and 1 is less than 3. Using probabilities, the probability of 1 or fewer people washing their hands is ${}_8C_0 \cdot 0.7^0 \cdot 0.3^8 + {}_8C_1 \cdot 0.7^1 \cdot 0.3^7 = 0.00129$, (0.001 if using the table) which is low, such as less than 0.05.

Chapter 6: Normal Probability Distributions

Section 6-1: The Standard Normal Distribution

1. The word "normal" has a special meaning in statistics. It refers to a specific bell-shaped distribution that can be described by Formula 6-1. The lottery digits do not have a normal distribution.

2.



3. The mean and standard deviation have values of $\mu = 0$ and $\sigma = 1$, respectively.

4. The notation z_α represents the z score that has an area of α to its right.

5. $P(x > 3) = 0.2(5-3) = 0.4$

6. $P(x < 4) = 0.2(4-0) = 0.8$

7. $P(2 < x < 3) = 0.2(3-2) = 0.2$

8. $P(2.5 < x < 4.5) = 0.2(4.5-2.5) = 0.4$

9. $P(z < 0.44) = 0.6700$

10. $P(z > -1.04) = 0.8508$

11. $P(-0.84 < z < 1.28) = P(z < 1.28) - P(z < -0.84) = 0.8997 - 0.2005 = 0.6992$ (Tech: 0.6993)

12. $P(-1.07 < z < 0.67) = P(z < 0.67) - P(z < -1.07) = 0.7486 - 0.1423 = 0.6063$

13. $z = 1.23$

14. $z = -0.51$

15. $z = -1.45$

16. $z = 0.82$

17. $P(z < -1.23) = 0.1093$

18. $P(z < -1.96) = 0.0250$

19. $P(z < 1.28) = 0.8997$

20. $P(z < 2.56) = 0.9948$

21. $P(z > 0.25) = 1 - P(z < 0.25) = 1 - 0.5987 = 0.4013$

22. $P(z > 0.18) = 1 - P(z < 0.18) = 1 - 0.5704 = 0.4286$

23. $P(z > -2.00) = 1 - P(z < -2.00) = 1 - 0.0228 = 0.9772$

24. $P(z > -3.05) = 1 - P(z < -3.05) = 1 - 0.0011 = 0.9989$

25. $P(2.00 < z < 3.00) = P(z < 3.00) - P(z < 2.00) = 0.9986 - 0.9772 = 0.0214$ (Tech: 0.0215)

26. $P(1.50 < z < 2.50) = P(z < 2.50) - P(z < 1.50) = 0.9938 - 0.9332 = 0.0606$

27. $P(-2.55 < z < -2.00) = P(z < -2.00) - P(z < -2.55) = 0.0228 - 0.0054 = 0.0174$

28. $P(-2.75 < z < -0.75) = P(z < -0.75) - P(z < -2.75) = 0.2266 - 0.0030 = 0.2236$

29. $P(-2.00 < z < 2.00) = P(z < 2.00) - P(z < -2.00) = 0.0228 - 0.9772 = 0.9544$ (Tech: 0.9545)

30. $P(-3.00 < z < 3.00) = P(z < 3.00) - P(z < -3.00) = 0.9987 - 0.0013 = 0.9974$ (Tech: 0.9973)

31. $P(-1.00 < z < 5.00) = P(z < 5.00) - P(z < -1.00) = 0.9999 - 0.1587 = 0.8412$ (Tech: 0.8413)

32. $P(-4.27 < z < 2.34) = P(z < 2.34) - P(z < -4.27) = 0.9904 - 0.0001 = 0.9903$

33. $P(z < 4.55) = 0.9999$ (Tech: 0.999997)

34. $P(z > -3.75) = 0.9999$

35. $P(z>0)=0.5000$

36. $P(z<0)=0.5000$

37. $P_{99}=2.33$

38. $P_{10}=-1.28$

39. $P_{2.0}=-2.05$ and $P_{98.0}=2.05$

40. $P_{3.0}=-1.88$ and $P_{97.0}=1.88$

41. $z_{0.10}=1.28$

42. $z_{0.02}=2.05$

43. $z_{0.04}=1.75$

44. $z_{0.15}=1.04$

45. $P(-1<z<1)=P(z<1)-P(z<-1)=0.8413-0.1587=0.6826=68.26\%$ (Tech: 68.27%)

46. $P(-2<z<2)=P(z<2)-P(z<-2)=0.9772-0.0228=0.9544=95.44\%$ (Tech: 95.45%)

47. $P(-3<z<3)=P(z<3)-P(z<-3)=0.9987-0.0013=0.9974=99.74\%$ (Tech: 99.73%)

48. $P(-3.5<z<3.5)=P(z<3.5)-P(z<-3.5)=0.9999-0.0001=0.9998=99.98\%$ (Tech: 99.95%)

49.a. $P(z>2)=1-P(z<2)=1-0.9772=0.0228=2.28\%$

b. $P(z<-2)=0.0228=2.28\%$

c. $P(-2<z<2)=P(z<2)-P(z<-2)=0.9772-0.0228=0.9544=95.44\%$ (Tech: 95.45%)

50. a. $\mu=2.5$ min. and $\sigma=5/\sqrt{12}=1.4$ min

b. The probability is $1/\sqrt{3}$ or 0.5774, and it is very different from the probability of 0.6827 that would be obtained by incorrectly using the standard normal distribution. The distribution does affect the results very much.

Section 6-2: Real Applications of Normal Distributions

1. a. $\mu=0$ and $\sigma=1$

b. The z scores are numbers without units of measurements.

2. a. The area equals the maximum probability value of 1.

b. The median is the middle value and for normally distributed scores that is also the mean, which is 3152.0 g.

c. The mode is also 3152.0 g.

d. The variance is the square of the standard deviation which is 480,803.6 g^2.

3. The standard normal distribution has a mean of 0 and a standard deviation of 1, but a nonstandard normal distribution has a different value for one or both of those parameters.

4. No, randomly generated digits have a uniform distribution, but not a normal distribution. The probability of a digit less than 3 is $3/10=0.3$.

5. $z_{x=118}=\dfrac{118-100}{15}=1.2$; which has an area of 0.8849 to the left.

6. $z_{x=90}=\dfrac{91-100}{15}=-0.6$; which has an area of 0.7257 to the right.

7. $z_{x=133}=\dfrac{133-100}{15}=2.2$; which has an area of 0.9861 to the left. $z_{x=79}=\dfrac{110-100}{15}=-1.4$; which has an area of 0.0808 to the left. The area between the two scores is $0.9861-0.0808=0.9053$.

8. $z_{x=124}=\dfrac{124-100}{15}=1.6$; which has an area of 0.9452 to the left. $z_{x=112}=\dfrac{112-100}{15}=0.8$; which has an area of 0.7881 to the left. The area between the two scores is $0.9452-0.7881=0.1571$.

9. $z=2.44$; so $x=2.44\cdot 15+100=136$

10. $z=1$; so $x=1\cdot 15+100=115$

11. $z=-2.07$; so $x=-2.07\cdot 15+100=69$

12. $z=1.33$; so $x=1.33\cdot 15+100=120$

13. $z_{x=21.0} = \frac{21.0-23.5}{1.1} = -2.27$; which has an area of 0.0115 to the left (Table: 0.0116).

14. $z_{x=24.0} = \frac{24.0-22.7}{1.0} = 1.30$; which has an area of 0.0968 to the right.

15. $z_{x=22.0} = \frac{22.0-22.7}{1.0} = -0.70$; which has an area of 0.2420 to the left and $z_{x=24.0} = \frac{24.0-22.7}{1.0} = 1.30$; which has an area of 0.9032 to the left. The area between the two scores is $0.9032-0.2420 = 0.6612$.

16. $z_{x=22.0} = \frac{22.0-23.5}{1.1} = -1.36$; which has an area of 0.0869 to the left and $z_{x=22.0} = \frac{24.0-23.5}{1.1} = 0.45$; which has an area of 0.6736 to the left. The area between the two scores is $0.6736-0.0869 = 0.5867$ (Tech: 0.5889).

17. $P_{90} = 1.28$; so the length is $1.28 \cdot 1.1 + 23.5 = 24.9$ in.

18. $Q_1 = -0.67$; so the length is $-0.67 \cdot 1. + 22.7 = 22.0$ in.

19. $P_1 = -2.33$; so the lower length is $-2.33 \cdot 1.1 + 23.5 = 20.9$ in. and $P_{99} = 2.33$; so the upper length is $2.33 \cdot 1.1 + 23.5 = 26.1$ in., so a length of 26 in. is not significantly high.

20. $P_{2.5} = -1.96$; so the lower length is $x = -1.96 \cdot 1.0 + 22.7 = 20.7$ in. and $P_{97.5} = 1.96$; so the upper length is $1.96 \cdot 1.0 + 22.7 = 24.7$ in., so a length of 20 in. is significantly low.

21. a. $z_{x=78} = \frac{78-63.7}{2.9} = 4.93$; which has an area of 0.9999 to the left. $z_{x=62} = \frac{62-63.7}{2.9} = -0.59$; which has an area of 0.2776 to the left. Therefore, the percentage of qualified women is $0.9999-0.2776 = 0.7223$, or 72.23% (Tech 72.11%). Yes, about 28% of women are not qualified because of their heights.
    b. The z score with 3% to the left is –1.88, which corresponds to a height of $-1.88 \cdot 2.9 + 63.7 = 58.2$ in. The z score with 3% to the right is 1.88 which corresponds to a height of $1.88 \cdot 2.9 + 63.7 = 69.2$ in.

22. a. $z_{x=77} = \frac{78-68.6}{2.8} = 3.00$; which has an area of 0.9987 to the left. $z_{x=64} = \frac{64-68.6}{2.8} = -1.64$; which has an area of 0.0505 to the left. Therefore, the percentage of qualified women is $0.9987-0.0505 = 0.9482$, or 94.82% (Tech 94.84%).
    b. The z score with 2.5% to the left is –1.96, which corresponds to a height of $-1.96 \cdot 2.8 + 68.6 = 63.1$ in. The z score with 2.5% to the right is 1.96 which corresponds to a height of $1.96 \cdot 2.8 + 68.6 = 74.1$ in.

23. a. The z score for men for the minimum height is $\frac{56-68.6}{2.8} = -4.5$ and the z score for men for the maximum height is $\frac{56-69.5}{2.4} = -2.36$. The area between the z scores is $0.0091-0.0001 = 0.0090$ or 0.90% (Tech: 0.92%). Because so few men can meet the height requirement, it is likely that most Mickey Mouse characters are women.
    b. The z score with 5% of men to the left is –1.65, which corresponds to a height of $-1.65 \cdot 2.8 + 68.6 = 64.0$ in. and the z score with 50% of men to the right is 0 which corresponds to a height of $0 \cdot 2.8 + 68.6 = 68.6$ in.

24. a. The z score for the minimum height is $\frac{51.6-68.6}{2.8} = -6.07$; which has an area of 0.0001, or 0.01% to the left meaning that practically no man can fit without bending (Tech: 0.00%).
    b. The door design is very inadequate, but the jet is relatively small and seats only six people. A much higher door would require such major changes in the design and cost of the jet, that the greater height is not practical.
    c. The z score for 40% to the left is –0.25, which corresponds to a height of $-0.25 \cdot 2.8 + 68.6 = 67.9$ in.

25. The z score for eye contact of 230.0 seconds is $\frac{230.0-184.0}{55.0}=0.84$; so 0.2005, or 20.05% (Tech: 20.15%) of people would make eye contact longer than 230 seconds. No, the proportion of schizophrenics is not at all likely to be as high as 0.2005, or about 20%.

26. a. The z score that has 95% of the area to the left is 1.67 which corresponds to a height of $1.67\cdot 1.2+21.4 = 23.4$ in. If there is clearance for 95% of males, there will certainly be clearance for all women in the bottom 5%

 b. The men's z score is $\frac{23.5-21.4}{1.2}=1.75$; which has an area to the left of 0.9599, or 95.99%. The women's z score is $\frac{23.5-19.6}{1.1}=3.55$; which has an area to the left of 0.9999, or 99.99% (Tech 99.98%). The table will fit almost everyone except about 4% of the men with the largest sitting knee heights.

27. The z score for the minimum weight is $\frac{140.0-171.1}{46.1}=-0.67$ and the z score for the maximum weight is $\frac{211.0-171.1}{46.1}=0.87$. The area between the z scores is $0.8078-0.2514=0.5564$, or 55.64% (Tech: 55.67%). Yes, about 44% of women were excluded.

28. a. The minimum weight has a z score of $\frac{5.60-5.67}{0.06}=-1.17$ g and the maximum weight has a z score of $\frac{5.7-5.67}{0.06}=1.17$ g. Therefore, the percentage of legal quarters rejected is $1-(0.8790-0.1210) = 0.242$, or 24.20% (Tech: 24.33%). That percentage is too high because roughly one-fourth of all legitimate quarters would be rejected.

 b. The z scores for the upper and lower 2.5% are 2 and –2 respectively. Therefore quarters with weights between $-2\cdot 0.06+5.67=5.55$ g and $2\cdot 0.06+5.67=5.79$ g should be accepted.

29. a. The z score for the minimum weight is $\frac{2495.0-3152.0}{693.4}=-0.95$; which has an area to the left of 0.1711 (Tech: 0.1717).

 b. The z score for 5% to the left is –1.645, which corresponds to a weight of $-1.645\cdot 693.4+3152.0 = 2011.4$ g (Tech: 2011.5 g).

 c. Birth weights are significantly low if they are 2011.4 g or less, and they are "low birth weights" if they are 2495 g or less. Birth weights between 2011.4 g and 2495 g are "low birth weights" but they are not significantly low.

30. a. The z score for a temperature of 100.6 is $\frac{100.6-98.2}{0.62}=3.87$; which corresponds to a an area of $1-0.9999=0.0001=0.01\%$ (Tech: 0.02%) to the right, which suggests a cutoff of 100.4°F is appropriate.

 b. The z score for a probability of 2% is 2.05 which corresponds to a temperature of $2.05\cdot 0.62+98.2 = 99.47$°F.

31. a. The z score for a 308 day pregnancy is $\frac{308-268}{15}=2.67$; which corresponds to a probability of 0.0038 or 0.38%. Either a very rare event occurred or the husband is not the father.

 b. The z score corresponding to 3% is –1.87 which corresponds to a pregnancy of $-1.87\cdot 15+268 = 240$ days.

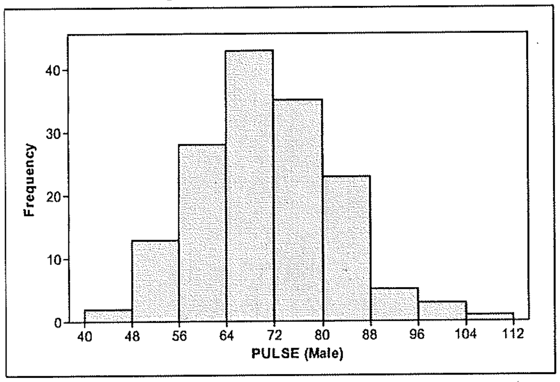
32. a. The z score for 174 lb. is $\frac{174-188.6}{38.9}=-0.38$; which has an area to the left of 0.3520 (Tech: 0.3537).

b. $3500/140=25$ people

c. $3500/188.6=18.6$, so 18 people

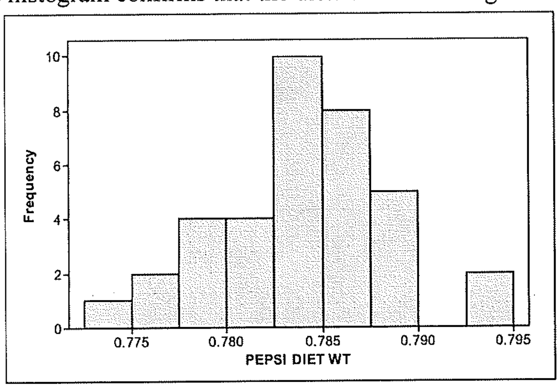
d. The mean weight is increasing over time, so safety limits must be periodically updated to avoid an unsafe condition.

33. a. The mean is 69.5817 (69.6 rounded) beats per minute and the standard deviation is 11.3315 (11.3 rounded) beats per minute. The histogram for the data confirms that the distribution is roughly normal.



b. The z score for the bottom 2.5% is –1.95, which corresponds to a pulse of $-1.95\cdot11.3315+69.5817$ $=47.4$ beats per minute. The z score for the top 2.5% is 1.95, which corresponds to a pulse of $1.95\cdot11.3315+69.5817=91.8$ beats per minute.

34. a. The mean is 0.8564900 (0.8565 rounded) grams and the standard deviation is 0.0517942 (0.0518 rounded) grams. The histogram confirms that the distribution of weights is roughly normal.



b. The z score for the bottom 0.5% is –2.59, which has a corresponding weight of $-2.59\cdot0.0517942$ $+0.8564900=0.7231$ grams. The z score for the top 0.5% is 2.59, which has a corresponding weight of $2.59\cdot0.0517942+0.8564900=0.9899$ grams.

35. a. The new mean is equal to the old mean plus the additional points which is $60+15=75$. The standard deviation is unchanged at 12 (since the same amount was added to each score.)

b. No, the conversion should also account for variation.

c. The z score for the bottom 70% is 0.52 which has a corresponding score of $60+0.52\cdot12=66.2$, (Tech: 66.3) and the z score for the top 10% is 1.28 which has a corresponding score of $60+1.28\cdot12=75.4$.

d. Using a scheme like the one in part (c), because variation is included in the curving process.

36. The z score for Q_1 is –0.67, and the z score for Q_3 is 0.67. The IQR is $0.67-(-0.67)=1.34$.

$1.5\cdot IQR=2.01$, so $Q_1-1.5\cdot IQR=-0.67-2.01=-2.68$ and $Q_3+1.5\cdot IQR=0.67+2.01=2.68$.

The percentage to the left of –2.68 is 0.0037 and the percentage to the right of 2.68 is 0.0037. Therefore, the percentage of an outlier is 0.0074 (Tech: 0.0070).

Section 6-3: Sampling Distributions and Estimators

1. a. In the long run, the sample proportions will have a mean of 0.512.
   b. The sample proportions will tend to have a distribution that is approximately normal.
2. a. without replacement
   b. (1) When selecting a relatively small sample from a large population, it makes no significant difference whether we sample with replacement or without replacement.
   (2) Sampling with replacement results in independent events that are unaffected by previous outcomes, and independent events are easier to analyze and they result in simpler calculations and formulas.
3. sample mean, sample variance, sample proportion
4. No, the data set is only one sample, but the sampling distribution of the mean is the distribution of the means from all samples, not the one sample mean obtained from the one sample in Data Set 4.
5. No, the sample is not a simple random sample from the population of all births worldwide. The proportion of boys born in China is substantially higher than in other countries.
6. a. The distribution of the sample means is approximately normal.
   b. The sample means target the mean annual income of all college presidents.
7. a. The population mean is $\mu = \dfrac{4+5+9}{3} = 6$, and the population variance is

   $$\sigma^2 = \frac{(4-6)^2 + (5-6)^2 + (9-6)^2}{3} = 4.7.$$

   b. The possible sample of size 2 are {(4, 4), (4, 5), (4, 9), (5, 4), (5, 5), (5, 9), (9, 4), (9, 5), (9, 9)} which have the following variances {0, 0.5, 12.5, 0.5, 0, 8, 12.5, 8, 0} respectively.

Sample Variance	Probability
0.0	3/9
0.5	2/9
8	2/9
12.5	2/9

   c. The sample variance's mean is $\dfrac{3\cdot 0 + 2\cdot 0.5 + 2\cdot 8 + 2\cdot 12.5}{9} = 4.7.$

   d. Yes. The mean of the sampling distribution of the sample variances (4.7) is equal to the value of the population variance (4.7) so the sample variances target the value of the population variance.
8. a. The population standard deviation (using the result from the previous problem) is $\sigma = \sqrt{4.7} = 2.168$.
   b. By taking the square root of the sample variances from the previous problem we get

Sample Standard Deviation	Probability
0.000	3/9
0.707	2/9
2.828	2/9
3.536	2/9

   c. The mean of the sample standard deviations is $\dfrac{3\cdot 0 + 2\cdot 0.707 + 2\cdot 2.828 + 2\cdot 3.536}{9} = 1.571.$

   d. No, the mean of the sampling distribution of the sample standard deviations is 1.571, and it is not equal to the value of the population standard deviation (2.160), so the sample standard deviations do not target the value of the population standard deviation.
9. a. The population median is 5.
   b. The possible sample of size 2 are {(4, 4), (4, 5), (4, 9), (5, 4), (5, 5), (5, 9), (9, 4), (9, 5), (9, 9)}, which have the following medians {4, 4.5, 6.5, 4.5, 5, 7, 6.5, 7, 9} with the following associated probabilities.

9. (continued)

Sample Median	Probability
4	1/9
4.5	2/9
5	1/9
6.5	2/9
7	2/9
9	1/9

c. The mean of the sampling distribution of the sampling median is $\frac{4+2\cdot 4.5+5+2\cdot 6.52\cdot 7+9}{9}=6.0.$

d. No, the mean of the sampling distribution of the sample medians is 6.0, and it is not equal to the value of the population median of 5.0, so the sample medians do not target the value of the population median.

10. a. The proportion of odd numbers is 2/3, or 0.7 (there are two odd numbers from the population of 4, 5, and 9).

b. The possible sample of size 2 are {(4, 4), (4, 5), (4, 9), (5, 4), (5, 5), (5, 9), (9, 4), (9, 5), (9, 9)}, which have the following proportion of odd numbers {0, 0.5, 0.5, 0.5, 1, 1, 0.5, 1, 1.

Sample Proportion	Probability
0.0	1/9
0.5	4/9
1.0	4/9

c. The mean of the sampling distribution of sample proportions is $\frac{0+4\cdot 0.5+4\cdot 1}{9}=\frac{2}{3}$, or 0.7.

d. Yes. The mean of the sampling distribution of the sample proportion of odd numbers is 2/3, and it is equal to the value of the population proportion of odd numbers of 2/3, so the sample proportions target the value of the population proportion

11. a. The possible samples of size 2 are {(34, 34), (34, 36), (34, 41), (34, 51), (36, 34), (36, 36), (36, 41), (36, 51), (41, 34), (41, 36), (41, 41), (41, 51), (51, 34), (51, 36), (51, 41), (51, 51)}, which have the following ranges and associated probabilities.

Sample Range	Probability
34	1/16
35	2/16
36	1/16
37.5	2/16
38.5	2/16
41	2/16
42.5	2/16
43.5	1/16
46	2/16
51	1/16

b. The mean of the population is $\frac{34+36+41+51}{4}=40.5$ and the mean of the sample means is

$\frac{34+2\cdot 35+36+2\cdot 37.5+2\cdot 38.5+2\cdot 41+2\cdot 42.5+43.5+2\cdot 46+51}{16}=40.5$ as well.

c. The sample means target the population mean. Sample means make good estimators of population means because they target the value of the population mean instead of systematically underestimating or overestimating it.

12. a. The possible samples of size 2 are {(34, 34), (34, 36), (34, 41), (34, 51), (36, 34), (36, 36), (36, 41), (36, 51), (41, 34), (41, 36), (41, 41), (41, 51), (51, 34), (51, 36), (51, 41), (51, 51)}, which have the following medians and associated probabilities.

Sample Median	Probability
34	1/16
35	2/16
36	1/16
37.5	2/16
38.5	2/16
41	2/16
42.5	2/16
43.5	1/16
46	2/16
51	1/16

b. The median of the population is $\frac{36+41}{2} = 38.5$, but the median of the sample medians is $\frac{38.5+41.0}{2}$ $= 40.5$. The two values are not equal.

c. The sample medians do not target the population median of 38.5, so the sample medians do not make good estimators of the population medians.

13. a. The possible samples of size 2 are {(34, 34), (34, 36), (34, 41), (34, 51), (36, 34), (36, 36), (36, 41), (36, 51), (41, 34), (41, 36), (41, 41), (41, 51), (51, 34), (51, 36), (51, 41), (51, 51)}, which have the following ranges and associated probabilities.

Sample Range	Probability
0	4/16
2	2/16
5	2/16
7	2/16
10	2/16
15	2/16
17	2/16

b. The range of the population is $51-34=17.0$, but the mean of the sample ranges is $\frac{4\cdot 0+2\cdot 2+2\cdot 5+2\cdot 7+2\cdot 10+2\cdot 15+2\cdot 17}{16} = 7.0$. The values are not equal.

c. The sample ranges do not target the population range of 17, so sample ranges do not make good estimators of the population range.

14. a. The possible samples of size 2 are {(34, 34), (34, 36), (34, 41), (34, 51), (36, 34), (36, 36), (36, 41), (36, 51), (41, 34), (41, 36), (41, 41), (41, 51), (51, 34), (51, 36), (51, 41), (51, 51)}, which have the following variances and associated probabilities.

Sample Variance (s^2)	Probability
0.0	4/16
2.0	2/16
12.5	2/16
24.5	2/16
50.0	2/16
112.5	2/16
144.5	2/16

14. (continued)

The mean of the sample variances is $\frac{4\cdot0+2\cdot2+2\cdot12.5+2\cdot24.5+2\cdot50+2\cdot112.5+2\cdot144.5}{16}=43.25.$

The two values are equal.

c. The sample variances do target the population variance of 43.25, so sample variances do make good estimators of the population variance.

15. The possible birth samples are {(b, b), (b, g), (g, b), (g, g)}.

Proportion of Girls	Probability
0	0.25
1 / 2	0.5
2/2	0.25

Yes. The proportion of girls in 2 births is 0.5, and the mean of the sample proportions is 0.5. The result suggests that a sample proportion is an unbiased estimator of the population proportion.

16. The possible birth samples are {bbb, bbg, bgb, gbb, ggg, ggb, gbg, bgg}.

Proportion of Girls	Probability
0	1/3
1/3	3/8
2/3	3/8
3/3	1/8

Yes. The proportion of girls in 3 births is 0.5 and the mean of the sample proportions is 0.5. The result suggests that a sample proportion is an unbiased estimator of the population proportion.

17. The possibilities are: both questions incorrect, one question correct (two choices), both questions correct.

a.

Proportion Correct	Probability
$\frac{0}{2}=0$	$\frac{4}{5}\cdot\frac{4}{5}=\frac{16}{25}$
$\frac{1}{2}=0.5$	$2\cdot\left(\frac{1}{5}\cdot\frac{4}{5}\right)=\frac{8}{25}$
$\frac{2}{2}=1$	$\left(\frac{1}{5}\cdot\frac{1}{5}\right)=\frac{1}{25}$

b. The mean is $\frac{16\cdot0+8\cdot0.5+1\cdot1}{25}=0.2.$

c. Yes, the sampling distribution of the sample proportions has a mean of 0.2 and the population proportion is also 0.2 (because there is 1 correct answer among 5 choices). Yes, the mean of the sampling distribution of the sample proportions is always equal to the population proportion.

18. a. The proportions of 0, 0.5, and 1 have the following probabilities.

Proportion with Yellow Pods	Probability
0	1/25
0.5	8/25
1	16/25

b. The mean is $\frac{1\cdot0+8\cdot0.5+16\cdot1}{25}=0.8.$

c. Yes, the population proportion is 0.8 and the mean of the sampling proportions is also 0.8. The mean of the sampling distribution of proportions is always equal to the population proportion.

19. The formula yields $P(0)=\dfrac{1}{2(2-2\cdot 0)!(2\cdot 0)!}=0.25$, $P(0.5)=\dfrac{1}{2(2-2\cdot 0.5)!(2\cdot 0.5)!}=0.5$, and $P(1)=\dfrac{1}{2(2-2\cdot 1)!(2\cdot 1)!}=0.25$, which describes the sampling distribution of the sample proportions. The formula is just a different way of presenting the same information in the table that describes the sampling distribution.
20. Sample values of the mean absolute deviation (MAD) do not usually target the value of the population MAD, so a MAD statistic is not good for estimating a population MAD. If the population of {4, 5, 9} from Example 5 is used, the sample MAD values of 0, 0.5, 2, and 2.5 have corresponding probabilities of 3/9, 2/9, 2/9, and 2/9. For these values, the population MAD is 2, but the sample MAD values have a mean of 1.1, so the mean of the sample MAD values is not equal to the population MAD.

Section 6-4: The Central Limit Theorem

1. Because the sample size, *n*, is greater than 30, the sampling distribution of the mean ages can be approximated by a normal distribution with mean μ and standard deviation $\sigma/\sqrt{40}$.
2. No, because the original population is normally distributed, the sample means will be normally distributed for any sample size, not just for $n>30$.
3. $\mu_{\bar{x}}$ represents the mean of all sample means and $\sigma_{\bar{x}}$ which represents the standard deviation of all sample means. For samples of 64 IQ scores, $\mu_{\bar{x}}=100$, and $\sigma_{\bar{x}}=15/\sqrt{64}=1.875$.
4. No. The sample of annual incomes will tend to have a distribution that is skewed to the right, no matter how large the sample is. If we compute the sample mean, we can consider that value to be one value in a normally distributed population.
5. a. $z_{x=80}=\dfrac{80.0-74.0}{12.5}=0.48$; which has a probability of 0.6844 to the left.

   b. $z_{x=80}=\dfrac{80.0-74.0}{12.5/\sqrt{16}}=1.92$; which has a probability of 0.9726 to the left (Tech: 0.8889).

   c. Because the original population has a normal distribution, the distribution of sample means is a normal distribution for any sample size.
6. a. $z_{x=70}=\dfrac{70.0-74.0}{12.5}=-0.32$; which has a probability of $1-0.3745=0.6255$ to the right.

   b. $z_{x=70}=\dfrac{70.0-74.0}{12.5/\sqrt{25}}=-1.6$; which has a probability of $1-0.0548=0.9452$ to the right.

   c. Because the original population has a normal distribution, the distribution of sample means is a normal distribution for any sample size.
7. a. $z_{x=72}=\dfrac{72.0-74.0}{12.5}=-0.16$ and $z_{x=76}=\dfrac{76.0-74.0}{12.5}=0.16$; which have a probability of $0.5636-0.4364=0.1272$ between them (Tech: 0.1271).

   b. $z_{x=72}=\dfrac{72.0-74.0}{12.5/\sqrt{4}}=-0.32$ and $z_{x=76}=\dfrac{76.0-74.0}{12.5/\sqrt{2}}=0.32$; which have a probability of $0.6255-0.3745=0.2510$ between them.

   c. Because the original population has a normal distribution, the distribution of sample means is normal for any sample size.
8. a. $z_{x=90}=\dfrac{90.0-74.0}{12.5}=1.28$ and $z_{x=78}=\dfrac{78.0-74.0}{12.5}=0.32$; which have a probability of $0.8997-0.6255=0.2742$ between them.

8. (continued)

b. $z_{x=90} = \dfrac{90.0-74.0}{12.5/\sqrt{16}} = 5.12$ and $z_{x=78} = \dfrac{78.0-74.0}{12.5/\sqrt{16}} = 1.28$; which have a probability of $0.9999-0.8997=0.1002$ between them (Tech: 0.0.1003).

c. Because the original population has a normal distribution, the distribution of sample means is normal for any sample size.

9. $z_{x=185} = \dfrac{185-189}{39/\sqrt{27}} = -0.53$; which has a probability of $1-0.2981=0.7019$ (Tech: 0.7030) to the right. The elevator does not appear to be safe because there is about a 70% chance that it will be overloaded whenever it is carrying 27 adult males.

10. $z_{x=185} = \dfrac{185-174}{39/\sqrt{27}} = 1.47$; which has a probability of $1-0.9292=0.0708$ (Tech: 0.0714) to the right. The elevator appears to be relatively safe because there is about a 7% chance of overloading when filled with 27 adult males. Using the outdated mean that is too low has the effect of making the elevator appear to be much safer than it really is.

11. a. The z score for the top 2% is 2.05, which corresponds to an IQ score of $2.05 \cdot 15 + 100 = 131$.

b. $z_{x=131} = \dfrac{131-100}{15/\sqrt{4}} = 4.13$; which has a probability of $1-0.9999=0.0001$ (Tech: 0.0000179) to the right.

c. No, it is possible that the 4 subjects have a mean of 132 while some of them have scores below the Mensa requirement of 131.

12. a. $z_{x=22} = \dfrac{22-18.2}{1} = 3.8$; which has a probability of 0.9999. So the percentage is 99.99%

b. $z_{x=18.5} = \dfrac{18.5-18.2}{1/\sqrt{36}} = 1.8$ which has a probability of 0.9641. No, when considering the diameters of manholes, we should use a design based on individual men, not samples of 36 men.

13. a. The mean weight of passengers is $3500/25 = 140$ lb.

b. $z_{x=140} = \dfrac{140-189}{39/\sqrt{25}} = -6.28$; which has a probability of $1-0.0001=0.9999$ (Tech: 0.999999998) to the right.

c. $z_{x=175} = \dfrac{175-189}{39/\sqrt{20}} = -1.61$; which has a probability of $1-0.0537=0.9463$ (Tech: 0.9458) to the right.

d. The new capacity of 20 passengers does not appear to be safe enough because the probability of overloading is too high.

14. a. $z_{x=5.790} = \dfrac{5.790-5.670}{0.062} = 1.94$ and $z_{x=5.550} = \dfrac{5.550-5.670}{0.062} = -1.94$; which have a probability of $0.9738-0.0262=0.9476$ (Tech: 0.9471) between them.

b. $z_{x=5.790} = \dfrac{5.790-5.670}{0.062/\sqrt{4}} = 3.87$ and $z_{x=5.550} = \dfrac{5.550-5.670}{0.062/\sqrt{4}} = -3.87$; which have a probability of $0.9999-0.0001=0.9998$ (Tech: 0.9999) between them.

c. Part (a) because individual rejected quarters could result in lost sales. The quarters are inserted one at a time, not in groups of 4.

15\. a. $z_{x=17} = \frac{17.0-14.4}{1.0} = 2.60$; which has a probability of $1-0.9953=0.0047$ to the right.

b. $z_{x=17} = \frac{17.0-14.4}{1.0/\sqrt{122}} = 28.7$; which has a probability of $1-0.9999=0.0001$ (Tech: 0.0000) to the right.

c. The result from part (a) is relevant because the seats are occupied by individuals.

16\. a. $z_{x=12.19} = \frac{12.19-12.00}{0.11} = 1.73$; which has a probability of $1-0.9582=0.0418$ (Tech: 0.0421) to the right.

b. $z_{x=12.19} = \frac{12.19-12.00}{0.11/\sqrt{36}} = 10.36$; which has a probability of $1-0.9999=0.0001$ (Tech: 0.0000) to the right.

c. It appears that the cans are filled with an amount greater than 12.00 oz. Instead of being cheated, consumers are getting a little more than 12.00 oz.

17\. a. $z_{x=211} = \frac{211-171}{46} = 0.87$ and $z_{x=140} = \frac{140-171}{46} = -0.67$; which have a probability of $0.8078-0.2514 = 0.5564$ between them (Tech: 0.5575).

b. $z_{x=211} = \frac{211-171}{46/\sqrt{25}} = 4.35$ and $z_{x=140} = \frac{140-171}{46/\sqrt{25}} = -3.37$; which have a probability of $0.9999-0.0004 = 0.9995$ between them (Tech: 0.9996).

c. Part (a) because the ejection seats will be occupied by individual women, not groups of women.

18\. a. $z_{x=140} = \frac{140-189}{39/\sqrt{50}} = -8.88$; which has a probability of $1-0.0001=0.9999$ to the right (Tech: 1.0000 when rounded to four decimal places).

b. $z_{x=174} = \frac{174-189}{39/\sqrt{14}} = -1.44$; which has a probability of $1-0.0449=0.9251$ to the right (Tech: 0.9249). Because there is a high probability of overloading, the new ratings do not appear to be safe when the boat is loaded with 14 male passengers.

19\. a. $z_{x=72} = \frac{72-68.6}{2.8} = 1.21$; which has a probability of 0.8869 (Tech: 0.8877).

b. $z_{x=72} = \frac{72-68.6}{2.8/\sqrt{100}} = 12.1$; which has a probability of 0.9999. (Tech: 1.0000 when rounded to four decimal places.)

c. The probability of Part (a) is more relevant because it shows that about 89% of male passengers will not need to bend. The result from part (b) gives us useful information about the comfort and safety of individual male passengers.

d. Because men are generally taller than women, a design that accommodates a suitable proportion of men will necessarily accommodate a greater proportion of women.

20\. $z_{x=167.6} = \frac{167.6-189.0}{39/\sqrt{37}} = -3.34$; which has a probability of $1-0.0004=0.9996$ to the right. There is a 0.9996 probability that the aircraft is overloaded. Because that probability is so high, the pilot should take action, such as removing excess fuel and/or requiring that some passengers disembark and take a later flight.

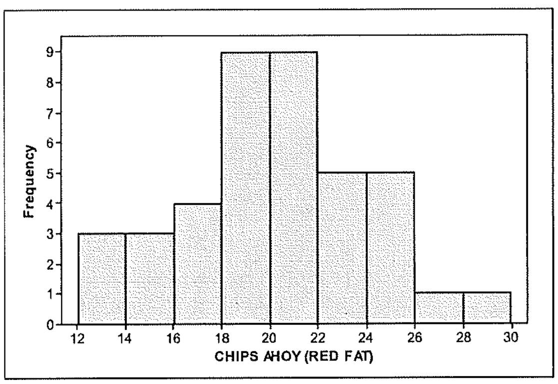
21\. a. Yes, the sampling is without replacement and the sample size of 50 is greater than 5% of the finite population size of 275. $\sigma_{\bar{x}} = \frac{16}{\sqrt{50}}\sqrt{\frac{275-50}{275-1}} = 2.0504584$

21. (continued)

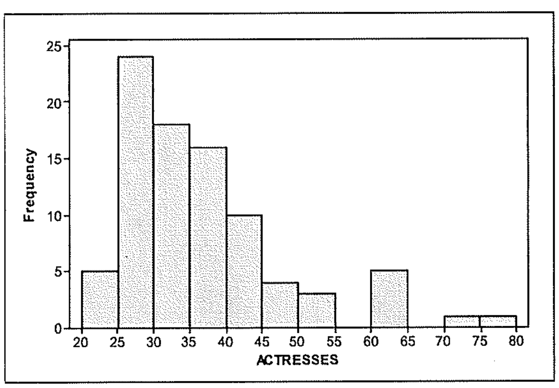
b. $z_{x=105} = \dfrac{105-95.5}{2.0504584} = 4.63$ and $z_{x=95.5} = \dfrac{95-95.5}{2.0504584} = -0.24$; which have a probability of $1-0.4053 = 0.5947$ between them (Tech: 0.5963).

Section 6-5: Assessing Normality

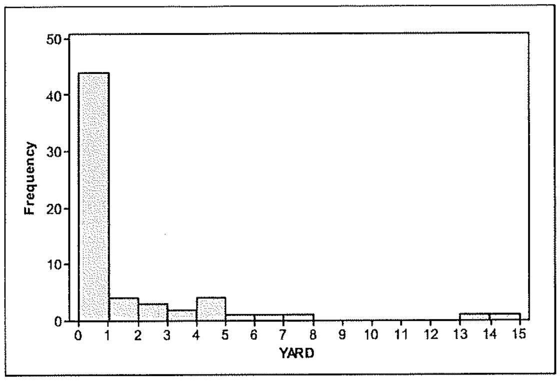
1. The histogram should be approximately bell-shaped, and the normal quantile plot should have points that approximate a straight-line pattern.
2. Either the points are not reasonably close to a straight-line pattern, or there is some systematic pattern that is not a straight-line pattern.
3. We must verify that the sample is from a population having a normal distribution. We can check for normality using a histogram, identifying the number of outliers, and constructing a normal quantile plot.
4. Because the histogram is roughly bell-shaped, we can conclude that the data are from a population having a normal distribution.
5. Normal, the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern.
6. Normal, the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern.
7. Not normal, the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern.
8. Not normal, the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern.
9. normal



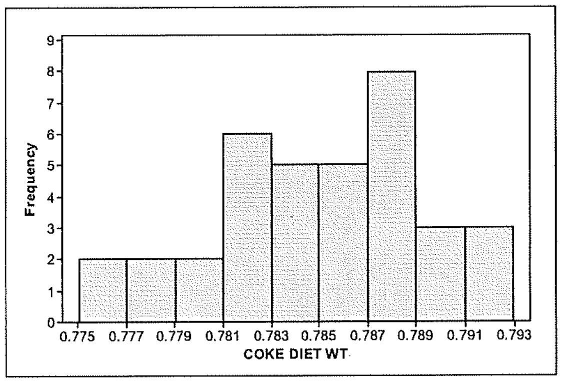
10 not normal



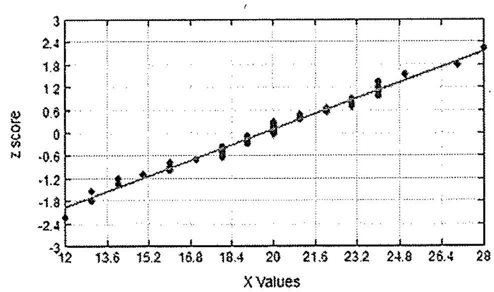
11. not normal



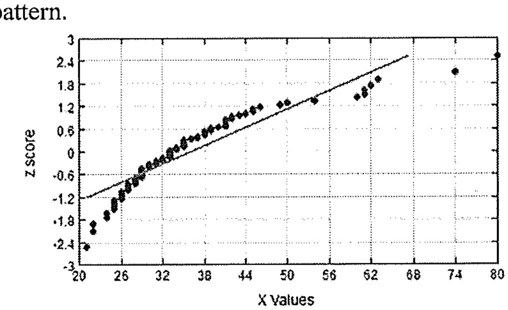
12. normal



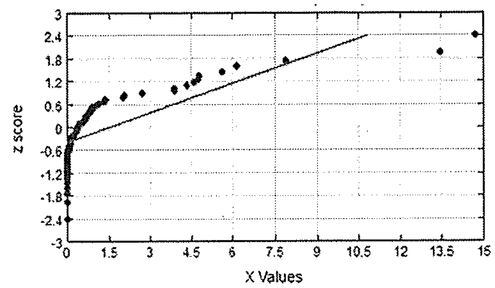
13. Normal, the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern.



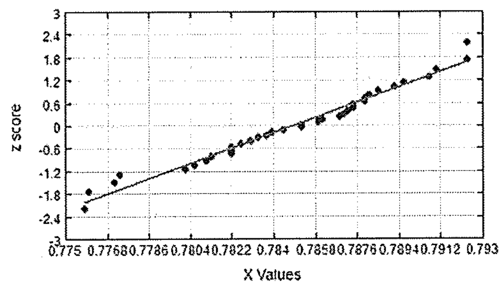
14. Not normal, the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern.



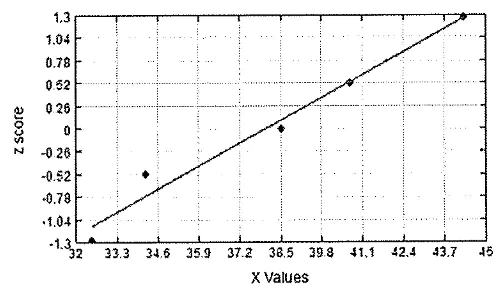
15. Not normal, the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern.



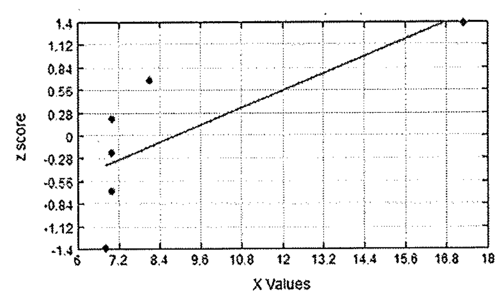
16. Normal, the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern.



17. Normal, the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern. The points have coordinates $(32.5,-1.28)$, $(34.2,-0.52)$, $(38.5,0)$, $(40.7,0.52)$, and $(44.3,1.28)$.

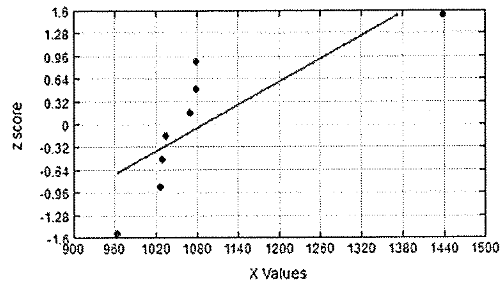


18. Not normal, the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern. The points have coordinates $(6.8,-1.38)$, $(7.0,-0.67)$, $(7.0,-0.21)$, $(7.0,0.21)$, $(8.1,0.67)$, and $(17.3,1.38)$.

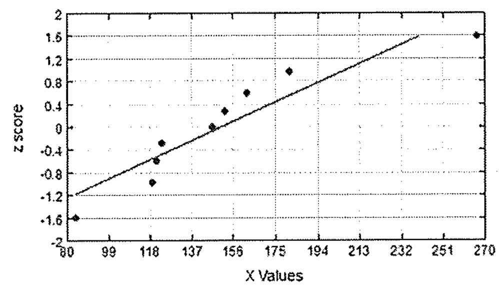


19. Not normal, the points are not reasonably close to a straight-line pattern and there appears to be a pattern that is not a straight-line pattern. The points have coordinates $(963,-1.53)$, $(1027,-0.89)$, $(1029,-0.49)$, $(1034,-0.16)$, $(1070,0.16)$, $(1079,0.49)$, $(1079,0.89)$, and $(1439,1.53)$.

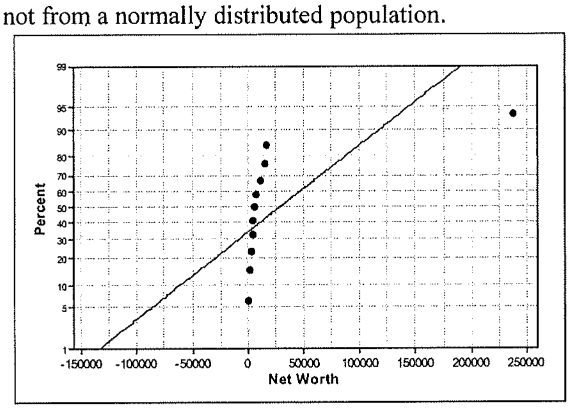
19. (continued)



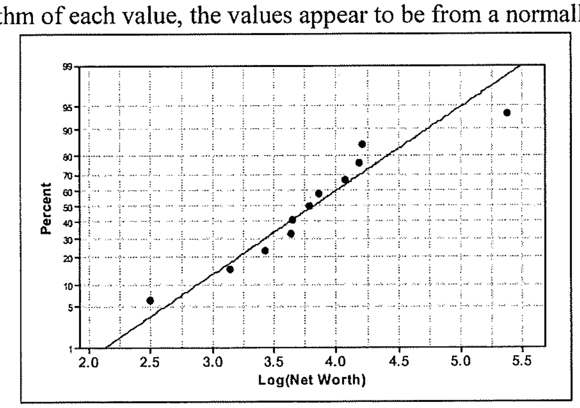
20. Normal, the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern. The points have coordinates $(84,-1.59)$, $(119,-0.97)$, $(121,-0.59)$, $(123,-0.28)$, $(146,0)$, $(152,0.28)$, $(162,0.59)$, $(181,0.97)$, and $(266,1.59)$.



21. a. Yes, the histogram or normal quantile plot will remain unchanged.
    b. Yes, the histogram or normal quantile plot will remain unchanged.
    c. No, the histogram and normal quantile plot will not indicate a normal distribution.
22. The original values are not from a normally distributed population.



After taking the logarithm of each value, the values appear to be from a normally distributed population.



The original values are from a population with a lognormal distribution.

Section 6-6: Normal as Approximation to Binomial

1. a. the area below (to the left of) 502.5
   b. the area between 501.5 and 502.5
   c. the area above (to the right of) 502.5
2. Yes, the circumstances correspond to 100 independent trials of a binomial experiment in which the probability of success is 0.2. Also, with $n = 100$, $p = 0.2$, and $q = 0.8$, the requirements of $np = 100 \cdot 0.2 = 20 \geq 5$ and $nq = 100 \cdot 0.8 = 80 \geq 5$ are both satisfied.
3. $p = 0.2$, $q = 0.8$, $\mu = 20$, $\sigma = 4$; The value of $\mu = 20$ shows that for people who make random guesses for the 100 questions, the mean number of correct answers is 20. For people who make 100 random guesses, the standard deviation of $\sigma = 4$ is a measure of how much the numbers of correct responses vary.
4. The histogram should be approximately normal or bell-shaped, because sample proportions tend to approximate a normal distribution.
5. The requirements for the normal approximation are satisfied with $np = 20 \cdot 0.512 = 10.2 \geq 5$ and $nq = 20 \cdot 0.488 = 9.8 \geq 5$. $z_{x=7.5} = \dfrac{7.5 - 20 \cdot 0.512}{\sqrt{20 \cdot 0.512 \cdot 0.488}} = -1.23$; which has a probability of 0.1093 (Tech: 0.1102) to the left.
6. The requirement of $np = 8 \cdot 0.512 = 4.1 \geq 5$ is not satisfied. The normal approximation should not be used.
7. The requirement of $np = 20 \cdot 0.2 = 4 \geq 5$ is not satisfied. The normal approximation should not be used.
8. The requirements for the normal approximation are satisfied with $np = 50 \cdot 0.2 = 10 \geq 5$ and $nq = 50 \cdot 0.8 = 40 \geq 5$. $z_{x=11.5} = \dfrac{11.5 - 50 \cdot 0.2}{\sqrt{50 \cdot 0.2 \cdot 0.8}} = 0.53$ and $z_{x=12.5} = \dfrac{12.5 - 50 \cdot 0.2}{\sqrt{50 \cdot 0.2 \cdot 0.8}} = 0.88$; which have a probability of $0.8106 - 0.7019 = 0.1087$ (Tech: 0.1096) between them.
9. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.23 = 23 \geq 5$ and $nq = 100 \cdot 0.77 = 77 \geq 5$. $z_{x=19.5} = \dfrac{19.5 - 100 \cdot 0.23}{\sqrt{100 \cdot 0.23 \cdot 0.77}} = -0.83$; which has a probability of 0.2033 (Tech: 0.2028) to the left. No, 20 is not a significantly low number of white cars. (Tech: Using the binomial distribution: 0.2047.)
10. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.18 = 18 \geq 5$ and $nq = 100 \cdot 0.82 = 82 \geq 5$. $z_{x=24.5} = \dfrac{24.5 - 100 \cdot 0.18}{\sqrt{100 \cdot 0.18 \cdot 0.82}} = 1.69$; which has a probability of $1 - 0.9545 = 0.0455$ (Tech: 0.2028) to the right. Yes, 25 is a significantly high number of black cars. (Tech: Using the binomial distribution: 0.0496.)
11. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.10 = 10 \geq 5$ and $nq = 100 \cdot 0.90 = 90 \geq 5$. $z_{x=13.5} = \dfrac{13.5 - 100 \cdot 0.10}{\sqrt{100 \cdot 0.10 \cdot 0.90}} = 1.17$ and $z_{x=14.5} = \dfrac{14.5 - 100 \cdot 0.10}{\sqrt{100 \cdot 0.10 \cdot 0.90}} = 1.50$; which have a probability of $0.9524 - 0.9332 = 0.0542$ (Tech: 0.0549) between them. Determination of whether 14 red cars is significantly high should be based on the probability of *14 or more red cars*, not the probability of exactly 14 red cars. (Tech: Using the binomial distribution: 0.0513.)
12. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.16 = 16 \geq 5$ and $nq = 100 \cdot 0.84 = 84 \geq 5$. $z_{x=9.5} = \dfrac{9.5 - 100 \cdot 0.16}{\sqrt{100 \cdot 0.16 \cdot 0.84}} = -1.77$ and $z_{x=10.5} = \dfrac{10.5 - 100 \cdot 0.16}{\sqrt{100 \cdot 0.16 \cdot 0.84}} = -1.50$; which have a probability of $0.0668 - 0.0384 = 0.0284$ (Tech: 0.0287) between them. Determination of whether 10gray cars is significantly high should be based on the probability of *10 or fewer red cars*, not the probability of exactly 10 red cars. (Tech: Using the binomial distribution: 0.0292.)

13. a. The requirements for the normal approximation are satisfied with $np = 879 \cdot 0.25 = 219.75 \ge 5$ and $nq = 879 \cdot 0.75 = 659.25 \ge 5$. $z_{x=230.5} = \dfrac{230.5 - 879 \cdot 0.25}{\sqrt{879 \cdot 0.25 \cdot 0.75}} = 0.84$ and $z_{x=231.5} = \dfrac{231.5 - 879 \cdot 0.25}{\sqrt{879 \cdot 0.25 \cdot 0.75}}$ $= 0.92$; which have a probability of $0.8212 - 0.7995 = 0.0217$ (Tech: 0.0212) between them. (Tech: Using the binomial distribution: 0.0209.)

    b. The requirements for the normal approximation are satisfied with $np = 879 \cdot 0.25 = 219.75 \ge 5$ and $nq = 879 \cdot 0.75 = 659.25 \ge 5$. $z_{x=230.5} = \dfrac{230.5 - 879 \cdot 0.25}{\sqrt{879 \cdot 0.25 \cdot 0.75}} = 0.84$; which has a probability of $1 - 0.7995 = 0.2005$ (Tech: 0.2012) to the right. The result of 231 overturned calls is not significantly high. (Tech: Using the binomial distribution: 0.2006.)

14. a. The requirements for the normal approximation are satisfied with $np = 879 \cdot 0.22 = 193.38 \ge 5$ and $nq = 879 \cdot 0.78 = 685.62 \ge 5$. $z_{x=230.5} = \dfrac{230.5 - 879 \cdot 0.22}{\sqrt{879 \cdot 0.22 \cdot 0.78}} = 3.02$ and $z_{x=231.5} = \dfrac{231.5 - 879 \cdot 0.22}{\sqrt{879 \cdot 0.22 \cdot 0.78}}$ $= 3.10$; which have a probability of $0.9990 - 0.9987 = 0.0003$ between them.

    b. The requirements for the normal approximation are satisfied with $np = 879 \cdot 0.22 = 193.38 \ge 5$ and $nq = 879 \cdot 0.78 = 685.62 \ge 5$. $z_{x=230.5} = \dfrac{230.5 - 879 \cdot 0.22}{\sqrt{879 \cdot 0.22 \cdot 0.78}} = 3.02$; which has a probability of $1 - 0.9987 = 0.0013$ to the right. The result of 231 overturned calls is significantly high. (Tech: Using the binomial distribution: 0.0015.)

15. a. The requirements for the normal approximation are satisfied with $np = 250 \cdot 0.51 = 127.5 \ge 5$ and $nq = 250 \cdot 0.49 = 122.5 \ge 5$. $z_{x=109.5} = \dfrac{109.5 - 250 \cdot 0.51}{\sqrt{250 \cdot 0.51 \cdot 0.49}} = -2.28$; which has a probability of 0.0113 (Tech: 0.0114) to the left. (Tech: Using the binomial distribution: 0.0113.)

    b. The result of 109 is significantly low.

16. a. The requirements for the normal approximation are satisfied with $np = 650 \cdot 0.12 = 78 \ge 5$ and $nq = 650 \cdot 0.88 = 572 \ge 5$. $z_{x=85.5} = \dfrac{85.5 - 650 \cdot 0.12}{\sqrt{650 \cdot 0.12 \cdot 0.88}} = 0.91$; which has a probability of $1 - 0.8186 = 0.1814$ (Tech: 0.1827) to the right. (Tech: Using the binomial distribution: 0.1819.)

    b. The result of 86 people with green eyes is not significantly high.

17. a. The requirements for the normal approximation are satisfied with $np = 929 \cdot 0.75 = 696.75 \ge 5$ and $nq = 929 \cdot 0.25 = 232.25 \ge 5$. $z_{x=704.5} = \dfrac{704.5 - 929 \cdot 0.75}{\sqrt{929 \cdot 0.75 \cdot 0.25}} = 0.59$; which has a probability of $1 - 0.7224 = 0.2776$ (Tech: 0.2785) to the right. (Tech: Using the binomial distribution: 0.2799.)

    b. The result of 705 peas with red flowers is not significantly high.

    c. The result of 705 peas with red flowers is not strong evidence against Mendel's assumption that $3/4$ of peas will have red flowers.

18. a. The requirements for the normal approximation are satisfied with $np = 1480 \cdot 0.292 = 432.16 \ge 5$ and $nq = 1480 \cdot 0.708 = 1047.84 \ge 5$. $z_{x=454.5} = \dfrac{454.5 - 1480 \cdot 0.292}{\sqrt{1480 \cdot 0.292 \cdot 0.708}} = 1.28$; which has a probability of $1 - 0.8997 = 0.1003$ (Tech: 0.1008) to the right. (Tech: Using the binomial distribution: 0.1012.)

    b. The result of 455 who have sleepwalked is not significantly high.

    c. The result of 455 does not provide strong evidence against the rate of 29.2%.

19. a. Using the normal approximation: $\mu = 1002 \cdot 0.61 = 611.22$, $\sigma = \sqrt{1002 \cdot 0.61 \cdot 0.39} = 15.4394$, and $z_{x=700.5} = \dfrac{700.5 - 611.22}{\sqrt{1002 \cdot 0.61 \cdot 0.39}} = 5.78$; which has a probability of 0.0001 (Tech 0.0000) to the right.

b. The result suggests that the surveyed people did not respond accurately.

20. a. Using normal approximation: $\mu = 420{,}095 \cdot 0.00034 = 142.83$, $\sigma = \sqrt{420{,}095 \cdot 0.000344 \cdot 0.999656}$ $= 11.9492$, and $z_{x=135.5} = \dfrac{135.5 - 142.83}{\sqrt{420{,}095 \cdot 0.000344 \cdot 0.999656}} = -0.61$; which has a probability of 0.2709 (Tech: 0.2697) to the left. (Tech: Using the binomial distribution: 0.2726.)

b. Media reports appear to be wrong.

21. (1) The requirements for the normal approximation are satisfied with $np = 11 \cdot 0.512 = 5.632 \geq 5$ and $nq = 11 \cdot 0.488 = 5.368 \geq 5$. $z_{x=6.5} = \dfrac{6.5 - 11 \cdot 0.512}{\sqrt{11 \cdot 0.512 \cdot 0.488}} = 0.52$ and $z_{x=7.5} = \dfrac{7.5 - 11 \cdot 0.512}{\sqrt{11 \cdot 0.512 \cdot 0.488}} = 1.13$; which have a probability of $0.8708 - 0.6985 = 0.1723$ between them.

(2) 0.1704

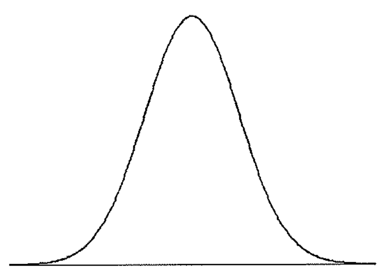
(3) 0.1726

No, the approximations are not off by very much.

22. The z score that corresponds to a 0.95 probability is 1.645. This means that we have to solve the equation $1.645 \cdot \sqrt{n \cdot 0.9005 \cdot 0.0995} + n \cdot 0.9005 = 213$ for n. This has a solution of 229 (Tech: 230) reservations.

Quick Quiz

1.



2. $z = P_9 = -1.34$

3. $P(z > -2.93) = 1 - P(z < -2.93) = 1 - 0.0017 = 0.9983$

4. $P(0.87 < z < 1.78) = P(z < 1.78) - P(z < 0.87) = 0.9625 - 0.8078 = 0.1547$ (Tech: 0.1546)

5. a. $\mu = 0$ and $\sigma = 1$

b. $\mu_{\bar{x}}$ represents the mean of all sample means, and $\sigma_{\bar{x}}$ represents the standard deviation of all sample means.

6. $z_{x=80} = \dfrac{80.0 - 70.2}{11.2} = 0.88$; which has a probability of 0.8106 (Tech: 0.8092) to the left.

7. $z_{x=60} = \dfrac{60.0 - 70.2}{11.2} = -0.91$ and $z_{x=80} = \dfrac{80.0 - 70.2}{11.2} = 0.88$; which have a probability of $0.8106 - 0.1814$ $= 0.6292$ (Tech: 0.6280) between them.

8. The z score for P_{90} is 1.28, which corresponds to a diastolic blood pressure of $1.28 \cdot 11.2 + 70.2 = 84.5$ mm Hg (Tech: 84.6 mm Hg).

9. $z_{x=75} = \dfrac{75.0 - 70.2}{11.2/\sqrt{16}} = 1.71$; which has a probability of 0.9564 (Tech: 0.9568) to the left.

10. The normal quantile plot suggests that diastolic blood pressure levels of women are normally distributed.

Review Exercises

1. a. The probability to the left of a z score of 1.54 is 0.9382.
   b. The probability to the right of a z score of –1.54 is $1-0.0618=0.9382$.
   c. The probability between z scores –1.33 and 2.33 is $0.9901-0.0918=0.8983$.
   d. The z score for Q_1 is –0.67.
   e. $z_{x=0.50}=\dfrac{0.50-0.0}{1/\sqrt{9}}=1.50$; which has a probability of $1-0.9332=0.0668$ to the right.

2. a. $z_{x=54}=\dfrac{54-59.7}{2.5}=-2.28$; which has a probability of 0.0113 to the left, or 1.13%.
   b. The z score for the lowest 95% is 1.645 which corresponds to a standing eye height of $1.645\cdot 2.5+59.7=63.8$ in.

3. a. $z_{x=70}=\dfrac{70-64.3}{2.6}=2.19$; which has a probability of $1-0.9857=0.0143$ to the right, or 1.43%. (Tech: 1.42%)
   b. The z score for the lowest 2% is –2.05 which corresponds to a standing eye height of $-20.05\cdot 2.6+64.3=59.0$ in.

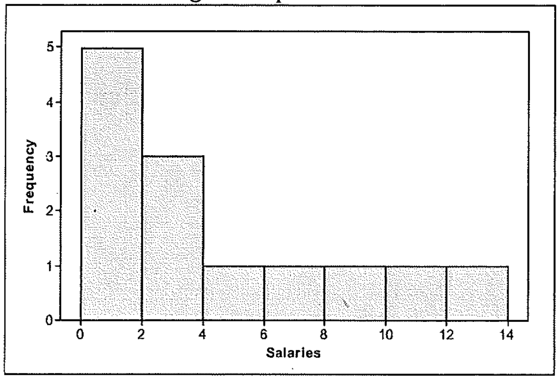
4. a. The distribution of samples means is normal.
   b. $\mu_{\bar{x}}=100$
   c. $\sigma_{\bar{x}}=15/\sqrt{64}=1.875$

5. a. An unbiased estimator is a statistic that targets the value of the population parameter in the sense that the sampling distribution of the statistic has a mean that is equal to the mean of the corresponding parameter.
   b. mean, variance and proportion
   c. true

6. a. $z_{x=72}=\dfrac{72-68.6}{2.8}=1.21$; which has a probability of 0.8869, or 88.69% (Tech: 88.77%) to the left. With about 11% of all men needing to bend, the design does not appear to be adequate, but the Mark VI monorail appears to be working quite well in practice.
   b. The z score for 99% is 2.33 which corresponds to a doorway height of $2.33\cdot 2.8+68.6=75.1$ in.

7. a. Because women are generally a little shorter than men, a doorway height that accommodates men will also accommodate women.
   b. $z_{x=72}=\dfrac{72-68.6}{2.8/\sqrt{60}}=9.41$; which has a probability of 0.9999, or 1 when rounded.
   c. Because the mean height of 60 men is less than 72 in., it does not follow that the 60 individual men all have heights less than 72 in. In determining the suitability of the door height for men, the mean of 60 heights is irrelevant, but the heights of individual men are relevant.

8. a. No, a histogram is far from bell shaped and a normal quantile plot reveals a pattern of points that is far from a straight-line pattern.



b. No, the sample size $(n = 13)$ does not satisfy the condition of $n > 30$ and the values do not appear to be from a population having a normal distribution.

9. The requirements for the normal approximation are satisfied with $np = 1064 \cdot 0.75 = 798 \geq 5$ and $nq = 1064 \cdot 0.25 = 266 \geq 5$. $z_{z=787.5} = \dfrac{787.5 - 1064 \cdot 0.75}{\sqrt{11064 \cdot 0.75 \cdot 0.25}} = -0.74$; which has a probability of 0.2296 (Tech: 0.2286) to the left. The occurrence of 787 offspring plants with long stem is not unusually low because its probability is not small. The results are consistent with Mendel's claimed proportion of $3/4$. (Tech: Using the binomial distribution: 0.2278.)

10. a. $z_{x=70} = \dfrac{70.0 - 63.7}{2.9} = 2.17$; which has a probability of $1 - 0.9850 = 0.0150$, or 1.50% (Tech: 1.49%) to the right.

b. The z score for the upper 2.5% is 1.96 which corresponds to a doorway height of $1.96 \cdot 2.9 + 63.7$ $= 69.4$ in.

Cumulative Review Exercises

1. a. The mean is $\bar{x} = \dfrac{0.6 + 0.9 + \cdots + 1.7 + 2.6 + 2.8 + 3.3 + 4.1 + \cdots + 22.5 + 23.4}{15} = \$6.02$ million, or \$6,020,000.

b. The median is \$2.80 million, or \$2,800,000.

c. $s = \sqrt{\dfrac{(0.6 - 6.02)^2 + (0.9 - 6.02)^2 + \ldots + (22.5 - 6.02)^2 + (23.4 - 6.02)^2}{15 - 1}} = \$7.47$ million, or \$7,470,000.

d. $s^2 = (7.47)^2 = 55.73$ (million dollars)^2

e. $z_{x=23.4} = \dfrac{23.4 - 6.02}{7.47} = 2.33$

f. ratio

g. discrete

2. a. Q_1: $L = \dfrac{25 \cdot 15}{100} = 3.75$, so $Q_1 = \$1.30$ million

Q_2: $L = \dfrac{50 \cdot 15}{100} = 7.5$, so $Q_2 = \$2.80$ million

Q_3: $L = \dfrac{75 \cdot 15}{100} = 11.25$, so $Q_2 = \$7.10$ million

2. (continued)

   b.

   c. The sample does not appear to be from a population having a normal distribution.

3. No, the distribution does not appear to be a normal distribution.

4. a. $\overline{B}$ is the event of selecting someone who does not have blue eyes.

   b. $P(\overline{B}) = 1 - 0.35 = 0.65$

   c. $0.35 \cdot 0.35 \cdot 0.35 = 0.0429$

   d. The requirements for the normal approximation are satisfied with $np = 100 \cdot 0.35 = 35 \geq 5$ and $nq = 100 \cdot 0.65 = 65 \geq 5$. $z_{x=39.5} = \dfrac{39.5 - 100 \cdot 0.35}{\sqrt{100 \cdot 0.35 \cdot 0.65}} = 0.94$; which has a probability of $1 - 0.8264 = 0.1736$ (Tech: 0.1727) to the right. (Tech: Using the binomial distribution: 0.1724.)

   e. No, 40 people with blue eyes is not significantly high.

5. a. $z_{x=10} = \dfrac{10 - 9.6}{0.5} = 0.80$; which has a probability of 0.7881 to the left.

   b. $z_{x=8.0} = \dfrac{8.0 - 9.6}{0.5} = -3.20$ and $z_{x=11.0} = \dfrac{11.0 - 9.6}{0.5} = 2.80$; which have a probability of $0.9974 - 0.0007 = 0.9967$ (Tech: 0.9968) between them.

   c. The z score for the top 5% is 1.645, which correspond to the length $1.645 \cdot 0.5 + 9.6 = 10.4$ in.

   d. $z_{x=9.8} = \dfrac{9.8 - 9.6}{0.5/\sqrt{25}} = 2.00$; which has a probability of $1 - 0.9772 = 0.0228$ to the right.

Chapter 7: Estimating Parameters and Determining Sample Sizes

Section 7-1: Estimating a Population Proportion

1. The confidence level (such as 95%) was not provided.
2. When using 14% to estimate the value of the population percentage for those who prefer chocolate pie, the maximum likely difference between 14% and the true population percentage is four percentage points, so the interval from $14\% - 4\% = 10\%$ to $14\% + 4\% = 18\%$ is likely to contain the true population percentage.
3. $\hat{p} = 0.14$ is the sample proportion; $\hat{q} = 0.86$ (found from evaluating $1 - \hat{p}$); $n = 1000$ is the sample size; $E = 0.04$ is the margin of error; p is the population proportion, which is unknown. The value of α is 0.05.
4. The 95% confidence interval will be wider than the 80% confidence interval. A confidence interval must be wider in order for us to be more confident that it captures the true value of the population proportion. (Think of estimating the age of a classmate. You might be 90% confident that she is between 20 and 30, but you might be 99.9% confident that she is between 10 and 40.)
5. $z_{0.05} = 1.645$
6. $z_{0.005} = 2.576$ (Table: 2.575)
7. $z_{0.0025} = 2.81$
8. $z_{0.01} = 2.33$
9. $\hat{p} = \dfrac{0.0434 + 0.217}{2} = 0.130$ and $E = \dfrac{0.217 - 0.0434}{2} = 0.087$; 0.130 ± 0.087
10. $\hat{p} = \dfrac{0.179 + 0.321}{2} = 0.250$ and $E = \dfrac{0.321 - 0.179}{2} = 0.071$; 0.250 ± 0.071
11. $0.0169 < p < 0.143$
12. $0.270 - 0.073 < p < 0.270 + 0.073$, or $0.197 < p < 0.343$
13. a. $\hat{p} = 33/362 = 0.0912$

    b. $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\dfrac{\left(\frac{33}{362}\right)\left(\frac{329}{362}\right)}{362}} = 0.0297$

    c. $\hat{p} - E < p < \hat{p} - E$

    $0.0912 - 0.0297 < p < 0.0912 + 0.0297$

    $0.0615 < p < 0.121$

    d. We have 95% confidence that the interval from 0.0615 to 0.121 actually does contain the true value of the population proportion of McDonald's drive-through orders that are not accurate.
14. a. $\hat{p} = 153/5924 = 0.0258$

    b. $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 2.58\sqrt{\dfrac{\left(\frac{153}{5924}\right)\left(\frac{771}{5924}\right)}{5924}} = 0.00531$

    c. $\hat{p} - E < p < \hat{p} - E$

    $0.0258 - 0.00531 < p < 0.0258 + 0.00531$

    $0.0205 < p < 0.0311$

    d. We have 99% confidence that the interval from 0.0205 to 0.0311 actually does contain the true value of the population proportion of all Eliquis users who experience nausea.
15. a. $\hat{p} = 717/5000 = 0.143$

    b. $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 1.645\sqrt{\dfrac{\left(\frac{717}{5000}\right)\left(\frac{4283}{5000}\right)}{5000}} = 0.00815$

15. (continued)

c. $\hat{p}-E < p < \hat{p}-E$

$0.143-0.00815 < p < 0.143+0.00815$

$0.135 < p < 0.152$

d. We have 90% confidence that the interval from 0.135 to 0.152 actually does contain the true value of the population proportion of returned surveys.

16. a. $\hat{p}=856/1228=0.697$

b. $E=z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=1.96\sqrt{\frac{\left(\frac{856}{1228}\right)\left(\frac{372}{1228}\right)}{1228}}=0.0257$

c. $\hat{p}-E < p < \hat{p}-E$

$0.697-0.0257 < p < 0.697+0.0257$

$0.671 < p < 0.723$

d. We have 95% confidence that the interval from 0.671 to 0.723 actually does contain the true value of the population proportion of medical malpractice lawsuits that are dropped or dismissed.

17. 95% CI: $\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=\frac{426}{860}\pm 1.96\sqrt{\frac{\left(\frac{426}{860}\right)\left(\frac{434}{860}\right)}{860}} \Rightarrow 0.462 < p < 0.529$; Because 0.512 is contained within the confidence interval, there is not strong evidence against 0.512 as the value of the proportion of boys in all births.

18. a. 99% CI: $\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=\frac{428}{580}\pm 2.58\sqrt{\frac{\left(\frac{428}{580}\right)\left(\frac{152}{580}\right)}{580}} \Rightarrow 0.691 < p < 0.785$, or $69.1\% < p < 78.5\%$

b. No, the confidence interval includes 75%, so the true percentage could easily equal 75%.

19. a. 99% CI: $\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=\frac{54}{318}\pm 2.58\sqrt{\frac{\left(\frac{54}{318}\right)\left(\frac{264}{318}\right)}{318}} \Rightarrow 0.116 < p < 0.224$, or $11.6\% < p < 22.4\%$

b. Because the two confidence intervals overlap, it is possible that Burger King and Wendy's have the same rate of orders that are not accurate. Neither restaurant appears to have a significantly better rate of accuracy of orders.

20. a. 95% CI: $\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=\frac{152}{227}\pm 1.96\sqrt{\frac{\left(\frac{52}{227}\right)\left(\frac{175}{227}\right)}{227}} \Rightarrow 0.174 < p < 0.284$, or $17.4\% < p < 28.4\%$

b. Because the two confidence intervals overlap, it is possible that the OxyContin treatment group and the placebo group have the same rate of nausea. Nausea does not appear to be an adverse reaction made worse with OxyContin.

21. a. 0.5

b. $\hat{p}=123/280=0.439$

c. 99% CI: $\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=\frac{123}{280}\pm 2.56\sqrt{\frac{\left(\frac{123}{280}\right)\left(\frac{157}{280}\right)}{280}} \Rightarrow 0.363 < p < 0.516$

d. If the touch therapists really had an ability to select the correct hand by sensing an energy field, their success rate would be significantly greater than 0.5, but the sample success rate of 0.439 and the confidence interval suggest that they do not have the ability to select the correct hand by sensing an energy field.

22. a. $3005(0.817)=2455$

b. 90% CI: $\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=0.817\pm 1.645\sqrt{\frac{(0.817)(0.183)}{3005}} \Rightarrow 0.805 < p < 0.829$, or $80.5\% < p < 82.9\%$

c. nothing

23. a. 90% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.000321 \pm 1.645\sqrt{\frac{(0.000321)(0.999679)}{420{,}095}} \Rightarrow 0.000276 < p < 0.000366$, or $0.0276\% < p < 0.0366\%$; (Using $x = 135$: $0.0276\% < p < 0.0367\%$)

b. No, because 0.0340% is included in the confidence interval.

24. a. 98% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.70 \pm 2.33\sqrt{\frac{(0.70)(0.30)}{1002}} \Rightarrow 0.666 < p < 0.733$; (Table: $0.666 < p < 0.734$)

b. No. Because 0.61 is not included in the confidence interval, it does not appear that the responses are consistent with the actual voter turnout.

25. Placebo group:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = \frac{7}{270} \pm 1.96\sqrt{\frac{\left(\frac{7}{270}\right)\left(\frac{263}{270}\right)}{270}} \Rightarrow 0.00697 < p < 0.0449$, or $0.697\% < p < 4.49\%$

Treatment group:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = \frac{8}{863} \pm 1.96\sqrt{\frac{\left(\frac{7}{863}\right)\left(\frac{855}{863}\right)}{863}} \Rightarrow 0.00288 < p < 0.0157$, or $0.288\% < p < 1.57\%$

Because the two confidence intervals overlap, there does not appear to be a significant difference between the rates of allergic reactions. Allergic reactions do not appear to be a concern for Lipitor users.

26. XSORT: 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = \frac{879}{945} \pm 1.96\sqrt{\frac{\left(\frac{879}{945}\right)\left(\frac{66}{945}\right)}{945}} \Rightarrow 0.914 < p < 0.946$, or $91.4\% < p < 94.6\%$

YSORT: 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = \frac{239}{291} \pm 1.96\sqrt{\frac{\left(\frac{239}{291}\right)\left(\frac{52}{291}\right)}{291}} \Rightarrow 0.777 < p < 0.865$, or $77.7\% < p < 86.5\%$

The two confidence intervals do not overlap. It appears that the success rate for the XSORT method is higher than the success rate for the YSORT method, but the big story here is that the XSORT method and the YSORT method both appear to be very effective because the success rates are well above the 50% rates expected with no treatments.

27. Sustained care:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.828 \pm 1.96\sqrt{\frac{(0.828)(0.172)}{198}} \Rightarrow 0.775 < p < 0.881$, or $77.5\% < p < 88.1\%$;

(Using $x = 164$: $77.5\% < p < 88.1\%$)

Standard care:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.628 \pm 1.96\sqrt{\frac{(0.628)(0.372)}{199}} \Rightarrow 0.561 < p < 0.695$, or $56.1\% < p < 69.5\%$;

The two confidence intervals do not overlap. It appears that the success rate is higher with sustained care.

28. Measured:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.258 \pm 1.96\sqrt{\frac{(0.2528)(0.742)}{198}} \Rightarrow 0.197 < p < 0.319$, or $19.7\% < p < 31.9\%$;

(Using $x = 51$: $19.7\% < p < 31.8\%$)

Reported:

95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.409 \pm 1.96\sqrt{\frac{(0.409)(0.591)}{198}} \Rightarrow 0.341 < p < 0.477$, or $34.1\% < p < 47.7\%$;

(Using $x = 81$: $34.1\% < p < 47.8\%$)

The two confidence intervals do not overlap. It appears that the success rate is higher with sustained care.

29. $\hat{p}=18/34=0.529$, 95% CI: $\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=\frac{18}{34}\pm 1.96\sqrt{\frac{\left(\frac{18}{34}\right)\left(\frac{16}{34}\right)}{34}}\Rightarrow 0.362<p<0.697$,
or $36.2\%<p<69.7\%$; Greater height does not appear to be an advantage for presidential candidates. If greater height is an advantage, then taller candidates should win substantially more than 50% of the elections, but the confidence interval shows that the percentage of elections won by taller candidates is likely to be anywhere between 36.2% and 69.7%.

30. $\hat{p}=19/100=0.19$ 95% CI: $\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=0.19\pm 1.96\sqrt{\frac{(0.19)(0.81)}{100}}\Rightarrow 0.113<p<0.267$,
or $11.3\%<p<26.7\%$; Because the confidence interval contains the claimed value of 16%, the sample data do not provide strong evidence against the claim that 16% of M&Ms are green.

31. a. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[2.575]^2\cdot 0.25}{0.03^2}=1842$ (Tech: 1844)

b. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[2.575]^2\cdot(0.10)(0.90)}{0.03^2}=664$

c. They don't change.

32. a. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[1.645]^2\cdot 0.25}{0.02^2}=1692$ (Tech: 1691)

b. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[1.645]^2\cdot(0.95)(0.05)}{0.02^2}=332$

c. Yes, the added knowledge results in a very substantial decrease in the required sample size.

33. a. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[1.96]^2\cdot 0.25}{0.05^2}=385$

b. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[1.96]^2\cdot(0.40)(0.60)}{0.05^2}=369$

c. No, the sample size doesn't change much.

34. a. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[2.575]^2\cdot 0.25}{0.04^2}=1037$

b. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[2.575]^2\cdot(0.26)(0.74)}{0.04^2}=798$

35. a. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[1.645]^2\cdot 0.25}{0.025^2}=1083$

b. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[1.645]^2\cdot(0.38)(0.62)}{0.025^2}=1021$

36. a. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[2.575]^2\cdot 0.25}{0.02^2}=4145$ (Tech: 4147)

b. $n=\frac{\left[z_{\alpha/2}\right]^2\hat{p}\hat{q}}{E^2}=\frac{[2.575]^2\cdot(0.43)(0.57)}{0.02^2}=4063$ (Tech: 4066)

c. No, the sample size doesn't change much.

37. a. $n=\frac{[z_{\alpha/2}]^2\hat{p}\hat{q}}{E^2}=\frac{[1.645]^2\cdot 0.25}{0.03^2}=752$

b. $n=\frac{[z_{\alpha/2}]^2\hat{p}\hat{q}}{E^2}=\frac{[1.645]^2\cdot(0.16)(0.84)}{0.03^2}=405$

c. No. A sample of the people you know is a convenience sample, not a simple random sample, so it is very possible that the results would not be representative of the population.

38. a. $n=\frac{[z_{\alpha/2}]^2\hat{p}\hat{q}}{E^2}=\frac{[2.575]^2\cdot 0.25}{0.02^2}=4145$ (Tech: 4147)

b. $n=\frac{[z_{\alpha/2}]^2\hat{p}\hat{q}}{E^2}=\frac{[2.575]^2\cdot(0.82)(0.18)}{0.02^2}=2447$ (Tech: 2449)

c. Randomly selecting adult women would result in an underestimate, because some women will give birth to their first child after the survey was conducted. It will be important to survey women who have completed the time during which they can give birth.

39. $n=\frac{N\hat{p}\hat{q}[z_{\alpha/2}]^2}{\hat{p}\hat{q}[z_{\alpha/2}]^2+(N-1)E^2}=\frac{2500(0.82)(0.18)[2.575]^2}{(0.82)(0.18)[2.575]^2+(2500-1)0.02^2}=1237$ (Tech: 1238)

40. Because we have 95% confidence that p is less than 0.451, we can safely conclude that fewer than 50% of adults have a Facebook page.

$$\hat{p}+z_{\alpha}\sqrt{\frac{\hat{p}\hat{q}}{n}}=0.43+1.645\sqrt{\frac{(0.43)(0.57)}{1487}}\Rightarrow p<0.451$$

41. a. The requirement of at least 5 successes and at least 5 failures is not satisfied, so the normal distribution cannot be used.

b. $3/40=0.075$

Section 7-2: Estimating a Population Mean

1. a. 13.05 Mbps $<\mu<$ 22.15 Mbps

b. The best point estimate of μ is $\bar{x}=\frac{13.046+22.15}{2}=17.60$ Mbps. The margin of error is

$$E=\frac{22.15-13.046}{2}=4.55 \text{ Mbps.}$$

c Because the sample size of 50 is greater than 30, we can consider the sample mean to be from a population with a normal distribution.

2. a. $\text{df}=50-1=49$

b. 2.00958 (Table: 2.009)

c. In general, the number of degrees of freedom for a collection of sample data is the number of sample values that can vary after certain restrictions have been imposed on all data values.

3. We have 95% confidence that the limits of 13.05 Mbps and 22.15 Mbps contain the true value of the mean of the population of all Verizon data speeds at the airports.

4. When we say that the confidence interval methods of this section are robust against departures from normality, we mean that these methods work reasonably well with distributions that are not normal, provided that departures from normality are not too extreme.

5. Neither the normal nor the t distribution applies.

6. $t_{\alpha/2}=1.671$

7. $z_{\alpha/2}=2.576$ (Table: 2.575)

8. $t_{\alpha/2}=1.972$

9. The sample size is greater than 30 and the data appear to be from a population that is normally distributed. 95% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 30.4 \pm 1.972 \cdot \frac{7.1}{\sqrt{205}} \Rightarrow 29.4 \text{ hg} < \mu < 31.4 \text{ hg}$; No, the results do not differ by much.
10. The sample size is greater than 30 and the data appear to be from a population that is normally distributed. 95% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 32.7 \pm 1.972 \cdot \frac{6.6}{\sqrt{205}} \Rightarrow 31.8 \text{ hg} < \mu < 36.6 \text{ hg}$; Yes, it appears that birth weights of boys are substantially greater than birth weights of girls.
11. The sample size is greater than 30 and the data appear to be from a population that is normally distributed. 95% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 98.20 \pm 1.983 \cdot \frac{0.62}{\sqrt{106}} \Rightarrow 98.08°\text{F} < \mu < 98.32°\text{F}$; Because the confidence interval does not contain 98.6°F, it appears that the mean body temperature is not 98.6°F, as is commonly believed.
12. The sample size is greater than 30. 90% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 2.1 \pm 1.685 \cdot \frac{4.8}{\sqrt{40}} \Rightarrow 0.8 \text{ lb} < \mu < 3.4 \text{ lb}$; Because the confidence interval does not include 0 or negative values, it does appear that the weight loss program is effective, with a positive loss of weight. Because the amount of weight lost is relatively small, the weight loss program does not appear to be very practical.
13. It is assumed that the 16 sample values appear to be from a normally distributed population. 98% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 98.9 \pm 2.602 \cdot \frac{42.3}{\sqrt{16}} \Rightarrow 71.4 \text{ min} < \mu < 126.4 \text{ min}$; The confidence interval includes the mean of 102.8 min that was measured before the treatment, so the mean could be the same after the treatment. This result suggests that the zopiclone treatment does not have a significant effect.
14. The sample size is greater than 30. 98% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 0.4 \pm 2.407 \cdot \frac{21.0}{\sqrt{49}}$ $\Rightarrow -6.8 \text{ mg/dL} < \mu < 7.6 \text{ mg/dL}$; Because the confidence interval includes the value of 0, it is very possible that the mean of the changes in LDL cholesterol is equal to 0, suggesting that the garlic treatment did not affect LDL cholesterol levels. It does not appear that garlic has a significant effect in reducing LDL cholesterol.
15. The sample appears to have a normal distribution. 95% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 2.6 \pm 2.262 \cdot \frac{1.07}{\sqrt{10}}$ $\Rightarrow 1.8 < \mu < 3.4$; The given numbers are just substitutes for the four DNA base names, so the numbers don't measure or count anything, and they are at the nominal level of measurement. The confidence interval has no practical use.
16. The sample appears to have a normal distribution. 90% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 6.35 \pm 1.833 \cdot \frac{2.325}{\sqrt{10}}$ $\Rightarrow 5.00 \ \mu\text{g} < \mu < 7.70 \ \mu\text{g}$; No, the samples obtained from California might be very different from those obtained in Arkansas.
17. The sample appears to have a normal distribution. 99% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 7.0 \pm 3.707 \cdot \frac{2.216}{\sqrt{7}}$ $\Rightarrow 5.0 < \mu < 9.0$; The results tell us nothing about the population of adult females.
18. The sample appears to have a normal distribution. 99% CI: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 6.6 \pm 2.787 \cdot \frac{1.88}{\sqrt{26}}$ $\Rightarrow 5.5 < \mu < 7.6$; No, male participants in speed dating are not a representative sample of the population of all adult males. Also, the values are subjective judgments by females doing speed dating.

19. The sample appears to have a normal distribution. 98% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 0.719 \pm 3.143 \cdot \frac{0.366}{\sqrt{7}}$
$\Rightarrow 0.284 \text{ ppm} < \mu < 1.153 \text{ ppm}$; Using the FDA guideline, the confidence interval suggests that there could be too much mercury in fish because it is possible that the mean is greater than 1 ppm. Also, one of the sample values exceeds the FDA guideline of 1 ppm, so at least some of the fish have too much mercury.
20. The data appear to have a distribution that is far from normal, so the confidence interval might not be a good estimate of the population mean. 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 6.5 \pm 1.729 \cdot \frac{3.51}{\sqrt{20}}$
$\Rightarrow 5.1 \text{ years} < \mu < 7.9 \text{ years}$ (Tech: $4.9 \text{ years} < \mu < 8.1 \text{ years}$); The confidence interval does not contain the value of 4 years, but six of the twenty values are 4, so it is common to earn a bachelor's degree in four years, but the typical college student uses more than 4 years.
21. The data appear to have a distribution that is far from normal, so the confidence interval might not be a good estimate of the population mean.
98% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 172 \pm 2.821 \cdot \frac{32.2}{\sqrt{10}} \Rightarrow 143.3 \text{ million dollars} < \mu < 200.7 \text{ million dollars}$; Because the amounts are from the ten wealthiest celebrities, the confidence interval doesn't tell us anything about the population of all celebrities.
22. The presence of five zeros suggests that the sample is not from a normally distributed population, so the normality requirement is violated and the confidence interval might not be a good estimate of the population mean.
99% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 32.6 \pm 2.861 \cdot \frac{20.33}{\sqrt{20}} \Rightarrow 19.5 \text{ mg} < \mu < 45.6 \text{ mg}$; People consume some brands much more often than others, but the 20 brands are all weighted equally in the calculations. This, along with the violation of the normality requirement, means the confidence interval might not be a good estimate of the population mean.
23. The sample appears to have a normal distribution. 90% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 3.92 \pm 1.761 \cdot \frac{0.549}{\sqrt{15}}$
$\Rightarrow 3.67 < \mu < 4.17$; Because all of the students were at the University of Texas at Austin, the confidence interval doesn't tell us anything about the population of college students in Texas.
24. The data appear to have a distribution that is not normal, so the confidence interval might not be a good estimate of the population mean. 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = -9.17 \pm 2.201 \cdot \frac{23.86}{\sqrt{12}} \Rightarrow -24.3 \text{ min} < \mu < 6.0 \text{ min}$;
Nine of 12 of the flights arrived early and 3 of them were late. The confidence interval includes 0 (on time), so the on-time performance looks reasonably good.
25. Both samples appear to have a normal distribution.
Males: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 69.58 \pm 1.976 \cdot \frac{0.916}{\sqrt{153}} \Rightarrow 67.8 \text{ bpm} < \mu < 71.4 \text{ bpm}$
Females: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 74.0 \pm 1.976 \cdot \frac{1.03}{\sqrt{147}} \Rightarrow 72.0 \text{ bpm} < \mu < 76.1 \text{ bpm}$
Although final conclusions about means of populations should not be based on the overlapping of confidence intervals, the intervals do not overlap, so adult females appear to have a mean pulse rate that is higher than the mean pulse rate of adult males.

26. Neither sample appears to have a normal distribution, so the normality requirement is violated and the confidence intervals might not be good estimates of the population means.

    Sprint: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 3.71 \pm 2.010 \cdot \frac{5.598}{\sqrt{50}} \Rightarrow 2.12 \text{ Mbps} < \mu < 5.30 \text{ Mbps}$

    T-Mobile: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 10.99 \pm 2.010 \cdot \frac{7.33}{\sqrt{50}} \Rightarrow 8.90 \text{ Mbps} < \mu < 13.07 \text{ Mbps}$

    Although final conclusions about means of populations should not be based on the overlapping of confidence intervals, the intervals do not overlap, so T-Mobile appears to have a mean speed that is higher than the mean for Sprint.

27. The samples both appear to have a normal distribution.

    McDonald's: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 179.3 \pm 2.010 \cdot \frac{62.94}{\sqrt{50}} \Rightarrow 161.4 \text{ sec} < \mu < 197.2 \text{ sec}$

    Burger King: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 153.3 \pm 2.010 \cdot \frac{49.79}{\sqrt{50}} \Rightarrow 139.1 \text{ sec} < \mu < 167.5 \text{ sec}$

    (Table: $139.2 \text{ sec} < \mu < 167.4 \text{ sec}$)

    Although final conclusions about means of populations should not be based on the overlapping of confidence intervals, the intervals do overlap, so there does not appear to be a significant difference between the mean dinner service times at McDonald's and Burger King.

28. The samples both appear to have a normal distribution.

    Chips Ahoy: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 23.95 \pm 2.023 \cdot \frac{2.552}{\sqrt{40}} \Rightarrow 23.1 \text{ chips} < \mu < 24.8 \text{ chips}$

    Keebler: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 30.41 \pm 2.035 \cdot \frac{3.456}{\sqrt{34}} \Rightarrow 29.2 \text{ chips} < \mu < 31.6 \text{ chips}$

    Although final conclusions about means of populations should not be based on the overlapping of confidence intervals, the intervals do not overlap, so Keebler appears to have cookies with a higher mean number of chocolate chips than Chips Ahoy.

29. The sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.575 \cdot 15}{4}\right]^2 = 94$. This does appear to be very practical.

30. The sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.326 \cdot 15}{3}\right]^2 = 136$. This does appear to be very practical.

31. The required sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.96 \cdot 1.0}{0.01}\right]^2 = 38{,}416$ (Tech: 38,415). This does appear to be very practical.

32. The sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.645 \cdot 17.65}{1.5}\right]^2 = 375$. Yes, the assumption seems reasonable.

33. The required sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.96 \cdot 17.7}{0.5}\right]^2 = 4815$ (Tech: 4814). Yes, the assumption seems reasonable.

34. a. $\sigma \approx \frac{104-36}{4} = 17.0$; The required sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.575 \cdot 17.0}{2}\right]^2 = 480$.

    b. The required sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.575 \cdot 12.5}{2}\right]^2 = 260$.

    c. The result from part (a) is substantially larger than the result from part (b). The result from part (b) is likely to be better because it uses s instead of the estimated σ obtained from the range rule of thumb.

35. a. $\sigma \approx \frac{104-40}{4} = 16.0$; The required sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.575 \cdot 16.0}{2}\right]^2 = 425$.

b. The required sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.575 \cdot 11.3}{2}\right]^2 = 212$.

c. The result from part (a) is substantially larger than the result from part (b). The result from part (b) is likely to be better because it uses s instead of the estimated σ obtained from the range rule of thumb.

36. a. $\sigma \approx \frac{99.6-96.5}{4} = 0.775$; The required sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.326 \cdot 0.775}{0.1}\right]^2 = 326$ (Table: 327).

b. The required sample size is $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.326 \cdot 0.62}{0.1}\right]^2 = 209$.

c. The result from part (a) is substantially larger than the result from part (b). The result from part (b) is likely to be better because it uses s instead of the estimated σ obtained from the range rule of thumb.

37. The sample size is greater than 30 and the data appear to be from a population that is normally distributed.

95% CI: $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 30.4 \pm 1.96 \cdot \frac{7.1}{\sqrt{205}} \Rightarrow 29.4 \text{ hg} < \mu < 31.4 \text{ hg}$

38. The sample size is greater than 30 and the data appear to be from a population that is normally distributed.

95% CI: $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 32.7 \pm 1.96 \cdot \frac{6.6}{\sqrt{205}} \Rightarrow 31.8 \text{ hg} < \mu < 36.6 \text{ hg}$

39. The second confidence interval is narrower, indicating that we have a more accurate estimate when the relatively large sample is from a relatively small finite population.

Large pop: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 0.8565 \pm 2.26 \cdot \frac{0.0518}{\sqrt{100}} \Rightarrow 0.8462 \text{ g} < \mu < 0.8668 \text{ g}$

Finite pop: 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{n-1}} = 0.8565 \pm 2.26 \cdot \frac{0.0518}{\sqrt{100}}\sqrt{\frac{465-100}{100-1}} \Rightarrow 0.8474 \text{ g} < \mu < 0.8656 \text{ g}$

Section 7-3: Estimating a Population Standard Deviation or Variance

1. $\sqrt{9027.8 \left(\text{cm}^3\right)^2} < \sqrt{\sigma^2} < \sqrt{33299.8 \left(\text{cm}^3\right)^2} \Rightarrow 95.0 \text{ cm}^3 < \sigma < 182.5 \text{ cm}^3$. We have 95% confidence that the limits of 95.0 cm^3 and 182.5 cm^3 contain the true value of the standard deviation of brain volumes.

2. The format implies that $s = 15.7$, but s is given as 14.3. In general, a confidence interval for σ does not have s at the center.

3. The dotplot does not appear to depict sample data from a normally distributed population. The large sample size does not justify treating the values as being from a normally distributed population. Because the normality requirement is not satisfied, the confidence interval estimate of s should not be constructed using the methods of this section.

4. The normality requirement for a confidence interval estimate of σ has a much stricter normality requirement than the loose normality requirement for a confidence interval estimate of μ. Departures from normality have a much greater effect on confidence interval estimates of σ than on confidence interval estimates of μ.

5. df = 24, $\chi_L^2 = 12.401$, and $\chi_R^2 = 39.364$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(25-1)0.24^2}{39.364}} < \sigma < \sqrt{\frac{(25-1)0.24^2}{12.401}}$$

95% CI: 0.19 mg < σ < 0.33 mg

6. df = 36, $\chi_L^2 = 21.336$, and $\chi_R^2 = 54.437$

(Table: $\chi_L^2 = 24.433$, and $\chi_R^2 = 59.342$)

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(37-1)0.04111^2}{54.437}} < \sigma < \sqrt{\frac{(37-1)0.04111^2}{21.336}}$$

95% CI: 0.1340 g < σ < 0.02141 g

(Table: 0.01284 g < σ < 0.0200 g)

7. df = 146, $\chi_L^2 = 105.761$, and $\chi_R^2 = 193.761$

(Table: $\chi_L^2 = 67.328$, and $\chi_R^2 = 140.169$)

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(147-1)65.4^2}{193.761}} < \sigma < \sqrt{\frac{(147-1)65.4^2}{105.741}}$$

99% CI: 56.8 < σ < 76.8

(Table: 66.7 < σ < 96.3)

8. df = 152, $\chi_L^2 = 110.846$, and $\chi_R^2 = 200.657$

(Table: $\chi_L^2 = 67.328$, and $\chi_R^2 = 140.169$)

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(153-1)65.4^2}{200.657}} < \sigma < \sqrt{\frac{(153-1)65.4^2}{110.846}}$$

99% CI: 6.18 cm < σ < 8.31 cm

(Table: 7.39 cm < σ < 10.68 cm)

9. df = 100, $\chi_L^2 = 74.222$, and $\chi_R^2 = 129.561$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(106-1)0.62^2}{129.561}} < \sigma < \sqrt{\frac{(106-1)0.62^2}{74.222}}$$

95% CI: 0.56°F < σ < 0.74°F

(Tech: 0.55°F < σ < 0.72°F)

10. df = 40, $\chi_L^2 = 26.509$, and $\chi_R^2 = 55.758$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(40-1)4.8^2}{55.758}} < \sigma < \sqrt{\frac{(40-1)4.8^2}{26.509}}$$

90% CI: 4.0 lb < σ < 5.9 lb

(Tech: 4.1 lb < σ < 5.8 lb)

The confidence interval gives us information about the variation among the amounts of lost weight, but it does not give us information about the effectiveness. of the diet.

11. df = 15, $\chi_L^2 = 5.229$, and $\chi_R^2 = 30.578$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(16-1)42.3^2}{30.578}} < \sigma < \sqrt{\frac{(16-1)42.3^2}{5.229}}$$

98% CI: 29.6 min < σ < 71.6 min

No, the confidence interval does not indicate whether the treatment is effective.

12. df = 50, $\chi_L^2 = 29.707$, and $\chi_R^2 = 76.154$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(49-1)21.0^2}{76.154}} < \sigma < \sqrt{\frac{(49-1)21.0^2}{29.707}}$$

98% CI: 16.7 mg/dL < σ < 26.7 mg/dL

(Tech: 16.9 mg/dL < σ < 27.4 mg/dL)

No, the confidence interval does not indicate whether the treatment is effective.

13. The sample appears to have a normal distribution.

df = 11, $\chi_L^2 = 4.575$, and $\chi_R^2 = 21.920$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(12-1)2.216^2}{21.920}} < \sigma < \sqrt{\frac{(12-1)2.216^2}{4.575}}$$

95% CI: 1.6 < σ < 3.8

14. The sample appears to have a normal distribution.

 df = 25, $\chi_L^2 = 13.120$, and $\chi_R^2 = 40.646$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(26-1)1.880^2}{40.646}} < \sigma < \sqrt{\frac{(26-1)1.880^2}{13.120}}$$

 95% CI: $1.5 < \sigma < 2.6$

15. The sample appears to have a normal distribution.

 df = 11, $\chi_L^2 = 3.816$, and $\chi_R^2 = 21.920$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(12-1)4.08^2}{21.920}} < \sigma < \sqrt{\frac{(12-1)4.08^2}{3.816}}$$

 95% CI: 2.9 mph $< \sigma <$ 6.9 mph

 Because traffic conditions vary considerably at different times during the day, the confidence interval is an estimate of the standard deviation of the population of speeds at 3:30 on a weekday, not other times.

16. a. The sample does not appear to have a normal distribution.

 df = 9, $\chi_L^2 = 2.700$, and $\chi_R^2 = 19.023$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(10-1)0.151^2}{19.023}} < \sigma < \sqrt{\frac{(10-1)0.151^2}{2.700}}$$

 95% CI: 0.33 min $< \sigma <$ 0.87 min

 b. The sample does not appear to have a normal distribution.

 df = 9, $\chi_L^2 = 2.700$, and $\chi_R^2 = 19.023$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(10-1)0.576^2}{19.023}} < \sigma < \sqrt{\frac{(10-1)0.576^2}{2.700}}$$

 95% CI: 1.25 min $< \sigma <$ 3.33 min

 c. The variation appears to be significantly lower with a single line. The single line appears to be better because customers are more satisfied if their waiting times are about the same.

17. a. The sample does not appear to have a normal distribution.

 df = 90, $\chi_L^2 = 65.647$, and $\chi_R^2 = 118.136$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(93-1)0.5276^2}{118.136}} < \sigma < \sqrt{\frac{(93-1)0.5276^2}{65.647}}$$

 95% CI: $0.47 < \sigma < 0.62$

 (Table: $0.46 < \sigma < 0.62$)

17. (continued)
    b. The sample does not appear to have a normal distribution.
    df = 90, $\chi_L^2 = 65.647$, and $\chi_R^2 = 118.136$
    $$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$
    $$\sqrt{\frac{(93-1)0.5608^2}{118.136}} < \sigma < \sqrt{\frac{(93-1)0.5608^2}{65.647}}$$
    95% CI: $0.49 < \sigma < 0.66$
    c. The amounts of variation are about the same.
18. a. The sample appears to have a normal distribution.
    df = 204, $\chi_L^2 = 166.338$, and $\chi_R^2 = 245.448$ (Using technology.)
    $$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$
    $$\sqrt{\frac{(205-1)706.3^2}{245.448}} < \sigma < \sqrt{\frac{(205-1)706.3^2}{166.338}}$$
    95% CI: 643.9 g $< \sigma <$ 782.1 g
    b. The sample appears to have a normal distribution.
    df = 194, $\chi_L^2 = 157.321$, and $\chi_R^2 = 234.465$ (Using technology.)
    $$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$
    $$\sqrt{\frac{(195-1)660.2^2}{234.465}} < \sigma < \sqrt{\frac{(195-1)660.2^2}{157.321}}$$
    95% CI: 600.5 g $< \sigma <$ 733.1 g
    c. The amounts of variation are about the same.
19. 19,205 is too large. There are not 19,205 statistics professors in the population, and even if there were, that sample size is too large to be practical.
20. 33,218 is too large for most sample designs.
21. The sample size is 48. No, with many very low incomes and a few high incomes, the distribution is likely to be skewed to the right and will not satisfy the requirement of a normal distribution.
22. The sample size is 336.
23. $\chi_L^2 = \frac{1}{2}\left[-z_{\alpha/2} + \sqrt{2k-1}\right]^2 = \frac{1}{2}\left[-2.575 + \sqrt{2 \cdot 152 - 1}\right]^2 = 109.993$ and
    $\chi_R^2 = \frac{1}{2}\left[z_{\alpha/2} + \sqrt{2k-1}\right]^2 = \frac{1}{2}\left[2.575 + \sqrt{2 \cdot 152 - 1}\right]^2 = 199.638$
    (Tech: Using $z_{\alpha/2} = 2.575829303$, $\chi_L^2 = 109.980$ and $\chi_R^2 = 199.655$; The approximate values are quite close to the actual critical values.
24. $n = \frac{1}{2}\left(\frac{z_{\alpha/2}}{d}\right)^2 = \frac{1}{2}\left(\frac{2.326}{0.15}\right)^2 = 121$ (Statdisk yields a sample size of 122.)

Section 7-4: Bootstrapping: Using Technology for Estimates

1. Without replacement, every sample would be identical to the original sample, so the proportions or means or standard deviations or variances would all be the same, and there would be no confidence "interval."

2. For the given sample, a bootstrap sample is another sample in which five of the taxi-out times are randomly selected with replacement.
3. Part (b): (There are only three elements, not five.), Part (d): (14 and 20 are not in the original sample), Part (e): (There are too many elements), are not possible bootstrap samples.
4. There is no universal exact number, but there should be at least 1000 bootstrap samples, and the use of 10,000 or more is common.
5. The proportions from the 10 samples (in ascending order) are: 0, 0, 0, 0, 0, 0.25, 0.25, 0.5, 0.5, and 0.5. $P_5 = 0.000$ and $P_{95} = 0.500$, so the 90% interval is $0.000 < p < 0.500$.
6. The proportions from the 10 samples (in ascending order) are: 0, 0.25, 0.25, 0.5, 0.5, 0.5, 0.5, 1.5, 0.75, and 0.75. $P_{10} = 0.125$ and $P_{90} = 0.750$, so the 80% interval is $0.125 < p < 0.750$.
7. a. The means from the 10 samples (in ascending order) are: –0.25, 0.5, 0.5, 0.75, 3, 3, 3, 5, 8.25, and 9. $P_{10} = 0.125$ and $P_{90} = 8.625$, so the 80% interval is $0.1 \text{ kg} < \mu < 8.6 \text{ kg}$.
   b. The standard deviations from the 10 samples (in ascending order) are: 1.5, 2.363, 2.886751, 2.887, 4, 5.5, 5.715, 5.715, 5.715476, and 6.976. $P_{10} = 1.931$ and $P_{90} = 6.346$, so the 80% interval is $1.9 \text{ kg} < \sigma < 6.3 \text{ kg}$.
8. a. The means from the 10 samples (in ascending order) are: 62, 70.5, 73.5, 77.5, 78.25, 81, 81, 85.25, 93, and 103.5. $P_{10} = 66.25$ and $P_{90} = 98.25$, so the 80% interval is $66.3 \text{ cW/kg} < \mu < 98.3 \text{ cW/kg}$.
   b. The standard deviations from the 10 samples (in ascending order) are: 15.5, 17.898, 27.713, 37.621, 42.429, 45, 47.067, 47.067, 51.772, and 57.449. $P_{10} = 16.699$ and $P_{90} = 54.610$, so the 80%interval is $16.7 \text{ cW/kg} < \sigma < 54.6 \text{ cW/kg}$.
9. Answers will vary, but here are typical answers.
   a. 90% CI: $-0.8 \text{ kg} < \mu < 7.8 \text{ kg}$
   b. 90% CI: $1.2 \text{ kg} < \sigma < 7.0 \text{ kg}$
10. Answers will vary, but here are typical answers.
   a. 90% CI: $50.0 \text{ cW/kg} < \mu < 118.3 \text{ cW/kg}$
   b. 90% CI: $9.8 \text{ cW/kg} < \sigma < 57.3 \text{ cW/kg}$
11. Answers will vary, but here are typical answers.
   a. 99% CI: $5.36 < \mu < 8.5$; This isn't dramatically different from $5.0 < \mu < 9.0$.
   b. 95% CI: $1.2 < \sigma < 2.9$; This isn't dramatically different from $1.6 < \sigma < 3.8$.
12. Answers will vary, but here are typical answers.
   a. 99% CI: $5.6 < \mu < 7.4$; This isn't dramatically different from $5.5 < \mu < 7.6$.
   b. 95% CI: $1.4 < \sigma < 2.2$; This isn't dramatically different from $1.5 < \sigma < 2.6$.
13. Answers will vary, but here is a typical result: 95% CI: $0.0608 < p < 0.123$. This is quite close to the confidence interval of $0.0615 < p < 0.121$ found in Exercise 13 from Section 7-1.
14. Answers will vary, but here is a typical result: 99% CI: $0.0208 < p < 0.0317$. This is quite close to the confidence interval of $0.0205 < p < 0.0311$ found in Exercise 14 from Section 7-1.
15. Answers will vary, but here is a typical result: 90% CI: $0.1356 < p < 0.152$. The result is essentially the same as the confidence interval of $0.135 < p < 0.152$ found in Exercise 15 from Section 7-1.
16. Answers will vary, but here is a typical result: 95% CI: $0.671 < p < 0.722$. The result is very close to the confidence interval of $0.671 < p < 0.723$ found in Exercise 16 from Section 7-1.
17. Answers will vary, but here is a typical result: 90% CI: $3.69 < \mu < 4.15$. This result is very close to the confidence interval $3.676 < \mu < 4.17$ found in Exercise 23 in Section 7-2.
18. Answers will vary, but here is a typical result: 99% CI: $21.6 \text{ mg} < \mu < 43.1 \text{ mg}$. This result is reasonably close to the confidence interval of $19.5 \text{ mg} < \mu < 45.6 \text{ mg}$ found in Exercise 22 in Section 7-2.

19. a. Answers will vary, but here is a typical result: 95% CI: 233.6 sec $< \mu <$ 245.1 sec.
    b. 95% CI: 234.4 sec $< \mu <$ 246.0 sec
    c. The result from the bootstrap method is reasonably close to the result found using the methods of Section 7-2.
20. a. Answers will vary, but here is a typical result: 95% CI: 9.0 sec $< \sigma <$ 30.9 sec.
    b. 95% CI: 17.0 sec $< \sigma <$ 25.4 sec
    c. The confidence interval from the bootstrap method is very different from the confidence interval found using the methods of Section 7-3. The bootstrap confidence interval is better. A histogram or normal quantile plot shows that the data are skewed to the left, so the normality requirement of Section 7-3 is not satisfied, and the confidence interval of 17.0 sec $< \mu <$ 25.4 sec is not likely to be very good.
21. a. Answers will vary, but here is a typical result: 95% CI: $2.5 < \sigma < 3.3$.
    b. 95% CI: $2.4 < \sigma < 3.7$
    c. The confidence interval from the bootstrap method is not very different from the confidence interval found using the methods of Section 7-3. Because a histogram or normal quantile plot shows that the sample appears to be from a population not having a normal distribution, the bootstrap confidence interval of $2.5 < \sigma < 3.3$ would be a better estimate of σ.
22. a. Answers will vary, but here is a typical result: 95% CI: $1.6 < \mu < 3.3$.
    b. 95% CI: $1.6 < \mu < 3.3$
    c. The confidence interval found from the bootstrap method is essentially the same as the confidence interval found using the methods of Section 7-2.
23. Answers will vary, but here is a typical result using 10,000 bootstrap samples: $2.5 < \sigma < 3.3$. This result is the same as the confidence interval found using 1000 bootstrap samples. In this case, increasing the number of bootstrap samples from 1000 to 10,000 does not have much of an effect on the confidence interval.
24. a. No. A histogram or normal quantile plot would show that the distribution of the sample data is far from normal.
    b. Yes. The sample means appear to have a distribution that is approximately normal.
    c. Yes. The sample standard deviations appear to have a distribution that is approximately normal.

Quick Quiz

1. $\hat{p} = \dfrac{0.692 + 0.748}{2} = 0.720$
2. We have 95% confidence that the limits of 0.692 and 0.748 contain the true value of the proportion of adults in the population who say that the law goes easy on celebrities.
3. $z_{0.01/2} = 2.576$ (Table: 2.575)
4. $40\% - 3.1\% < p < 40\% + 3.1\% \Rightarrow 36.9\% < p < 43.1\%$
5. $n = \dfrac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \dfrac{[1.96]^2 (0.25)}{0.04^2} = 601$
6. $n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\dfrac{2.326 \cdot 15}{3}\right]^2 = 136$
7. There is a loose requirement that the sample values are from a normally distributed population.
8. The degrees of freedom is the number of sample values that can vary after restrictions have been imposed on all of the values. For the sample data described in Exercise 7, $\text{df} = 12 - 1 = 11$.
9. $t_{0.05/2} = 2.201$
10. No, the use of the χ^2 distribution has a fairly strict requirement that the data must be from a normal distribution. The bootstrap method could be used to find a 95% confidence interval estimate of σ.

Review Exercises

1. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.40 \pm 1.96\sqrt{\frac{(0.40)(0.60)}{2036}} \Rightarrow 0.379 < p < 0.421$, or $37.9\% < p < 42.1\%$; Because we have 95% confidence that the limits of 37.9% and 42.1% contain the true percentage for the population of adults, we can safely say that fewer than 50% of adults prefer to get their news online.

2. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.645]^2 (0.25)}{0.04^2} = 423$

3. a. $\bar{x} = 2.926$

   b. 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 2.926 \pm 2.201 \cdot \frac{0.278027}{\sqrt{12}} \Rightarrow 2.749 < \mu < 3.102$

   c. We have 95% confidence that the limits of 2.749 and 3.102 contain the value of the population mean μ.

4. $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.575 \cdot 15}{4}\right]^2 = 94$

5. a. student t distribution

   b. normal distribution

   c. None of the three distributions is appropriate, but a confidence interval could be constructed by using bootstrap methods.

   d. χ^2 (chi-square distribution)

   e. normal distribution

6. a. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.96]^2 (0.25)}{0.03^2} = 1068$

   b. $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.96 \cdot 47}{5}\right]^2 = 340$

   c. You must take the larger sample of 1068.

7. The sample appears to be normally distributed. 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 143 \pm 2.201 \cdot \frac{259.78}{\sqrt{12}}$

   $\Rightarrow -22.1 \text{ sec} < \mu < 308.1 \text{ sec}$.

8. The sample appears to be normally distributed. $\text{df} = 11$, $\chi_L^2 = 3.816$, and $\chi_R^2 = 21.920$

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(12-1)259.78^2}{21.920}} < \sigma < \sqrt{\frac{(12-1)259.78^2}{3.816}}$$

   95% CI: $184.0 \text{ sec} < \sigma < 441.1 \text{ sec}$

9. Answers will vary, but here is a typical result: $7.1 \text{ sec} < \mu < 293.7 \text{ sec}$.

10. a. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.02 \pm 1.96\sqrt{\frac{(0.02)(0.98)}{1000}} \Rightarrow 0.0113 < p < 0.0287$

    b. Answers will vary, but here is a typical result: $0.0120 < p < 0290$.

    c. The confidence intervals are quite close.

Cumulative Review Exercises

1. $\bar{x} = \frac{(-46)+(-32)+\cdots+(-21)+(-19)+\cdots+28+103}{12} = -3.6 \text{ min}$, $Q_2 = \frac{(-23)+(-21)}{2} = -20.0$ min,

   $s = \sqrt{\frac{(-46-(-3.6))^2+(-32-(-3.6))^2+\cdots+(28-(-3.6))^2+(103-(-3.6))^2}{12-1}} = 39.9 \text{ min}$,

   range $= 103-(-46) = 149.0$ min.
2. Using the range rule of thumb, the limit separating significantly low values is $\mu - 2\sigma = -3.6 - 2(39.9) = -83.4$ min and the limit separating significantly high values is $\mu + 2\sigma = -3.6 + 2(39.9) = 76.2$ min. Because 103 min exceeds 76.2 min, the arrival delay time of 103 min is significantly high.
3. ratio level of measurement; continuous data.
4. The sample appears to not be normally distributed, so the confidence interval might not be a good estimate of the population mean. 95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = -3.6 \pm 2.201 \cdot \frac{39.9}{\sqrt{12}} \Rightarrow -28.9 \text{ min} < \mu < 21.7 \text{ min}$.
5. a. $z_{x=15} = \frac{15.0-(-5.0)}{30.4} = 0.66$; which has a probability of $1 - 0.7454 = 0.02546$ (Tech: 0.553) to the right.

   b. The z score for the lower 75% is 0.67, which correspond to a time of $0.67 \cdot 30.4 + (-5.0) = 15.4$ min (Tech: 15.5 min).
6. $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.96 \cdot 30.4}{5}\right]^2 = 143$ flights
7. 99% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.803 \pm 2.575\sqrt{\frac{(0.803)(0.197)}{1000}} \Rightarrow 0.771 < p < 0.835$, or $77.1\% < p < 83.5\%$
8. The graphs suggest that the population has a distribution that is skewed to the right instead of being normal. The histogram shows that some taxi-out times can be very long, and can occur with heavy traffic, but little or no traffic cannot make the taxi-out time very low. There is a minimum time required, regardless of traffic conditions. Construction of a confidence interval estimate of a population standard deviation has a fairly strict requirement that the sample data are from a normally distributed population, and the graphs show that this strict normality requirement is not satisfied.

Chapter 8: Hypothesis Testing

Section 8-1: Basics of Hypothesis Testing

1. Rejection of the claim about aspirin is more serious because it is a drug used for medical treatments. The wrong aspirin dosage could cause more serious adverse reactions than a wrong vitamin C dosage. It would be wise to use a smaller significance level for testing the claim about the aspirin.
2. Estimates and hypothesis tests are both methods of inferential statistics, but they have different objectives. We could use the sample body temperatures to construct a confidence interval estimate of the population mean, but hypothesis testing is used to test some claim made about the value of the mean body temperature.
3. a. H_0: $\mu = 174.1$ cm
   b. H_1: $\mu \neq 174.1$ cm
   c. Reject the null hypothesis or fail to reject the null hypothesis.
   d. No, in this case, the original claim becomes the null hypothesis. For the claim that the mean height of men is equal to 174.1 cm, we can either reject that claim or fail to reject it, but we cannot state that there is sufficient evidence to *support* that claim.
4. The P-value of 0.001 is preferred because it corresponds to the sample evidence that most strongly supports the alternative hypothesis that the method is effective.
5. a. $p > 0.5$ (more than a majority)
   b. H_0: $p = 0.5$; H_1: $p > 0.5$
6. a. $p < 0.95$
   b. H_0: $p = 0.95$; H_1: $p < 0.95$
7. a. $\mu = 69$ bpm
   b. H_0: $\mu = 69$ bpm; H_1: $\mu \neq 69$ bpm
8. a. $\sigma = 11$ bpm
   b. H_0: $\sigma = 11$ bpm; H_1: $\sigma > 11$ bpm
9. There is sufficient evidence to support the claim that most adults would erase all of their personal information online if they could.
10. There is sufficient evidence to support the claim that fewer than 95% of adults have a cell phone.
11. There is not sufficient evidence to warrant rejection of the claim that the mean pulse rate (in beats per minute) of adult males is 69 bpm.
12. There is not sufficient evidence to support the claim that the standard deviation of pulse rates of adult males is more than 11 bpm.
13. $z = \dfrac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \dfrac{0.59 - 0.50}{\sqrt{\frac{(0.59)(0.41)}{565}}} = 4.28$ (if using $x = 0.59 \cdot 565 = 333$, $z = 4.25$)
14. $z = \dfrac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \dfrac{0.87 - 0.95}{\sqrt{\frac{(0.95)(0.05)}{1128}}} = -12.33$ (or $z = -12.38$ if using $x = 0.87 \cdot 1128 = 981$)
15. $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{69.6 - 69}{11.3/\sqrt{153}} = 0.657$
16. $\chi^2 = \dfrac{(153-1)s^2}{\sigma^2} = \dfrac{(40-1)11.3^2}{11^2} = 160.404$
17. a. right-tailed
    b. P-value $= P(z > 1.00) = 0.1587$
    c. $0.1587 > 0.05$; Fail to reject H_0.
18. a. left-tailed
    b. P-value $= P(z < -2.50) = 0.0062$
    c. $0.0062 < 0.05$; Reject H_0.
19. a. two-tailed
    b. P-value $= 2 \cdot P(z > 2.01) = 0.0444$
    c. $0.0444 < 0.05$; Reject H_0.
20. a. two-tailed
    b. P-value $= 2 \cdot P(z < -1.94) = 0.0524$
    c. $0.0524 > 0.05$; Fail to reject H_0.

21. a. Critical value: $z = 1.645$
    b. $1.00 < 1.645$; Fail to reject H_0.
22. a. Critical value: $z = -1.645$
    b. $-2.50 < -1.645$; Reject H_0.
23. a. Critical values: $z = \pm 1.645$
    b. $-2.01 < -1.645$; Reject H_0.
24. a. Critical values: $z = \pm 1.96$
    b. $-1.96 < -1.94 < 1.96$; Fail to reject H_0.
25. a. $0.3257 > 0.05$; Fail to reject H_0.
    b. There is not sufficient evidence to support the claim that more than 58% of adults would erase all of their personal information online if they could.
26. a. $0.0003 < 0.05$; Reject H_0.
    b. There is sufficient evidence to support the claim that fewer than 90% of adults have a cell phone.
27. a. $0.0095 < 0.05$; Reject H_0.
    b. There is sufficient evidence to warrant rejection of the claim that the mean pulse rate (in beats per minute) of adult males is 72 bpm.
28. a. $0.3045 > 0.05$; Fail to reject H_0.
    b. There is not sufficient evidence to support the claim that the standard deviation of pulse rates of adult males is more than 11 bpm.
29. Type I error: In reality $p = 0.1$, but we reject the claim that $p = 0.1$. Type II error: In reality $p \neq 0.1$, but we fail to reject the claim that $p = 0.1$.
30. Type I error: In reality $p = 0.35$, but we reject the claim that $p = 0.35$. Type II error: In reality $p \neq 0.35$, but we fail to reject the claim that $p \neq 0.35$.
31. Type I error: In reality $p = 0.87$, but we support the claim that $p > 0.87$. Type II error: In reality $p > 0.87$, but we fail to support that conclusion.
32. Type I error: In reality $p = 0.25$, but we support the claim that $p < 0.25$. Type II error: In reality $p < 0.25$, but we fail to support that conclusion.
33. The power of 0.96 shows that there is a 96% chance of rejecting the null hypothesis of $p = 0.08$ when the true proportion is actually 0.18. That is, if the proportion of Chantix users who experience abdominal pain is actually 0.18, then there is a 96% chance of supporting the claim that the proportion of Chantix users who experience abdominal pain is greater than 0.08.
34. a. From $p = 0.5$, $\hat{p} = 0.5 + 1.645\sqrt{\frac{(0.5)(0.5)}{64}} = 0.6028125$

    From $p = 0.65$, $z = \frac{0.6028125 - 0.65}{\sqrt{\frac{(0.65)(0.35)}{64}}} = -0.791$; Power $= P(z > -0.791) = 0.7852$ (Tech: 0.7857)

    b. Assuming that $p = 0.5$, as in the null hypothesis, the critical value of $z = 1.645$ corresponds to $\hat{p} = 0.6028125$, so any sample proportion greater than 0.6028125 causes us to reject the null hypothesis, as shown in the shaded critical region of the top graph. If p is actually 0.65, then the null hypothesis of $p = 0.5$ is false, and the actual probability of rejecting the null hypothesis is found by finding the area greater than $\hat{p} = 0.6028125$ in the bottom graph, which is the shaded area. That is, the shaded area in the bottom graph represents the probability of rejecting the false null hypothesis.
35. From $p = 0.5$, $\hat{p} = 0.5 + 1.645\sqrt{\frac{(0.5)(0.5)}{n}}$; from $p = 0.55$, since $P(z > -0.842) = 0.8000$,

    $\hat{p} = 0.55 - 0.842\sqrt{\frac{(0.55)(0.45)}{n}}$

35. (continued)

So: $0.5+1.645\sqrt{\frac{(0.5)(0.5)}{n}}=0.55-0.842\sqrt{\frac{(0.55)(0.45)}{n}}$

$$0.5\sqrt{n}+1.645\sqrt{0.25}=0.55\sqrt{n}-0.842\sqrt{0.2475}$$

$$0.05\sqrt{n}=1.645\sqrt{0.25}+0.842\sqrt{0.2475}$$

$$n=\left(\frac{1.645\sqrt{0.25}+0.842\sqrt{0.2475}}{0.05}\right)^2=617$$

Section 8-2: Testing a Claim About a Proportion

1. a. $0.53 \cdot 510 = 270$
   b. $\hat{p}=0.53$; The symbol $\hat{p}$ is used to represent a sample proportion.
2. $H_0: p=0.5$; $H_1: p>0.5$
3. The method based on a confidence interval is not equivalent to the P-value method and the critical value method.
4. a. The first requirement is violated because the sample is a voluntary response sample instead of being a simple random sample. Because a requirement is violated, the methods of this section should not be used to test the claim.
   b. If the P-value is very low (such as less than or equal to 0.05), "the null must go" means that we should reject the null hypothesis.
   c. The statement that "if the P is high, the null will fly" suggests that with a high P-value, the null hypothesis has been proved or is supported, but we should never make such a conclusion.
   d. Choosing a significance level with a number like 0.0483 would make it seem like you're scheming to reach a desired conclusion.
5. a. left-tailed.
   b. $z=-4.46$
   c. P-value $=0.000004$
   d. $H_0: p=0.1$; Reject the null hypothesis.
   e. There is sufficient evidence to support the claim that less than 10% of treated subjects experience headaches.
6. a. right-tailed.
   b. $z=2.92$
   c. P-value $=0.0017$
   d. $H_0: p=0.25$; Reject the null hypothesis.
   e. There is sufficient evidence to support the claim that more than 1/4 of adults feel comfortable in a self-driving vehicle.
7. a. two-tailed.
   b. $z=-1.69$
   c. P-value $=0.091$
   d. $H_0: p=0.92$; Fail to reject the null hypothesis.
   e. There is not sufficient evidence to warrant rejection of the claim that 92% of adults own cell phones.
8. a. two-tailed.
   b. $z=1.33$
   c. P-value $=0.1840$
   d. $H_0: p=0.5$; Fail to reject the null hypothesis.
   e. There is not sufficient evidence to warrant rejection of the claim that half of us say that we should replace passwords with biometric security.
9. $H_0: p=0.10$; $H_1: p\neq 0.10$; Test statistic: $z=\frac{\frac{33}{362}-0.10}{\sqrt{\frac{(0.10)(0.90)}{362}}}=-0.56$;

   P-value $=2\cdot P(z<-0.56)=0.5755$ (Tech: 0.5751); Critical values: $z=\pm 1.96$;

   Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the rate of inaccurate orders is equal to 10%. With 10% of the orders being inaccurate, it appears that McDonald's should work to lower that rate.

10. H_0: $p = 0.03$; H_1: $p \neq 0.03$; Test statistic: $z = \dfrac{\frac{153}{5924} - 0.03}{\sqrt{\frac{(0.03)(0.97)}{5924}}} = -1.88$;

    P-value $= 2 \cdot P(z < -1.88) = 0.0602$ (Tech: 0.0597); Critical values: $z = \pm 1.96$;

    Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that 3% of Eliquis users develop nausea. The rate of nausea appears to be quite low, so it is not a problematic adverse reaction.

11. H_0: $p = 0.5$; H_1: $p \neq 0.5$; Test statistic: $z = \dfrac{\frac{481}{882} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{882}}} = 2.69$;

    P-value $= 2 \cdot P(z > 2.69) = 0.0072$ (Tech: 0.0071); Critical values: $z = \pm 2.575$ (Tech: $z = \pm 2.576$);

    Reject H_0. There is sufficient evidence to reject the claim that the proportion of those in favor is equal to 0.5. The result suggests that the politician is wrong in claiming that the responses are random guesses equivalent to a coin toss.

12. H_0: $p = 0.24$; H_1: $p \neq 0.24$; Test statistic: $z = \dfrac{\frac{27}{100} - 0.24}{\sqrt{\frac{(0.24)(0.76)}{100}}} = 0.70$;

    P-value $= 2 \cdot P(z > 0.70) = 0.4840$ (Tech: 0.4824); Critical values: $z = \pm 1.96$;

    Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that 24% of M&Ms are blue. Corrective action is not warranted. Even if they were making blue M&Ms at a rate significantly different from 24%, that would not be a big consumer issue.

13. H_0: $p = 0.20$; H_1: $p > 0.20$; Test statistic: $z = \dfrac{\frac{52}{227} - 0.20}{\sqrt{\frac{(0.20)(0.80)}{227}}} = 1.10$;

    P-value $= P(z > 1.10) = 0.1357$ (Tech: 0.1367); Critical value: $z = 1.645$;

    Fail to reject H_0. There is not sufficient evidence to support the claim that more than 20% of OxyContin users develop nausea. However, with $\hat{p} = 0.229$, we see that a large percentage of OxyContin users experience nausea, so that rate does appear to be very high.

14. H_0: $p = 0.5$; H_1: $p > 0.5$; Test statistic: $z = \dfrac{\frac{856}{1228} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{1228}}} = 13.81$;

    P-value $= P(z > 13.81) = 0.0001$ (Tech: 0.0000); Critical value: $z = 2.33$;

    Reject H_0. There is sufficient evidence to support the claim that most medical malpractice lawsuits are dropped or dismissed. This should be comforting to physicians.

15. H_0: $p = 0.15$; H_1: $p < 0.15$; Test statistic: $z = \dfrac{\frac{717}{5000} - 0.15}{\sqrt{\frac{(0.15)(0.85)}{5000}}} = -1.31$;

    P-value $= P(z < -1.31) = 0.1357$ (Tech: 0.1367); Critical value: $z = -2.33$;

    Fail to reject H_0. There is not sufficient evidence to support the claim that the return rate is less than 15%.

16. H_0: $p = 0.10$; H_1: $p < 0.10$; Test statistic: $z = \dfrac{\frac{27}{300} - 0.10}{\sqrt{\frac{(0.10)(0.90)}{300}}} = -0.58$;

    P-value $= P(z < -0.58) = 0.2810$ (Tech: 0.2819); Critical value: $z = -1.645$;

Fail to reject H_0. There is not sufficient evidence to support the claim that less than 10% of the test results are wrong. The sample results suggest that the test is wrong too often to be considered very reliable.

17. $H_0: p = 0.512$; $H_1: p \neq 0.512$; Test statistic: $z = \dfrac{\frac{426}{860} - 0.512}{\sqrt{\frac{(0.512)(0.488)}{860}}} = -0.98$;

$P\text{-value} = 2 \cdot P(z < -0.98) = 0.3270$ (Tech: 0.3286); Critical values: $z = \pm 1.96$;

Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that 51.2% of newborn babies are boys. The results do not *support* the belief that 51.2% of newborn babies are boys; the results merely show that there is not strong evidence against the rate of 51.2%.

18. $H_0: p = 0.25$; $H_1: p \neq 0.25$; Test statistic: $z = \dfrac{\frac{152}{580} - 0.25}{\sqrt{\frac{(0.25)(0.25)}{580}}} = 0.67$;

$P\text{-value} = 2 \cdot P(z > 0.67) = 0.5028$ (Tech: 0.5021); Critical values: $z = \pm 2.575$ (Tech: $z = \pm 2.576$);

Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that 25% of offspring peas are yellow. There is not sufficient evidence to conclude that Mendel's rate of 25% is wrong.

19. $H_0: p = 0.80$; $H_1: p < 0.80$; Test statistic: $z = \dfrac{\frac{74}{98} - 0.80}{\sqrt{\frac{(0.80)(0.20)}{98}}} = -1.11$;

$P\text{-value} = P(z < -1.11) = 0.1335$ (Tech: 0.1332); Critical value: $z = -1.645$;

Fail to reject H_0. There is not sufficient evidence to support the claim that the polygraph results are correct less than 80% of the time. However, based on the sample proportion of correct results in 75.5% of the 98 cases, polygraph results do not appear to have the high degree of reliability that would justify the use of polygraph results in court, so polygraph test results should be prohibited as evidence in trials.

20. $H_0: p = 1/3$; $H_1: p < 1/3$; Test statistic: $z = \dfrac{\frac{231}{879} - \frac{1}{3}}{\sqrt{\frac{(1/3)(2/3)}{879}}} = -4.44$;

$P\text{-value} = P(z < -4.44) = 0.0001$ (Tech: 0.0000); Critical value: $z = -2.33$;

Reject H_0. There is sufficient evidence to support the claim that fewer than $1/3$ of the challenges are successful. Players don't appear to be very good at recognizing referee errors.

21. $H_0: p = 0.5$; $H_1: p \neq 0.5$; Test statistic: $z = \dfrac{\frac{123}{280} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{280}}} = -2.03$;

$P\text{-value} = 2 \cdot P(z < -2.03) = 0.0424$ (Tech: 0.0422); Critical values: $z = \pm 1.645$;

Reject H_0. There is sufficient evidence to warrant rejection of the claim that touch therapists use a method equivalent to random guesses. However, their success rate of $123/280$, or 43.9%, indicates that they performed *worse* than random guesses, so they do not appear to be effective.

22. $H_0: p = 0.5$; $H_1: p \neq 0.5$; Test statistic: $z = \dfrac{\frac{123}{280} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{280}}} = -2.03$;

$P\text{-value} = 2 \cdot P(z < -2.03) = 0.0424$ (Tech: 0.0422); Critical values: $z = \pm 2.575$ (Tech: $z = \pm 2.576$);

Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that touch therapists use a method equivalent to random guesses. After changing the significance level from 0.10 to 0.01, the conclusion does change.

23. H_0: $p = 0.00034$; H_0: $p \neq 0.00034$; Test statistic: $z = \dfrac{\frac{135}{420{,}095} - 0.000340}{\sqrt{\frac{(0.000340)(0.99966)}{420{,}095}}} = -0.66$;

   P-value $= 2 \cdot P(z < -0.66) = 0.5092$ (Tech: 0.5122); Critical values: $z = \pm 2.81$;

   Fail to reject H_0. There is not sufficient evidence to support the claim that the rate is different from 0.0340%. Cell phone users should not be concerned about cancer of the brain or nervous system.

24. H_0: $p = 0.01$; H_1: $p \neq 0.01$; Test statistic: $z = \dfrac{\frac{20}{1234} - 0.01}{\sqrt{\frac{(0.01)(0.99)}{1234}}} = 2.19$;

   P-value $= 2 \cdot P(z > 2.19) = 0.0286$ (Tech: 0.0284); Critical values: $z = \pm 1.96$;

   Reject H_0. There is sufficient evidence to warrant rejection of the claim that 1% of sales are overcharges. Because 1.62% of the sampled items are overcharges, it appears that the error rate is worse with scanners, not better.

25. H_0: $p = 0.5$; H_1: $p > 0.5$; Test statistic: $z = \dfrac{\frac{28}{49} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{49}}} = 1.00$;

   P-value $= P(z > 1.00) = 0.1587$; Critical value: $z = 1.645$;

   Fail to reject H_0. There is not sufficient evidence to support the claim that the probability of an NFC team Super Bowl win is greater than one-half.

26. H_0: $p = 0.5$; H_1: $p > 0.5$; Test statistic: $z = \dfrac{\frac{39}{71} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{71}}} = 0.83$;

   P-value $= P(z > 0.83) = 0.2033$ (Tech: 0.2031); Critical value: $z = 1.645$;

   Fail to reject H_0. There is not sufficient evidence to support the claim that among smokers who try to quit with nicotine patch therapy, the majority are smoking one year after the treatment. There isn't sufficient evidence to conclude that the nicotine patch therapy is not effective.

27. H_0: $p = 0.5$; H_1: $p \neq 0.5$; Test statistic: $z = \dfrac{\frac{252}{460} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{460}}} = 2.05$;

   P-value $= 2 \cdot P(z > 2.05) = 0.0404$ (Tech: 0.0402); Critical values: $z = \pm 1.96$;

   Reject H_0. There is sufficient evidence to warrant rejection of the claim that the coin toss is fair in the sense that neither team has an advantage by winning it. The coin toss rule does not appear to be fair. This helps explain why the overtime rules were changed.

28. H_0: $p = 0.5$; H_1: $p < 0.5$; Test statistic: $z = \dfrac{\frac{6062}{12{,}000} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{12{,}000}}} = 1.13$;

   P-value $= P(z > 1.13) = 0.8708$ (Tech: 0.8712); Critical value: $z = -1.645$;

   Fail to reject H_0. There is not sufficient evidence to support the claim that less than 0.5 of the deaths occur the week before Thanksgiving. Based on these results, there is no indication that people can temporarily postpone their death to survive Thanksgiving. (With 50.5% of the deaths occurring before Thanksgiving, there is no way that the claim of a proportion *less than* 0.5 could be supported.)

29. $H_0: p = 0.5$; $H_1: p > 0.5$; Test statistic: $z = \dfrac{0.64 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{21{,}346}}} = 40.91$ (using $x = 13{,}661, z = 40.90$);

P-value $= P(z > 40.91) = 0.0001$ (Tech: 0.0000); Critical value: $z = 2.33$;

Reject H_0. There is sufficient evidence to support the claim that most people believe that the Loch Ness monster exists. Because the sample is a voluntary-response sample, the conclusion about the population might not be valid.

30. $H_0: p = 0.80$; $H_1: p \neq 0.80$; Test statistic: $z = \dfrac{0.828 - 0.80}{\sqrt{\frac{(0.80)(0.20)}{198}}} = 0.98$ (using $x = 164, z = 0.99$);

P-value $= 2 \cdot P(z > 0.98) = 0.3270$ (Tech: 0.3246); Critical values: $z = \pm 2.575$ (Tech: $z = \pm 2.576$);

Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that 80% of patients stop smoking when given sustained care. With a success rate around 80%, it appears that sustained care is effective.

31. $H_0: p = 0.791$; $H_1: p < 0.791$; Test statistic: $z = \dfrac{0.39 - 0.791}{\sqrt{\frac{(0.791)(0.209)}{870}}} = -29.09$ (using $x = 339, z = -29.11$);

P-value $= P(z < -29.09) = 0.0001$ (Tech: 0.0000) (using $x = 339$, 0.3222, Tech: 0.3198);

Critical value: $z = -2.33$;

Reject H_0. There is sufficient evidence to support the claim that the percentage of selected Americans of Mexican ancestry is less than 79.1%, so the jury selection process appears to be biased.

32. $H_0: p = 0.75$; $H_1: p > 0.75$; Test statistic: $z = \dfrac{0.817 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{3005}}} = 8.48$;

P-value $= P(z > 8.48) = 0.0001$ (Tech: 0.0000); Critical value: $z = -2.33$;

Reject H_0. There is sufficient evidence to support the claim that more than $3/4$ of adults use at least one prescription medication. With more than $3/4$ of adults using at least one prescription medication, it appears that prescription use among adults is high.

33. $H_0: p = 0.5$; $H_1: p \neq 0.5$;

Normal approximation:

$$z = \frac{\frac{9}{10} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{10}}} = 2.53; \quad P\text{-value} = 2 \cdot P(z > 2.53) = 0.0114$$

Exact:

$$P\text{-value} = 2 \cdot \left({}_{10}C_9 (0.5)^9 (0.5^1) + {}_{10}C_{10} (0.5)^{10} (0.5^0) \right) = 0.0215$$

Continuity Correction:

$$P\text{-value} = 2 \cdot \left({}_{10}C_9 (0.5)^9 (0.5^1) + {}_{10}C_{10} (0.5)^{10} (0.5^0) \right) - \frac{1}{2} \left({}_{10}C_9 (0.5)^9 (0.5^1) \right) = 0.0117$$

$H_0: p = 0.4$; $H_1: p \neq 0.4$;

Normal approximation:

$$z = \frac{\frac{9}{10} - 0.4}{\sqrt{\frac{(0.4)(0.6)}{10}}} = 3.23; \quad P\text{-value} = 2 \cdot P(z > 3.23) = 0.0012$$

Exact:

$$P\text{-value} = 2 \cdot \left({}_{10}C_9 (0.4)^9 (0.6^1) + {}_{10}C_{10} (0.4)^{10} (0.6^0) \right) = 0.0034$$

33. (continued)

Continuity Correction:

$$P\text{-value} = 2\cdot\left({}_{10}C_9\,(0.4)^9\left(0.6^1\right) + {}_{10}C_{10}\,(0.4)^{10}\left(0.6^0\right)\right) - \frac{1}{2}\left({}_{10}C_9\,(0.4)^9\left(0.6^1\right)\right) = 0.0018$$

$H_0: p = 0.5$; $H_1: p > 0.5$;

Normal approximation:

$$z = \frac{\frac{545}{1009} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{1009}}} = 2.53;\quad P\text{-value} = P(z > 2.53) = 0.0057 \text{ (Tech: 0.0054)}$$

Exact:

$$P\text{-value} = {}_{1009}C_{545}\,(0.5)^{545}\left(0.5^{464}\right) + \cdots + {}_{1009}C_{1009}\,(0.5)^{1009}\left(0.5^0\right) = 0.0059$$

Continuity Correction:

$$P\text{-value} = {}_{1009}C_{545}\,(0.5)^{545}\left(0.5^{464}\right) + \cdots + {}_{1009}C_{1009}\,(0.5)^{1009}\left(0.5^0\right) - \frac{1}{2}\left({}_{1009}C_{545}\,(0.5)^{545}\left(0.5^{464}\right)\right)$$
$$= 0.0054$$

The P-values agree reasonably well with the large sample size of $n = 1009$. The normal approximation to the binomial distribution works better as the sample size increases.

34. a. $H_0: p = 0.10$; $H_1: p \neq 0.10$;. Test statistic: $z = \frac{0.119 - 0.1}{\sqrt{\frac{(0.1)(0.9)}{1000}}} = 2.00$; Critical values: $z = \pm 1.96$;

Reject H_0. There is sufficient evidence to warrant rejection of the claim that the proportion of zeros is 0.1.

b. $H_0: p = 0.10$; $H_1: p \neq 0.10$; Test statistic: $z = \frac{0.119 - 0.1}{\sqrt{\frac{(0.1)(0.9)}{1000}}} = 2.00$;

$P\text{-value} = 2\cdot P(z > 2.00) = 0.0456$ (Tech: 0.0452);

Reject H_0. There is sufficient evidence to warrant rejection of the claim that the proportion of zeros is 0.1.

c. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.119 \pm 1.96\sqrt{\frac{(0.119)(0.881)}{1000}} \Rightarrow 0.0989 < p < 0.139$; Because 0.1 is contained within the confidence interval, fail to reject $H_0: p = 0.10$. There is not sufficient evidence to warrant rejection of the claim that the proportion of zeros is 0.1.

d. The traditional and P-value methods both lead to rejection of the claim, but the confidence interval method does not lead to rejection of the claim.

35. a. From $p = 0.40$, $\hat{p} = 0.4 - 1.645\sqrt{\frac{(0.4)(0.6)}{50}} = 0.286$

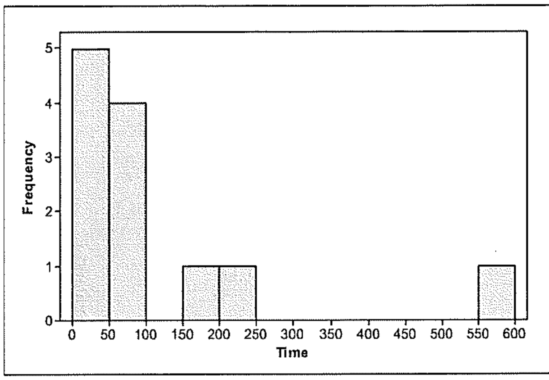
From $p = 0.25$, $z = \frac{0.286 - 0.25}{\sqrt{\frac{(0.25)(0.75)}{50}}} = 0.588$; Power $= P(z < 0.588) = 0.7224$ (Tech: 0.7219)

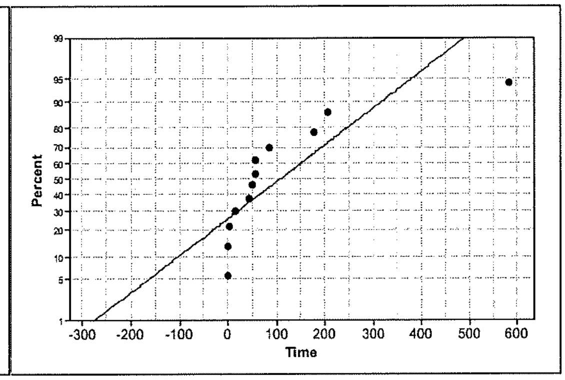
b. $1 - 0.7224 = 0.2776$ (Tech: 0.2781)

c. The power of 0.7224 shows that there is a reasonably good chance of making the correct decision of rejecting the false null hypothesis. It would be better if the power were even higher, such as greater than 0.8 or 0.9.

Section 8-3: Testing a Claim About a Mean

1. The requirements are (1) the sample must be a simple random sample, and (2) either or both of these conditions must be satisfied: The population is normally distributed or $n > 30$. There is not enough information given to determine whether the sample is a simple random sample. Because the sample size is not greater than 30, we must check for normality, but the value of 583 sec appears to be an outlier, and a normal quantile plot or histogram suggests that the sample does not appear to be from a normally distributed population.





2. df denotes the number of degrees of freedom. For the sample of 12 times, $\text{df} = 12 - 1 = 11$.
3. A t test is a hypothesis test that uses the Student t distribution, such as the method of testing a claim about a population mean as presented in this section. The t test methods are much more likely to be used than the z test methods because the t test does not require a known value of σ, and realistic hypothesis tests of claims about μ typically involve a population with an unknown value of σ.
4. For a 0.05 significance level used in a one-tailed test, use a 90% confidence level. The given confidence interval does contain the value of 90 sec, so it is possible that the value of μ is equal to 90 sec or some lower value, so there is not sufficient evidence to support the claim that the mean is greater than 90 sec.
5. $P\text{-value} = 0.1301$ (Table: $0.10 < P\text{-value} < 0.20$)
6. $P\text{-value} = 0.0437$ (Table: $0.250 < P\text{-value} < 0.05$)
7. $P\text{-value} = 0.2379$ (Table: $P\text{-value} > 0.20$)
8. $P\text{-value} = 0.4905$ (Table: $P\text{-value} > 0.10$)
9. H_0: $\mu = 4.00$ Mbps; H_1: $\mu < 4.00$ Mbps; Test statistic: $t = -0.366$; $P\text{-value} = 0.3579$;
   Critical value (with $\alpha = 0.05$): $t = -1.667$ (Table: -1.676);
   Fail to reject H_0. There is not sufficient evidence to support the claim that the Sprint airport data speeds are from a population having a mean less than 4.00 Mbps.
10. H_0: $\mu = 98.6°\text{F}$; H_1: $\mu \neq 98.6°\text{F}$; Test statistic: $t = -7.102$; $P\text{-value} < 0.0001$;
    Critical value (with $\alpha = 0.05$): $t = \pm 1.968$ (Table: $z = \pm 1.987$);
    Reject H_0. There is sufficient evidence to warrant rejection of the claim that the mean body temperature is equal to $98.6°\text{F}$.
11. H_0: $\mu = 0$ min; H_1: $\mu \neq 0$ min; Test statistic: $t = -8.720$; $P\text{-value} = 0.0000$;
    Critical value (with $\alpha = 0.05$): $t = \pm 1.970$;
    Reject H_0. There is sufficient evidence to warrant rejection of the claim that the mean prediction error is equal to zero. The predictions do not appear to be very accurate.
12. H_0: $\mu = 2.5$ mi; H_1: $\mu > 2.5$ mi; Test statistic: $t = 0.759$; $P\text{-value} = 0.2242$;
    Critical value (with $\alpha = 0.05$): $t = 1.648$;
    Fail to reject H_0. There is not sufficient evidence to support the claim that the mean tornado length is greater than 2.5 miles.

13. The data appear to follow a normal distribution and $n > 30$.
    H_0: $\mu = 4.00$; H_1: $\mu \neq 4.00$;
    Test statistic: $t = \dfrac{3.91 - 4.00}{0.53/\sqrt{93}} = -1.638$; Critical values: $t = \pm 1.987$ (Table: $t \approx \pm 1.676$);
    P-value $= 0.1049$ (Table: P-value > 0.10);
    Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the population of student course evaluations has a mean equal to 4.00.
14. The data appear to follow a normal distribution and $n > 30$.
    H_0: $\mu = 7.00$; H_1: $\mu < 7.00$;
    Test statistic: $t = \dfrac{6.19 - 7.00}{1.99/\sqrt{199}} = -5.742$; Critical value: $t = -2.345$ (Table: $t \approx -2.345$);
    P-value $= 0.0000$ (Table: P-value < 0.005);
    Reject H_0. There is sufficient evidence to support the claim that the population of ratings is less than 7.00.
15. The data cannot be verified to follow a normal distribution, but $n > 30$.
    H_0: $\mu = 0$; H_1: $\mu > 0$;
    Test statistic: $t = \dfrac{0.4 - 0}{21.0/\sqrt{49}} = -0.133$; Critical value: $t = 1.677$ (Table: $t \approx 1.676$);
    P-value $= 0.4472$ (Table: P-value > 0.10);
    Fail to reject H_0. There is not sufficient evidence to support the claim that with garlic treatment, the mean change in LDL cholesterol is greater than 0. The results suggest that the garlic treatment is not effective in reducing LDL cholesterol levels.
16. The data appear right-skewed, but $n > 30$.
    H_0: $\mu = 5.00$ km; H_1: $\mu \neq 5.00$ km;
    Test statistic: $t = \dfrac{5.82 - 5.00}{4.93/\sqrt{600}} = 4.074$; Critical values: $t = \pm 2.584$ (Table: $t \approx \pm 2.586$);
    P-value $= 0.0001$ (Table: P-value < 0.10);
    Reject H_0. There is sufficient evidence to warrant rejection of the claim that the population of earthquake depths has a mean equal to 5.00 km.
17. The data cannot be verified to follow a normal distribution, but $n > 30$.
    H_0: $\mu = 0$; H_1: $\mu > 0$;
    Test statistic: $t = \dfrac{3.0 - 0.0}{4.9/\sqrt{40}} = 3.872$; Critical value: $t = 2.426$;
    P-value $= 0.0002$ (Table: P-value < 0.005);
    Reject H_0. There is sufficient evidence to support the claim that the mean weight loss is greater than 0. Although the diet appears to have statistical significance, it does not appear to have practical significance, because the mean weight loss of only 3.0 lb does not seem to be worth the effort and cost.
18. The data cannot be verified to follow a normal distribution, and $n < 30$, so proceed with caution.
    H_0: $\mu = 48.0$ words; H_1: $\mu > 48.0$ words;
    Test statistic: $t = \dfrac{53.3 - 48.0}{15.7/\sqrt{10}} = 1.068$; Critical value: $t = 2.821$;
    P-value $= 0.1568$ (Table: P-value > 0.10);
    Fail to reject H_0. There is not sufficient evidence to support the claim that the mean number of defined words per page is greater than 48.0 words. There is not enough evidence to support the claim that there are more than 70,000 defined words in the dictionary.

19. The data appear right-skewed, but $n > 30$.
H_0: $\mu = 12.00$ oz; H_1: $\mu \neq 12.00$ oz;
Test statistic: $t = \dfrac{12.19 - 12.00}{0.11/\sqrt{36}} = 10.364$; Critical values: $t = \pm 2.030$;
P-value = 0.0000 (Table: P-value < 0.10);
Reject H_0. There is sufficient evidence to warrant rejection of the claim that the mean volume is equal to 12.00 oz. Because the mean appears to be greater than 12.00 oz, consumers are not being cheated because they are getting slightly more than 12.00 oz.

20. The data cannot be verified to follow a normal distribution and $n < 30$, so proceed with caution.
H_0: $\mu = 102.8$ min; H_1: $\mu < 102.8$ min;
Test statistic: $t = \dfrac{98.9 - 102.8}{42.3/\sqrt{16}} = -0.369$; Critical value (with $\alpha = 0.05$): $t = -1.753$;
P-value = 0.3587 (Table: P-value > 0.10);
Fail to reject H_0. There is not sufficient evidence to support the claim that after treatment with zopiclone, subjects have a mean wake time less than 102.8 min. These results suggest that the zoplicone treatment is not effective.

21. The sample data meet the loose requirement of having a normal distribution.
H_0: $\mu = 14\ \mu g/g$; H_1: $\mu < 14\ \mu g/g$;
Test statistic: $t = \dfrac{11.05 - 14.0}{6.46/\sqrt{10}} = -1.444$; Critical value: $t = -1.883$;
P-value = 0.0913 (Table: P-value > 0.05);
Fail to reject H_0. There is not sufficient evidence to support the claim that the mean lead concentration for all such medicines is less than 14 $\mu g/g$.

22. The sample data meet the loose requirement of having a normal distribution.
H_0: $\mu = 60$ sec; H_1: $\mu \neq 60$ sec;
Test statistic: $t = \dfrac{62.67 - 60.00}{9.48/\sqrt{15}} = 0.530$; Critical values: $t = \pm 2.145$;
P-value = 0.6043 (Table: P-value > 0.20);
Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the times are from a population with a mean equal to 60 seconds. Although some of the individual estimates are off by a large amount, the group of 15 students yielded the mean of 62.7 sec, which is not significantly different from 60 sec, so as a group they appear to be reasonably good at estimating one minute.

23. The data do not appear to follow a normal distribution and $n < 30$, so proceed with caution.
H_0: $\mu = 1000$ hic; H_1: $\mu < 1000$ hic;
Test statistic: $t = \dfrac{704 - 1000}{273/\sqrt{6}} = -2.661$; Critical value: $t = -3.365$;
P-value = 0.0224 (Table: $0.1 < P\text{-value} < 0.025$);
Fail to reject H_0. There is not sufficient evidence to support the claim that the population mean is less than 1000 hic. There is not strong evidence that the mean is less than 1000 hic, and one of the booster seats has a measurement of 1210 hic, which does not satisfy the specified requirement of being less than 1000 hic.

24. The sample data meet the loose requirement of having a normal distribution.
H_0: $\mu = 162$ cm; H_1: $\mu > 162$ cm;
Test statistic: $t = \dfrac{117.25 - 162}{1.84/\sqrt{16}} = 33.082$; Critical value: $t = 2.602$;
P-value = 0.0000 (Table: P-value < 0.005);

24. (continued)

Reject H_0. There is sufficient evidence to support the claim that supermodels have heights with a mean that is greater than the mean height of 162 cm for women in the general population. Supermodels appear to be taller than typical women.

25. The data appear to follow a normal distribution and $n > 30$.

H_0: $\mu = 75$ bpm; H_1: $\mu < 75$ bpm;

Test statistic: $t = \dfrac{74.04 - 75.00}{12.54/\sqrt{147}} = -0.927$; Critical value: $t = -1.655$ (Table: $t \approx -1.660$);

P-value $= 0.1777$ (Table: P-value > 0.10);

Fail to reject H_0. There is not sufficient evidence to support the claim that the mean pulse rate of adult females is less than 75 bpm.

26. The data appear to follow a normal distribution and $n > 30$.

H_0: $\mu = 2.50$; H_1: $\mu > 2.50$;

Test statistic: $t = \dfrac{2.50 - 2.5723}{0.6505/\sqrt{600}} = 2.721$; Critical value: $t = 2.333$ (Table: $t \approx 2.334$);

P-value $= 0.0033$ (Table: P-value < 0.005);

Reject H_0. There is sufficient evidence to support the claim that the sample is from a population with a mean greater than 2.50.

27. The data appear to follow a normal distribution and $n > 30$.

H_0: $\mu = 90$ mm Hg; H_1: $\mu < 90$ mm Hg;

Test statistic: $t = \dfrac{70.163 - 90}{11.22/\sqrt{147}} = -21.435$; Critical value: $t = -1.655$ (Table: $t \approx -1.660$);

P-value $= 0.0000$ (Table: P-value < 0.005);

Reject H_0. There is sufficient evidence to support the claim that the adult female population has a mean diastolic blood pressure level less than 90 mm Hg. The conclusion addresses the mean of a population, not individuals, so we cannot conclude that there are no female adults in the sample with hypertension.

28. The data appear to follow a normal distribution and $n > 30$.

H_0: $\mu = 90$ mm Hg; H_1: $\mu < 90$ mm Hg;

Test statistic: $t = \dfrac{71.32 - 90}{11.994/\sqrt{153}} = -19.265$; Critical value: $t = -1.655$ (Table: $t \approx -1.660$);

P-value $= 0.0000$ (Table: P-value < 0.005);

Reject H_0. There is sufficient evidence to support the claim that the adult male population has a mean diastolic blood pressure level less than 90 mm Hg. The conclusion addresses the mean of a population, not individuals, so we cannot conclude that there are no male adults in the sample with hypertension.

29. The data appear to follow a normal distribution and $n > 30$.

H_0: $\mu = 4.00$; H_1: $\mu \neq 4.00$;

Test statistic: $z = \dfrac{3.91 - 4.00}{0.53/\sqrt{93}} = -1.64$; Critical values: $z = \pm 1.96$;

P-value $= 0.1015$ (Table: 0.1010);

Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the population of student course evaluations has a mean equal to 4.00.

The null and alternative hypotheses are the same and the conclusions are the same. Results are not affected very much by the knowledge of σ.

30. The data appear to follow a normal distribution and $n > 30$.
H_0: $\mu = 7.00$; H_1: $\mu < 7.00$;
Test statistic: $z = \dfrac{6.19 - 7.00}{1.99/\sqrt{199}} = -5.74$; Critical value: $z = -2.33$;
P-value = 0.0000 (Table: 0.0001);
Reject H_0. There is sufficient evidence to support the claim that the population of ratings is less than 7.00. The null and alternative hypotheses are the same and the conclusions are the same. Results are not affected very much by the knowledge of σ.

31. $A = \dfrac{1.645(8 \cdot 499 + 3)}{8 \cdot 499 + 1} = 1.645823942$ and $t = \sqrt{499\left(e^{1.645823942^2/449} - 1\right)} = 1.648$; The approximation yields a critical t score of 1.648, which is the same as the value of 1.648 found from technology. The approximation appears to work quite well.

32. The power of 0.4943 shows that the chance of recognizing that $\mu < 7$ hours is not very high when in reality $\mu = 6.0$ hours. It would be better if the power was higher, such as 0.8 or greater. $\beta = 1 - 0.4943 = 0.5057$, so there is better than a 50% chance of failing to recognize that $\mu < 7$ hours when in reality $\mu = 6$.

Section 8-4: Testing a Claim About a Standard Deviation of Variance

1. The sample must be a simple random sample and the sample must be from a normally distributed population. The normality requirement for a hypothesis test of a claim about a standard deviation is much more strict, meaning that the distribution of the population must be much closer to a normal distribution.

2. H_0: $\sigma = 0.115$ oz; H_1: $\sigma \neq 0.115$ oz; Test statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(10-1)0.2684^2}{0.115^2} = 49.024$ (49.031 if using the original data values.); The sampling distribution of the test statistic is the χ^2 (chi-square) distribution.

3. a. Reject H_0.
   b. Reject the claim that the new filling process results in volumes with the same standard deviation of 0.115 oz.
   c. It appears that with the new filling process, the variation among volumes has increased, so the volumes are not as consistent. The new filling process appears to be inferior to the original filling process.

4. All of the values in the confidence interval are greater than the standard deviation of 0.115 oz from the original filling process, so it appears that the new filling process results in a larger standard deviation. With the new filling process, the cans would be filled with volumes having more variation.

5. H_0: $\sigma = 10$ bpm; H_1: $\sigma \neq 10$ bpm; Test statistic: $\chi^2 = 194.0888$; P-value = 0.0239;
Reject H_0. There is sufficient evidence to warrant rejection of the claim that pulse rates of men have a standard deviation equal to 10 beats per minute. Using the normal range of 60 to 100 beats per minute is not very good for estimating σ in this case.

6. H_0: $\sigma = 10$ bpm; H_1: $\sigma \neq 10$ bpm; Test statistic: $\chi^2 = 229.718$ or $\chi^2 = \dfrac{(147-1)12.5^2}{10^2} = 228.125$;
P-value < 0.0001;
Reject H_0. There is sufficient evidence to warrant rejection of the claim that pulse rates of women have a standard deviation equal to 10 beats per minute. Using the normal range of 60 to 100 beats per minute is not very good for estimating σ in this case.

7. H_0: $\sigma = 2.08°\text{F}$; H_1: $\sigma < 2.08°\text{F}$; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(106-1)0.62^2}{2.08^2} = 9.329$;

   P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 74.252$ (Table: $\chi^2 \approx 70.065$);
   Reject H_0. There is sufficient evidence to support the claim that body temperatures have a standard deviation less than $2.08°\text{F}$. It is very highly unlikely that the conclusion in the hypothesis test in Example 5 from Section 8-3 would change because of a standard deviation from a different sample.

8. H_0: $\sigma = 660.2$ hg; H_1: $\sigma \neq 660.2$ hg; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(30-1)829.5^2}{660.2^2} = 45.780$;

   P-value $= 0.0493$ (Table: P-value > 0.02); Critical values: $\chi^2 = 13.121$ and $\chi^2 = 52.336$;
   Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that birth weights of girls have the same standard deviation as the birth weights of boys.

9. H_0: $\sigma = 27.8$ lb; H_1: $\sigma \neq 27.8$ lb; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)18.6^2}{27.8^2} = 8.505$;

   P-value $= 0.0000$ (Table: P-value < 0.005); Critical values: $\chi^2 = 8.907$ and $\chi^2 = 32.852$;
   Reject H_0. There is sufficient evidence to warrant rejection of the claim that cans with thickness 0.0109 in. have axial loads with the same standard deviation as the axial loads of cans that are 0.0111 in. thick. The thickness of the cans does appear to affect the variation of the axial loads.

10. H_0: $\sigma = 14.1$; H_1: $\sigma < 14.1$; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(27-1)9.3^2}{14.1^2} = 11.311$;

    P-value $= 0.0056$ (Table: P-value < 0.01); Critical value: $\chi^2 = 12.198$;
    Reject H_0. There is sufficient evidence to support the claim that the last class has less variation. The lower variation implies that the scores are closer together, but it does not imply that the scores are higher, so the lower standard deviation does not suggest that the class is doing better.

11. H_0: $\sigma = 0.15$ oz; H_1: $\sigma > 0.15$ oz; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(27-1)0.17^2}{0.15^2} = 33.396$;

    P-value $= 0.1509$ (Table: P-value > 0.10); Critical value: $\chi^2 = 38.885$;
    Fail to reject H_0. There is not sufficient evidence to support the claim that the machine dispenses amounts with a standard deviation greater than the standard deviation of 0.15 oz specified in the machine design.

12. H_0: $\sigma = 7460$ words; H_1: $\sigma > 7460$ words; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(56-1)7871^2}{7460^2} = 61.227$;

    P-value $= 0.2625$ (Table: P-value > 0.10); Critical value: $\chi^2 = 82.292$ (Table: $\chi^2 \approx 76.145$);
    Fail to reject H_0. There is not sufficient evidence to support the claim that males have a standard deviation that is greater than the standard deviation of 7460 words for females.

13. H_0: $\sigma = 32.2$ ft; H_1: $\sigma > 32.2$ ft; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(12-1)52.441^2}{32.2^2} = 29.176$;

    P-value $= 0.0021$; Critical value: $\chi^2 = 19.675$;
    Reject H_0. There is sufficient evidence to support the claim that the new production method has errors with a standard deviation greater than 32.2 ft. The variation appears to be greater than in the past, so the new method appears to be worse, because there will be more altimeters that have larger errors. The company should take immediate action to reduce the variation.

14. H_0: $\sigma = 1.8$ min; H_1: $\sigma < 1.8$ min; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)0.4767^2}{1.8^2} = 0.631$;

P-value = 0.0001 (Table: P-value < 0.005); Critical value: $\chi^2 = 3.325$;

Reject H_0. There is sufficient evidence to support the claim that with a single waiting line, the waiting times have a standard deviation less than 1.8 min. Because the variation among waiting times appears to be reduced with the single waiting line, customers are happier because their waiting times are closer to being the same. Customers are not annoyed by being stuck in an individual line that takes much more time than other individual lines.

15. H_0: $\sigma = 55.3$ sec; H_1: $\sigma \neq 55.3$ sec; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(8-1)96.240^2}{55.93^2} = 27.726$;

P-value = 0.0084 (Table: P-value < 0.01); Critical values: $\chi^2 = 0.989$ and $\chi^2 = 20.278$;

Reject H_0. There is sufficient evidence to warrant rejection of the claim that service times at McDonald's have the same variation as service times at Wendy's. Drive-through service times during dinner times appear to vary more at McDonald's than those at Wendy's. Given the similar composition of the menus, McDonald's should consider methods for reducing the variation.

16. H_0: $\sigma = 0.0230$ g; H_1: $\sigma \neq 0.0230$ g; Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(9-1)0.01588^2}{0.0230^2} = 3.814$;

P-value = 0.2529 (Table: P-value > 0.20); Critical values: $\chi^2 = 1.344$ and $\chi^2 = 21.955$;

Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that wheat pennies are manufactured so that their weights have a standard deviation equal to 0.0230 g. It appears that the Mint specification is being met.

17. The data appear to be normally distributed. H_0: $\sigma = 55.93$ sec; H_1: $\sigma \neq 55.93$;

Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(50-1)62.938^2}{55.93^2} = 62.049$; P-value = 0.1996 (Table: P-value > 0.10);

Critical values: $\chi^2 = 27.249$ and $\chi^2 = 78.132$ (Table: $\chi^2 \approx 27.249$ and $\chi^2 \approx 78.132$);

Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that service times at McDonald's have the same variation as service times at Wendy's. Drive-through service times during dinner times appear have about the same variation at McDonald's and Wendy's. No action is warranted.

18. The data appear to be normally distributed. H_0: $\sigma = 0.0230$ g; H_1: $\sigma \neq 0.0230$ g;

Test statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(37-1)0.016480^2}{0.0230^2} = 18.483$; P-value = 0.0137 (Table: P-value > 0.10);

Critical values: $\chi^2 = 17.887$ and $\chi^2 = 61.581$ (Table: $\chi^2 \approx 20.707$ and $\chi^2 \approx 66.766$);

Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the pennies are manufactured so that their weights have a standard deviation equal to 0.0230 g. It appears that the Mint specification is being met.

19. Critical $\chi^2 = \frac{1}{2}\left(2.33 + \sqrt{2 \cdot 55 - 1}\right)^2 = 81.54$ (or 81.494 if using $z = 2.326348$ found from technology), which is reasonably close to the value of 22.465 obtained from STATDISK and Minitab.

20. Critical $\chi^2 = 55\left(1 - \frac{2}{9 \cdot 55} + \left(2.33\sqrt{\frac{2}{9 \cdot 55}}\right)\right)^3 = 82.360$ (or 82.309 if using $z = 2.326348$ found from technology), which is very close to the value of 82.292 obtained from STATDISK and Minitab.

Quick Quiz

1. a. t distribution
   b. Normal distribution
   c. Chi-square distribution
2. a. two-tailed
   b. left-tailed
   c. right-tailed
3. a. $H_0: p = 0.5$; $H_1: p > 0.5$
   b. Test statistic: $z = \dfrac{0.53 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{532}}} = 1.38$ (using $x = 282$, $z = 1.39$);
   c. Fail to reject H_0.
   d. There is not sufficient evidence to support the claim that the majority of Internet users aged 18–29 use Instagram.
4. P-value $= 0.10$
5. true
6. false
7. false
8. No, all critical values of χ^2 are always positive.
9. The t test requires that the sample is from a normally distributed population, and the test is robust in the sense that the test works reasonably well if the departure from normality is not too extreme. The χ^2 (chi-square) test is not robust against a departure from normality, meaning that the test does not work well if the population has a distribution that is far from normal.
10. The only true statement is the one given in part (a).

Review Exercises

1. a. false
   b. true
   c. false
   d. false
   e. false
2. $H_0: p = 0.5$; $H_1: p > 0.5$; Test statistic: $z = \dfrac{\frac{40}{41} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{41}}} = 6.09$;

   P-value $= P(z > 6.06) = 0.0001$ (Tech: 0.0000); Critical value: $z = 2.33$;

   Reject H_0. There is sufficient evidence to support the claim that the ballot selection method favors Democrats.
3. The data do not appear to follow a normal distribution, but $n > 30$.
   $H_0: \mu = 30$ years; $H_1: \mu > 30$ years;

   Test statistic: $t = \dfrac{36.2 - 30.0}{11.5/\sqrt{87}} = 5.029$; Critical value: $t = 2.370$ (Table: $t \approx 2.368$);

   P-value $= 0.0000$ (Table: P-value < 0.005);

   Reject H_0. There is sufficient evidence to support the claim that the mean age of actresses when they win Oscars is greater than 30 years.
4. The data cannot be verified to have a normal distribution, but $n > 30$.
   $H_0: \mu = 5.4$ million cells per microliter; $H_1: \mu < 5.4$ million cells per microliter;

   Test statistic: $t = \dfrac{4.932 - 5.4}{0.504/\sqrt{40}} = -5.873$; Critical value: $t = -2.426$;

   P-value $= 0.0000$ (Table: P-value < 0.005);

   Reject H_0. There is sufficient evidence to support the claim that the sample is from a population with a mean less than 5.4 million cells per microliter. The test deals with the distribution of sample means, not individual values, so the result does not suggest that each of the 40 males has a red blood cell count below 5.4 million cells per microliter.

5. H_0: $p = 0.43$; H_1: $p \neq 0.43$; Test statistic: $z = \dfrac{\frac{308}{611} - 0.43}{\sqrt{\frac{(0.43)(0.57)}{611}}} = 3.70$;

$P\text{-value} = 2 \cdot P(z > 3.70) = 0.0002$; Critical values: $z = \pm 1.96$;

Reject H_0. There is sufficient evidence to warrant rejection of the claim that the percentage who believe that they voted for the winning candidate is equal to 43%. There appears to be a substantial discrepancy between how people said that they voted and how they actually did vote.

6. The sample data meet the requirement of having a normal distribution.
H_0: $\mu = 20.16$; H_1: $\mu < 20.16$;

Test statistic: $t = \dfrac{18.76 - 20.16}{1.186/\sqrt{10}} = -3.732$; Critical value: $t = -2.821$;

$P\text{-value} = 0.0023$ (Table: $P\text{-value} < 0.005$);

Reject H_0. There is sufficient evidence to support the claim that the population of recent winners has a mean BMI less than 0.16. Recent winners appear to be significantly smaller than those from the 1920s and 1930s.

7. The sample data meet the requirement of having a normal distribution.

H_0: $\sigma = 1.34$; H_1: $\sigma \neq 1.34$; Test statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(10-1)1.186^2}{1.34^2} = 7.053$;

$P\text{-value} = 0.7368$ (Table: $P\text{-value} > 0.20$); Critical values: $\chi^2 = 1.735$ and $\chi^2 = 23.589$;

Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the recent winners have BMI values with variation different from that of the 1920s and 1930s.

8. a. A type I error is the mistake of rejecting a null hypothesis when it is actually true. A type II error is the mistake of failing to reject a null hypothesis when in reality it is false.
   b. Type I error: In reality, the mean BMI is equal to 20.16, but we support the claim that the mean BMI is less than 20.16. Type II error: In reality, the mean BMI is less than 20.16, but we fail to support that claim.

Cumulative Review Exercises

1. a. $\bar{x} = \dfrac{23+26+27+28+29+32+34+38+43+44+45+48+51+51}{14} = 37.1$ deaths

   b. $Q_2 = \dfrac{34+38}{2} = 36.0$ deaths

   c. $s = \sqrt{\dfrac{(23-37.1)^2+(26-37.1)^2+\cdots+(51-37.1)^2+(51-37.1)^2}{14-1}} = 9.8$ deaths

   d. $s^2 = 9.8^2 = 96.8 \text{ deaths}^2$

   e. range $= 51 - 23 = 28.0$ deaths

   f. The pattern of the data over time is not revealed by the statistics. A time-series graph would be very helpful in understanding the pattern over time.

2. a. ratio
   b. discrete
   c. quantitative
   d. No, the data are from recent and consecutive years, so they are not randomly selected.

3. 99% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 37.1 \pm 3.012 \cdot \dfrac{9.84}{\sqrt{14}} \Rightarrow 29.1 \text{ deaths} < \mu < 45.0 \text{ deaths}$; We have 99% confidence that the limits of 29.1 deaths and 45.0 deaths contain the value of the population mean.

4. The sample data meet the loose requirement of having a normal distribution.
H_0: $\mu = 72.6$ deaths; H_1: $\mu < 72.6$ deaths;

Test statistic: $t = \dfrac{37.07 - 72.6}{9.84/\sqrt{14}} = -13.509$; Critical value: $t = -2.650$;

P-value $= 0.0000$ (Table: P-value < 0.005);

Reject H_0. There is sufficient evidence to support the claim that the mean number of annual lightning deaths is now less than the mean of 72.6 deaths from the 1980s. Possible factors: Shift in population from rural to urban areas; better lightning protection and grounding in electric and cable and phone lines; better medical treatment of people struck by lightning; fewer people use phones attached to cords; better weather predictions.

5. Because the vertical scale starts at 50 and not at 0, the difference between the number of males and the number of females is exaggerated, so the graph is deceptive by creating the false impression that males account for nearly all lightning strike deaths. A comparison of the numbers of deaths shows that the number of male deaths is roughly 4 times the number of female deaths, but the graph makes it appear that the number of male deaths is around 25 times the number of female deaths.

6. H_0: $p = 0.5$; H_1: $p > 0.5$; Test statistic: $z = \dfrac{\frac{232}{287} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{287}}} = 10.45$;

P-value $= P(z > 10.45) = 0.0001$ (Tech: 0.0000); Critical value: $z = 2.33$;

Reject H_0. There is sufficient evidence to support the claim that the proportion of male deaths is greater than $1/2$. More males are involved in certain outdoor activities such as construction, fishing, and golf.

7. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = \dfrac{232}{287} \pm 1.96\sqrt{\dfrac{\left(\frac{232}{287}\right)\left(\frac{55}{287}\right)}{287}} \Rightarrow 0.762 < p < 0.854$; Because the entire confidence interval is greater than 0.5, it does not seem feasible that males and females have equal chances of being killed by lightning.

8. a. $0.8 \cdot 0.8 \cdot 0.8 = 0.512$
   b. $0.2 \cdot 0.2 \cdot 0.2 = 0.008$
   c. $1 - 0.2 \cdot 0.2 \cdot 0.2 = 0.992$
   d. ${}_5C_3\,(0.8)^3\left(0.2^2\right) = 0.205$
   e. $\mu = np = 50 \cdot 0.8 = 40$ males; $\sigma = \sqrt{npq} = \sqrt{50 \cdot 0.8 \cdot 0.2} = 2.8$ males
   f. Yes, using the range rule of thumb, values above $\mu + 2\sigma = 40.0 + 2(2.8) = 45.6$ are considered significantly high. Since 46 is greater than 45.6, 46 males would be a significantly high number in a group of 50.

Chapter 9: Inferences from Two Samples

Section 9-1: Two Proportions

1. The samples are simple random samples that are independent. For each of the two groups, the number of successes is at least 5 and the number of failures is at least 5. (Depending on what we call a success, the four numbers are 33, 115, 201,229 and 200,745 and all of those numbers are at least 5.) The requirements are satisfied.

2. $n_1 = 201{,}229, \hat{p}_1 = \dfrac{33}{201{,}299} = 0.000163992, \hat{q}_1 = 1 - 0.000163992 = 0.999836;$

   $n_2 = 200{,}745, \hat{p}_2 = \dfrac{115}{200{,}745} = 0.000572866, \hat{q}_2 = 1 - 0.000572866 = 0.999427;$

   $\overline{p} = \dfrac{33+115}{201{,}299 + 200{,}745} = 0.000368183, \overline{q} = 1 - 0.000368183 = 0.999632$

3. a. $H_0: p_1 = p_2; H_1: p_1 < p_2$
   b. There is sufficient evidence to support the claim that the rate of polio is less for children given the Salk vaccine than it is for children given a placebo. The Salk vaccine appears to be effective.

4. a. hypothesis test
   b. The *P*-value method and the critical value method are equivalent in the sense that they will always lead to the same conclusion, but the confidence interval method is not equivalent to them.
   c. 0.90, or 90%
   d. Because the confidence interval limits do not contain 0, there appears to be a significant difference between the two proportions. Because the confidence interval consists of negative values only, it appears that the first proportion is less than the second proportion. There is sufficient evidence to support the claim that the rate of polio is less for children given the Salk vaccine than it is for children given a placebo.

5. $H_0: p_1 = p_2; H_1: p_1 > p_2;$ population$_1$ = vinyl gloves, population$_2$ = latex gloves;
   Test statistic: $z = 12.82$; *P*-value = 0.0000; Critical value: $z = 2.33$; Reject H_0. There is sufficient evidence to support the claim that vinyl gloves have a greater virus leak rate than latex gloves.

6. $H_0: p_1 = p_2; H_1: p_1 > p_2;$ population$_1$ = surgery, population$_2$ = splints;
   Test statistic: $z = 3.12$; *P*-value = 0.0009; Critical value: $z = 2.33$; Reject H_0. There is sufficient evidence to support the claim that the success rate is better with surgery

For Exercises 7–22, assume that the data fit the requirements for the statistical methods for two proportions unless otherwise indicated.

7. a. $H_0: p_1 = p_2; H_1: p_1 > p_2;$ population$_1$ = cars, population$_2$ = trucks;
   Test statistic: $z = -0.95$; *P*-value = 0.8280 (Table: 0.5289); Critical value: $z = 1.645$; Fail to reject H_0. There is not sufficient evidence to support the claim that car owners violate license plate laws at a higher rate than owners of commercial trucks.

$$\overline{p} = \frac{239+45}{2049+334} = \frac{284}{2383}; \overline{q} = 1 - \frac{284}{2383} = \frac{2099}{2383};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\overline{p}\overline{q}}{n_1} + \dfrac{\overline{p}\overline{q}}{n_2}}} = \frac{\left(\dfrac{239}{2049} - \dfrac{45}{334}\right) - 0}{\sqrt{\dfrac{\left(\dfrac{284}{2383}\right)\left(\dfrac{2099}{2383}\right)}{2049} + \dfrac{\left(\dfrac{284}{2383}\right)\left(\dfrac{2099}{2383}\right)}{334}}} = -0.95$$

7. (continued)

   b. 90% CI: $-0.0510 < p_1 - p_2 < 0.0148$; Because the confidence interval limits contain 0, there is not a significant difference between the two proportions. There is not sufficient evidence to support the claim that car owners violate license plate laws at a higher rate than owners of commercial trucks.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{239}{2049} - \frac{45}{334}\right) \pm 1.645\sqrt{\frac{\left(\frac{239}{2049}\right)\left(\frac{1810}{2049}\right)}{2049} + \frac{\left(\frac{45}{334}\right)\left(\frac{289}{334}\right)}{334}}$$

8. a. $H_0: p_1 = p_2$; $H_1: p_1 \neq p_2$; population$_1$ = Burger King, population$_2$ = McDonald's;
   Test statistic: $z = -3.06$; P-value $= 0.0022$; Critical values: $z = \pm 1.96$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that Burger King and McDonald's have the same accuracy rates.

$$\bar{p} = \frac{264 + 329}{318 + 362} = \frac{593}{680};\ \bar{q} = 1 - \frac{593}{680} = \frac{87}{680};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{264}{318} - \frac{329}{362}\right) - 0}{\sqrt{\frac{\left(\frac{593}{680}\right)\left(\frac{87}{680}\right)}{318} + \frac{\left(\frac{593}{680}\right)\left(\frac{87}{680}\right)}{318}}} = -3.06$$

   b. 95% CI: $-0.129 < p_1 - p_2 < -0.0247$; Because the confidence interval limits do not contain 0, there appears to be a significant difference between the two proportions.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{264}{318} - \frac{329}{362}\right) \pm 1.96\sqrt{\frac{\left(\frac{264}{318}\right)\left(\frac{54}{318}\right)}{318} + \frac{\left(\frac{329}{362}\right)\left(\frac{33}{362}\right)}{362}}$$

   c. McDonald's appears to be better with a significantly higher accuracy rate.

9. a. $H_0: p_1 = p_2$; $H_1: p_1 > p_2$; population$_1$ = sustained care, population$_2$ = standard care;
   Test statistic: $z = 2.64$; P-value $= 0.0041$; Critical value: $z = 2.33$; Reject H_0. There is sufficient evidence to support the claim that the rate of success for smoking cessation is greater with the sustained care program.

$$\bar{p} = \frac{51 + 30}{198 + 199} = \frac{81}{397};\ \bar{q} = 1 - \frac{81}{397} = \frac{316}{397};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{51}{198} - \frac{30}{199}\right) - 0}{\sqrt{\frac{\left(\frac{81}{397}\right)\left(\frac{316}{397}\right)}{198} + \frac{\left(\frac{81}{397}\right)\left(\frac{316}{397}\right)}{199}}} = 2.64$$

   b. 98% CI: $0.0135 < p_1 - p_2 < 0.200$ (Table: $0.0134 < p_1 - p_2 < 0.200$); Because the confidence interval limits do not contain 0, there is a significant difference between the two proportions. Because the interval consists of positive numbers only, it appears that the success rate for the sustained care program is greater than the success rate for the standard care program.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{51}{198} - \frac{30}{199}\right) \pm 2.33\sqrt{\frac{\left(\frac{51}{198}\right)\left(\frac{147}{198}\right)}{198} + \frac{\left(\frac{30}{199}\right)\left(\frac{169}{199}\right)}{199}}$$

   c. Based on the samples, the success rates of the programs are 25.8% (sustained care) and 15.1% (standard care). That difference does appear to be substantial, so the difference between the programs does appear to have practical significance.

10. a. H_0: $p_1 = p_2$; H_1: $p_1 \neq p_2$; population$_1$ = men, population$_2$ = women;
Test statistic: $z = 1.23$; P-value = 0.2199 (Table: 0.2186); Critical values: $z = \pm 1.96$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that men and women have equal success in challenging calls.

$$\bar{p} = \frac{1027+509}{2441+1273} = \frac{256}{619}; \bar{q} = 1 - \frac{256}{619} = \frac{363}{619};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{1027}{2441} - \frac{509}{1273}\right) - 0}{\sqrt{\frac{\left(\frac{256}{619}\right)\left(\frac{363}{619}\right)}{2441} + \frac{\left(\frac{256}{619}\right)\left(\frac{363}{619}\right)}{1273}}} = 1.23$$

b. 95% CI: $-0.0124 < p_1 - p_2 < 0.0542$; Because the confidence interval limits contain 0, there is not a significant difference between the two proportions. There is not sufficient evidence to warrant rejection of the claim that men and women have equal success in challenging calls.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{1027}{2441} - \frac{509}{1273}\right) \pm 1.96\sqrt{\frac{\left(\frac{1027}{2441}\right)\left(\frac{1414}{2441}\right)}{2441} + \frac{\left(\frac{509}{1273}\right)\left(\frac{764}{1273}\right)}{1273}}$$

c. It appears that men and women have equal success in challenging calls.

11. a. H_0: $p_1 = p_2$; H_1: $p_1 > p_2$; population$_1$ = over age 55, population$_2$ = under age 25;
Test statistic: $z = 6.44$; P-value = 0.0000 (Table: 0.0001); Critical value: $z = 2.33$; Reject H_0. There is sufficient evidence to support the claim that the proportion of people over 55 who dream in black and white is greater than the proportion of those under 25.

$$\bar{p} = \frac{68+13}{306+298} = \frac{81}{604}; \bar{q} = 1 - \frac{81}{604} = \frac{523}{604};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{68}{306} - \frac{13}{298}\right) - 0}{\sqrt{\frac{\left(\frac{81}{604}\right)\left(\frac{523}{604}\right)}{306} + \frac{\left(\frac{81}{604}\right)\left(\frac{523}{604}\right)}{298}}} = 6.44$$

b. 98% CI: $0.117 < p_1 - p_2 < 0.240$; Because the confidence interval limits do not include 0, it appears that the two proportions are not equal. Because the confidence interval limits include only positive values, it appears that the proportion of people over 55 who dream in black and white is greater than the proportion of those under 25.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{68}{306} - \frac{13}{298}\right) \pm 2.33\sqrt{\frac{\left(\frac{68}{306}\right)\left(\frac{238}{306}\right)}{306} + \frac{\left(\frac{13}{298}\right)\left(\frac{285}{298}\right)}{298}}$$

c. The results suggest that the proportion of people over 55 who dream in black and white is greater than the proportion of those under 25, but the results cannot be used to verify the cause of that difference.

12. a. H_0: $p_1 = p_2$; H_1: $p_1 \neq p_2$; population$_1$ = OxyContin, population$_2$ = placebo;
Test statistic: $z = 1.78$; P-value = 0.0757 (Table: 0.0750); Critical values: $z = \pm 1.96$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a difference between the rates of nausea for those treated with OxyContin and those given a placebo.

12. (continued)

$$\bar{p}=\frac{52+5}{227+45}=\frac{57}{272};\ \bar{q}=1-\frac{57}{272}=\frac{215}{272};$$

$$z=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}=\frac{\left(\frac{52}{227}-\frac{5}{45}\right)-0}{\sqrt{\frac{\left(\frac{57}{272}\right)\left(\frac{215}{272}\right)}{227}+\frac{\left(\frac{57}{272}\right)\left(\frac{215}{272}\right)}{45}}}=1.78$$

b. 95% CI: $0.0111 < p_1 - p_2 < 0.225$; Because the confidence interval limits do not contain 0, there is a significant difference between the two proportions.

$$(\hat{p}_1-\hat{p}_2)\pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}=\left(\frac{52}{227}-\frac{5}{45}\right)\pm 1.96\sqrt{\frac{\left(\frac{52}{227}\right)\left(\frac{175}{227}\right)}{227}+\frac{\left(\frac{5}{45}\right)\left(\frac{40}{45}\right)}{45}}$$

c. The conclusions from parts (a) and (b) are different. The conclusion from part (a) results from a hypothesis test instead of an estimate of the difference between the two rates of nausea, so there does not appear to be sufficient evidence to conclude that there is a difference between the rates of nausea for those treated with OxyContin and those given a placebo.

13. a. $H_0\colon p_1 = p_2$; $H_1\colon p_1 > p_2$; population$_1$ = wearing seatbelt, population$_2$ = not wearing seatbelt; Test statistic: $z = 6.11$; P-value = 0.0000 (Table: 0.0001); Critical value: $z = 1.645$; Reject H_0. There is sufficient evidence to support the claim that the fatality rate is higher for those not wearing seat belts.

$$\bar{p}=\frac{31+16}{2823+7765}=\frac{47}{10{,}588};\ \bar{q}=1-\frac{47}{10{,}588}=\frac{10{,}541}{10{,}588};$$

$$z=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}=\frac{\left(\frac{31}{2823}-\frac{16}{7765}\right)-0}{\sqrt{\frac{\left(\frac{47}{10{,}588}\right)\left(\frac{10{,}541}{10{,}588}\right)}{2823}+\frac{\left(\frac{47}{10{,}588}\right)\left(\frac{10{,}541}{10{,}588}\right)}{7765}}}=6.11$$

b. 90% CI: $0.00559 < p_1 - p_2 < 0.0123$; Because the confidence interval limits do not include 0, it appears that the two fatality rates are not equal. Because the confidence interval limits include only positive values, it appears that the fatality rate is higher for those not wearing seat belts.

$$(\hat{p}_1-\hat{p}_2)\pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}=\left(\frac{31}{2823}-\frac{16}{7765}\right)\pm 1.645\sqrt{\frac{\left(\frac{31}{2823}\right)\left(\frac{2972}{2823}\right)}{2823}+\frac{\left(\frac{16}{7765}\right)\left(\frac{7749}{7765}\right)}{7765}}$$

c. The results suggest that the use of seat belts is associated with fatality rates lower than those associated with not using seat belts.

14. c. $H_0\colon p_1 = p_2$; $H_1\colon p_1 \neq p_2$; population$_1$ = day, population$_2$ = night; Test statistic: $z = 18.26$; P-value = 0.0000 (Table: 0.0002); Critical values: $z = \pm 2.576$ (Table: $z = \pm 2.575$); Reject H_0. There is sufficient evidence to warrant rejection of the claim that the survival rates are the same for day and night.

$$\bar{p}=\frac{11{,}604+4139}{58{,}593+28{,}155}=\frac{15{,}743}{86{,}748};\ \bar{q}=1-\frac{15{,}743}{86{,}748}=\frac{71{,}005}{86{,}748};$$

$$z=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}=\frac{\left(\frac{11{,}604}{58{,}593}-\frac{4139}{28{,}155}\right)-0}{\sqrt{\frac{\left(\frac{15{,}743}{86{,}748}\right)\left(\frac{71{,}005}{86{,}748}\right)}{58{,}593}+\frac{\left(\frac{15{,}743}{86{,}748}\right)\left(\frac{71{,}005}{86{,}748}\right)}{28{,}155}}}=18.26$$

14. (continued)

b. 99% CI: $0.0441 < p_1 - p_2 < 0.0579$; Because the confidence interval limits do not contain 0, there appears to be a significant difference between the two proportions. There is sufficient evidence to warrant rejection of the claim that the survival rates are the same for day and night.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} =$$

$$\left(\frac{11{,}604}{58{,}593} - \frac{4139}{28{,}155}\right) \pm 2.576\sqrt{\frac{\left(\frac{11{,}604}{58{,}593}\right)\left(\frac{46{,}989}{58{,}593}\right)}{58{,}593} + \frac{\left(\frac{4139}{28{,}155}\right)\left(\frac{54{,}016}{28{,}155}\right)}{28{,}155}}$$

c. The data suggest that for in-hospital patients who suffer cardiac arrest, the survival rate is not the same for day and night. It appears that the survival rate is higher for in-hospital patients who suffer cardiac arrest during the day.

15. a. $H_0: p_1 = p_2$; $H_1: p_1 \neq p_2$; population$_1$ = echinacea, population$_2$ = placebo; Test statistic: $z = 0.57$; P-value = 0.5720 (Table: 0.5868); Critical values: $z = \pm 1.96$; Fail to reject H_0. There is not sufficient evidence to support the claim that Echinacea treatment has an effect.

$$\bar{p} = \frac{40+88}{45+103} = \frac{32}{37};\ \bar{q} = 1 - \frac{32}{37} = \frac{5}{37};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{40}{45} - \frac{88}{103}\right) - 0}{\sqrt{\frac{\left(\frac{32}{37}\right)\left(\frac{5}{37}\right)}{45} + \frac{\left(\frac{32}{37}\right)\left(\frac{5}{37}\right)}{103}}} = 0.57$$

b. 95% CI: $-0.0798 < p_1 - p_2 < 0.149$; Because the confidence interval limits do contain 0, there is not a significant difference between the two proportions. There is not sufficient evidence to support the claim that Echinacea treatment has an effect.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{40}{45} - \frac{88}{103}\right) \pm 1.96\sqrt{\frac{\left(\frac{40}{45}\right)\left(\frac{5}{45}\right)}{45} + \frac{\left(\frac{88}{103}\right)\left(\frac{15}{103}\right)}{103}}$$

c. Echinacea does not appear to have a significant effect on the infection rate. Because it does not appear to have an effect, it should not be recommended.

16. a. $H_0: p_1 = p_2$; $H_1: p_1 < p_2$; population$_1$ = used bednet, population$_2$ = did not use bednet; Test statistic: $z = -2.44$; P-value = 0.0074; Critical value: $z = -2.33$; Reject H_0. There is sufficient evidence to support the claim that the incidence of malaria is lower for infants who use the bednets.

$$\bar{p} = \frac{15+27}{343+294} = \frac{6}{91};\ \bar{q} = 1 - \frac{6}{91} = \frac{85}{91};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{15}{343} - \frac{27}{294}\right) - 0}{\sqrt{\frac{\left(\frac{6}{91}\right)\left(\frac{85}{91}\right)}{343} + \frac{\left(\frac{6}{91}\right)\left(\frac{85}{91}\right)}{294}}} = -2.44$$

b. 98% CI: $-0.0950 < p_1 - p_2 < -0.00125$; Because the confidence interval does not include 0 and it includes only negative values, it appears that the rate of malaria is lower for infants who use the bednets.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{15}{343} - \frac{27}{294}\right) \pm 2.33\sqrt{\frac{\left(\frac{15}{343}\right)\left(\frac{328}{343}\right)}{343} + \frac{\left(\frac{27}{294}\right)\left(\frac{167}{294}\right)}{294}}$$

16. (continued)

c. The bednets appear to be effective.

17. a. $H_0: p_1 = p_2$; $H_1: p_1 < p_2$; population$_1$ = used left ear, population$_2$ = used right ear;
Test statistic: $z = -7.94$; P-value = 0.0000 (Table: 0.0001); Critical value: $z = -2.33$; Reject H_0.
There is sufficient evidence to support the claim that the rate of right-handedness for those who prefer to use their left ear for cell phones is less than the rate of right-handedness for those who prefer to use their right ear for cell phones.

$$\bar{p} = \frac{166+436}{216+452} = \frac{301}{334}; \bar{q} = 1 - \frac{301}{334} = \frac{33}{334};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{166}{216} - \frac{436}{452}\right) - 0}{\sqrt{\frac{\left(\frac{301}{334}\right)\left(\frac{33}{334}\right)}{216} + \frac{\left(\frac{301}{334}\right)\left(\frac{33}{334}\right)}{452}}} = -7.94$$

b. 98% CI: $-0.266 < p_1 - p_2 < -0.126$; Because the confidence interval limits do not contain 0, there is a significant difference between the two proportions. Because the interval consists of negative numbers only, it appears that the claim is supported. The difference between the populations does appear to have practical significance.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{166}{216} - \frac{436}{452}\right) \pm 2.33\sqrt{\frac{\left(\frac{166}{216}\right)\left(\frac{50}{216}\right)}{216} + \frac{\left(\frac{436}{452}\right)\left(\frac{16}{452}\right)}{452}}$$

18. a. $H_0: p_1 = p_2$; $H_1: p_1 < p_2$; population$_1$ = single bill, population$_2$ = multiple bills;
Test statistic: $z = -1.85$; P-value = 0.0324 (Table: 0.0322); Critical value: $z = -1.645$; Reject H_0.
There is sufficient evidence to support the claim that when given a single large bill, a smaller proportion of women in China spend some or all of the money when compared to the proportion of women in China given the same amount in smaller bills.

$$\bar{p} = \frac{60+68}{75+75} = \frac{64}{75}; \bar{q} = 1 - \frac{64}{75} = \frac{11}{75};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{60}{75} - \frac{68}{75}\right) - 0}{\sqrt{\frac{\left(\frac{64}{75}\right)\left(\frac{11}{75}\right)}{75} + \frac{\left(\frac{64}{75}\right)\left(\frac{11}{75}\right)}{75}}} = -1.85$$

b. 90% CI: $-0.201 < p_1 - p_2 < -0.0127$; Because the confidence interval does not include 0 and it includes only negative values, it appears that the first proportion is less than the second proportion. There is sufficient evidence to support the claim that when given a single large bill, a smaller proportion of women in China spend some or all of the money when compared to the proportion of women in China given the same amount in smaller bills.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{60}{75} - \frac{68}{75}\right) \pm 1.645\sqrt{\frac{\left(\frac{60}{75}\right)\left(\frac{15}{75}\right)}{75} + \frac{\left(\frac{68}{75}\right)\left(\frac{7}{75}\right)}{75}}$$

c. Because the P-value = 0.0324 (Table: 0.0322), the difference is significant at the 0.05 significance level, but not at the 0.01 significance level. The conclusion does change.

19. a. $H_0: p_1 = p_2$; $H_1: p_1 > p_2$; population$_1$ = oxygen, population$_2$ = placebo;
Test statistic: $z = 9.97$; P-value = 0.0000 (Table: 0.0001); Critical value: $z = 2.33$; Reject H_0. There is sufficient evidence to support the claim that the cure rate with oxygen treatment is higher than the cure rate for those given a placebo. It appears that the oxygen treatment is effective.

19. (continued)

$$\bar{p}=\frac{116+29}{150+148}=\frac{145}{298};\ \bar{q}=1-\frac{145}{298}=\frac{153}{298};$$

$$z=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}=\frac{\left(\frac{116}{150}-\frac{29}{148}\right)-0}{\sqrt{\frac{\left(\frac{145}{298}\right)\left(\frac{153}{298}\right)}{150}+\frac{\left(\frac{145}{298}\right)\left(\frac{153}{298}\right)}{148}}}=9.97$$

b. 98% CI: $0.467 < p_1 - p_2 < 0.687$; Because the confidence interval limits do not include 0, it appears that the two cure rates are not equal. Because the confidence interval limits include only positive values, it appears that the cure rate with oxygen treatment is higher than the cure rate for those given a placebo. It appears that the oxygen treatment is effective.

$$(\hat{p}_1-\hat{p}_2)\pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}=\left(\frac{116}{150}-\frac{29}{148}\right)\pm 2.33\sqrt{\frac{\left(\frac{116}{150}\right)\left(\frac{34}{150}\right)}{150}+\frac{\left(\frac{29}{148}\right)\left(\frac{119}{148}\right)}{148}}$$

c. The results suggest that the oxygen treatment is effective in curing cluster headaches.

20. a. H_0: $p_1 = p_2$; H_1: $p_1 \neq p_2$; population$_1$ = aspirin, population$_2$ = placebo;
Test statistic: $z = -5.19$; P-value = 0.0000 (Table: 0.0002); Critical values: $z = \pm 1.96$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that aspirin has no effect on myocardial infarctions.

$$\bar{p}=\frac{139+239}{11{,}037+11{,}034}=\frac{18}{1051};\ \bar{q}=1-\frac{18}{1051}=\frac{1033}{1051};$$

$$z=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}=\frac{\left(\frac{139}{11{,}037}-\frac{239}{11{,}034}\right)-0}{\sqrt{\frac{\left(\frac{18}{1051}\right)\left(\frac{1033}{1051}\right)}{11{,}037}+\frac{\left(\frac{18}{1051}\right)\left(\frac{1033}{1051}\right)}{11{,}034}}}=-5.19$$

b. 95% CI: $-0.0125 < p_1 - p_2 < -0.00564$; Because the confidence interval limits do not contain 0, there is a significant difference between the two proportions. There is sufficient evidence to warrant rejection of the claim that aspirin has no effect on myocardial infarctions.

$$(\hat{p}_1-\hat{p}_2)\pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}=\left(\frac{139}{11{,}037}-\frac{239}{11{,}034}\right)\pm 1.96\sqrt{\frac{\left(\frac{139}{11{,}037}\right)\left(\frac{10{,}898}{11{,}037}\right)}{11{,}037}+\frac{\left(\frac{239}{11{,}034}\right)\left(\frac{10{,}79}{11{,}034}\right)}{11{,}034}}$$

c. It appears that aspirin has an effect. Because the treatment group had a lower rate of myocardial infarctions, the aspirin treatment appears to be associated with a lower rate of myocardial infarctions. It should be noted that the treatment group and placebo group included only male physicians, so the results may or may not apply to the general population.

21. a. H_0: $p_1 = p_2$; H_1: $p_1 < p_2$; population$_1$ = male, population$_2$ = female;
Test statistic: $z = -1.17$; P-value = 0.1214 (Table: 0.1210); Critical value: $z = -2.33$; Fail to reject H_0. There is not sufficient evidence to support the claim that the rate of left-handedness among males is less than that among females.

21. (continued)

$$\bar{p}=\frac{23+65}{240+520}=\frac{11}{95};\ \bar{q}=1-\frac{11}{95}=\frac{84}{95};$$

$$z=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}=\frac{\left(\frac{23}{240}-\frac{65}{520}\right)-0}{\sqrt{\frac{\left(\frac{11}{95}\right)\left(\frac{84}{95}\right)}{240}+\frac{\left(\frac{11}{95}\right)\left(\frac{84}{95}\right)}{520}}}=-1.17$$

b. 98% CI: $-0.0848 < p_1-p_2 < -0.0264$ (Table: $-0.0849 < p_1-p_2 < -0.0265$); Because the confidence interval limits include 0, there does not appear to be a significant difference between the rate of left-handedness among males and the rate among females. There is not sufficient evidence to support the claim that the rate of left-handedness among males is less than that among females.

$$(\hat{p}_1-\hat{p}_2)\pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}=\left(\frac{23}{240}-\frac{65}{520}\right)\pm 2.33\sqrt{\frac{\left(\frac{23}{240}\right)\left(\frac{217}{240}\right)}{240}+\frac{\left(\frac{65}{520}\right)\left(\frac{455}{520}\right)}{520}}$$

c. The rate of left-handedness among males does not appear to be less than the rate of left-handedness among females.

22. a. H_0: $p_1=p_2$; H_1: $p_1>p_2$; population$_1$ = helicopter, population$_2$ = ground;
Test statistic: $z=10.75$; P-value $=0.0000$ (Table: 0.0001); Critical value: $z=2.33$; Reject H_0. There is sufficient evidence to support the claim that the rate of fatalities is higher for patients transported by helicopter.

$$\bar{p}=\frac{7813+17{,}775}{61{,}909+161{,}566}=\frac{25{,}588}{223{,}475};\ \bar{q}=1-\frac{25{,}588}{223{,}475}=\frac{197{,}887}{223{,}475};$$

$$z=\frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}=\frac{\left(\frac{7813}{61{,}909}-\frac{17{,}775}{161{,}566}\right)-0}{\sqrt{\frac{\left(\frac{25{,}588}{223{,}475}\right)\left(\frac{197{,}887}{223{,}475}\right)}{61{,}909}+\frac{\left(\frac{25{,}588}{223{,}475}\right)\left(\frac{197{,}887}{223{,}475}\right)}{161{,}566}}}=10.75$$

b. 98% CI: $0.0126 < p_1-p_2 < 0.0198$; Because the confidence interval limits do not contain 0, there is a significant difference between the two proportions. Because the entire range of values in the confidence interval consists of positive numbers, it appears that the rate of fatalities is higher for patients transported by helicopter.

$$(\hat{p}_1-\hat{p}_2)\pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}=$$

$$\left(\frac{7813}{61{,}909}-\frac{17{,}775}{161{,}566}\right)\pm 2.33\sqrt{\frac{\left(\frac{7813}{61{,}909}\right)\left(\frac{54{,}096}{61{,}909}\right)}{61{,}909}+\frac{\left(\frac{17{,}775}{161{,}566}\right)\left(\frac{143{,}791}{161{,}566}\right)}{161{,}566}}$$

c. The fatality rate for the helicopter sample is 0.126, or 12.6%, and the fatality rate for the ground services sample is 0.110, or 11.0%. The large sample sizes result in a significant difference, but it does not appear that the difference has very much practical significance. Also, it is possible that the most serious of the serious traumatic injuries led to helicopter transportation, and that could partly explain the higher rate of fatalities with helicopter transportation.

23. $n=\frac{z_{\alpha/2}^2}{2E^2}=\frac{1.96^2}{2\cdot 0.03^2}=2135$; The samples should include 2135 men and 2135 women.

24. a. The method of this section requires that both samples must have at least 5 successes and 5 failures, but the group not exposed to yawning includes a frequency of 4, which violates that requirement.

b. P-value = 0.3729 (Table: 0.3745); which is not close to the P-value of 0.5128 from Fisher's exact test. $H_0: p_1 = p_2$; $H_1: p_1 > p_2$; population$_1$ = yawning, population$_2$ = not yawning;

$$\bar{p} = \frac{10+4}{34+16} = \frac{7}{25};\ \bar{q} = 1 - \frac{7}{25} = \frac{18}{25};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{10}{34} - \frac{4}{16}\right) - 0}{\sqrt{\frac{\left(\frac{7}{25}\right)\left(\frac{18}{25}\right)}{34} + \frac{\left(\frac{7}{25}\right)\left(\frac{18}{25}\right)}{16}}} = 0.32$$

25. a. 95% CI: $0.0227 < p_1 - p_2 < 0.217$; population$_1$ = first sample, population$_2$ = second sample; Because the confidence interval limits do not contain 0, it appears that $p_1 = p_2$ can be rejected.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{112}{200} - \frac{88}{200}\right) \pm 1.96\sqrt{\frac{\left(\frac{112}{200}\right)\left(\frac{88}{200}\right)}{200} + \frac{\left(\frac{88}{200}\right)\left(\frac{112}{200}\right)}{200}}$$

b. First sample: 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = \frac{112}{200} \pm 1.96\sqrt{\frac{\left(\frac{112}{200}\right)\left(\frac{88}{200}\right)}{200}} \Rightarrow 0.491 < p_1 < 0.629$

Second sample: 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = \frac{88}{200} \pm 1.96\sqrt{\frac{\left(\frac{88}{200}\right)\left(\frac{112}{200}\right)}{200}} \Rightarrow 0.371 < p_2 < 0.509$

Because the confidence intervals do overlap, it appears that $p_1 = p_2$ cannot be rejected.

c. $H_0: p_1 = p_2$; $H_1: p_1 \neq p_2$; population$_1$ = first sample, population$_2$ = second sample;
Test statistic: $z = 2.40$; P-value = 0.0164; Critical values: $z = \pm 1.96$; Reject H_0. There is sufficient evidence to reject $p_1 = p_2$.

$$\bar{p} = \frac{112+88}{200+200} = \frac{1}{2};\ \bar{q} = 1 - \frac{1}{2} = \frac{1}{2};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{112}{200} - \frac{88}{200}\right) - 0}{\sqrt{\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{200} + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{200}}} = 2.40$$

d. Reject $p_1 = p_2$. The least effective method is using the overlap between the individual confidence intervals.

26. Hypothesis test: $H_0: p_1 = p_2$; $H_1: p_1 \neq p_2$; population$_1$ = first sample, population$_2$ = second sample;
Test statistic: $z = -1.9615$; P-value = 0.0498 (Table: 0.05); Critical values: $z = \pm 1.96$; Reject H_0. There is sufficient evidence to reject $p_1 = p_2$.

$$\bar{p} = \frac{10+1404}{20+2000} = \frac{7}{10};\ \bar{q} = 1 - \frac{7}{10} = \frac{3}{10};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\frac{10}{20} - \frac{1404}{2000}\right) - 0}{\sqrt{\frac{\left(\frac{7}{10}\right)\left(\frac{3}{10}\right)}{20} + \frac{\left(\frac{7}{10}\right)\left(\frac{3}{10}\right)}{2000}}} = -1.9615$$

26. (continued)
95% CI: $-0.422 < p_1 - p_2 < 0.0180$; population$_1$ = first sample, population$_2$ = second sample; which suggests that we should not reject $p_1 = p_2$ (because 0 is included).

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{10}{20} - \frac{1404}{2000}\right) \pm 1.96\sqrt{\frac{\left(\frac{10}{20}\right)\left(\frac{10}{20}\right)}{20} + \frac{\left(\frac{1404}{2000}\right)\left(\frac{596}{2000}\right)}{2000}}$$

The hypothesis test and confidence interval lead to different conclusions about the equality of $p_1 = p_2$.

Section 9-2: Two Means: Independent Samples

1. Only part (c) describes independent samples.
2. a. Because the confidence interval does not include 0, it appears that there is a significant difference between the mean level of hemoglobin in women and the mean level of hemoglobin in men.
   b. We have 95% confidence that the interval from 1.62 g/dL to 1.62 g/dL actually contains the value of the difference between the two population means $(\mu_1 - \mu_2)$.
   c. $1.62 \text{ g/dL} < \mu_1 - \mu_2 < 1.62 \text{ g/dL}$
3. a. yes
   b. yes
   c. 98%
4. The critical values of $t = \pm 2.201$ are more conservative than $t = \pm 2.093$ in the sense that rejection of the null hypothesis requires a *greater difference* between the sample means. The sample evidence must be stronger with $t = \pm 2.201$.
5. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 < \mu_2$; population$_1$ = Diet Coke, population$_2$ = regular Coke;
   Test statistic: $t = -22.092$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $t = -1.672$ (Table: $t = -1.690$); Reject H_0. There is sufficient evidence to support the claim that the contents of cans of Diet Coke have weights with a mean that is less than the mean for regular Coke.
   $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(0.78479 - 0.81682) - 0}{\sqrt{\frac{0.00439^2}{36} + \frac{0.00751^2}{36}}} = -22.092 \text{ (df} = 35)$$
   b. 90% CI: $-0.03445 \text{ lb} < \mu_1 - \mu_2 < -0.02961 \text{ lb}$ (Table: $-0.03448 \text{ lb} < \mu_1 - \mu_2 < -0.02958 \text{ lb}$);
   $$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (0.78479 - 0.81682) \pm 1.690\sqrt{\frac{0.00439^2}{36} + \frac{0.00751^2}{36}} \text{ (df} = 35)$$
   c. The contents in cans of Diet Coke appear to weigh less, probably due to the sugar present in regular Coke but not Diet Coke.
6. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = regular Coke, population$_2$ = regular Pepsi;
   Test statistic: $t = -3.995$; P-value = 0.0001 (Table: P-value < 0.01); Critical values: $t = \pm 1.996$ (Table: $t = \pm 2.030$); Reject H_0. There is sufficient evidence to warrant rejection of the claim that cans of regular Coke and regular Pepsi have the same mean volume.
   $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(12.19 - 12.29) - 0}{\sqrt{\frac{0.11^2}{36} + \frac{0.09^2}{36}}} = -3.995 \text{ (df} = 35)$$

6. (continued)

b. 95% CI: $-0.15 \text{ oz} < \mu_1 - \mu_2 < -0.05 \text{ oz}$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (12.19 - 12.29) \pm 2.030\sqrt{\frac{0.11^2}{36} + \frac{0.09^2}{36}} \text{ (df = 35)}$$

c. There does appear to be a difference between the mean volume of Coke and the mean volume of Pepsi. The difference is only about 0.1 oz, so it does not have practical significance.

7. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 < \mu_2$; population$_1$ = red background, population$_2$ = red background; Test statistic: $t = -2.979$; P-value = 0.0021 (Table: P-value < 0.005); Critical value: $t = -2.392$ (Table: $t = -2.441$); Reject H_0. There is sufficient evidence to support the claim that blue enhances performance on a creative task.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(3.39 - 3.97) - 0}{\sqrt{\frac{0.97^2}{35} + \frac{0.63^2}{36}}} = -2.979 \text{ (df = 34)}$$

b. 98% CI: $-1.05 < \mu_1 - \mu_2 < -0.11$ (Table: $-1.06 < \mu_1 - \mu_2 < -0.10$); The confidence interval consists of negative numbers only and does not include 0, so the mean creativity score with the red background appears to be less than the mean creativity score with the blue background. It appears that blue enhances performance on a creative task.

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (3.39 - 3.97) \pm 2.41\sqrt{\frac{0.97^2}{35} + \frac{0.63^2}{36}} \text{ (df = 34)}$$

8. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = red background, population$_2$ = red background; Test statistic: $t = -2.647$; P-value = 0.0101 (Table: P-value < 0.02); Critical values: $t = \pm 1.995$ (Table: $t = \pm 2.032$); Reject H_0. There is sufficient evidence to warrant rejection of the claim that the samples are from populations with the same mean. Color does appear to have an effect on word recall scores. Red appears to be associated with higher word memory recall scores.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(15.89 - 12.31) - 0}{\sqrt{\frac{5.90^2}{35} + \frac{5.48^2}{36}}} = -2.647 \text{ (df = 34)}$$

b. 95% CI: $-0.88 < \mu_1 - \mu_2 < 6.28$ (Table: $-0.83 < \mu_1 - \mu_2 < 6.33$); The confidence interval includes positive numbers only, so the mean score with a red background appears to be greater than the mean score with a blue background.

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (15.89 - 12.31) \pm 2.032\sqrt{\frac{5.90^2}{35} + \frac{5.48^2}{36}} \text{ (df = 34)}$$

c. The background color does appear to have an effect on word recall scores. Red appears to be associated with higher word memory recall scores.

9. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 > \mu_2$; population$_1$ = magnet treatment, population$_2$ = sham treatment; Test statistic: $t = 0.132$; P-value = 0.4480 (Table: P-value > 0.10); Critical value: $t = 1.691$ (Table: $t = 1.729$); Fail to reject H_0. There is not sufficient evidence to support the claim that the magnets are effective in reducing pain.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(0.49 - 0.44) - 0}{\sqrt{\frac{0.96^2}{20} + \frac{1.4^2}{20}}} = 0.132 \text{ (df = 19)}$$

9. (continued)

b. 90% CI: $-0.59 < \mu_1 - \mu_2 < 0.69$ (Table: $-0.61 < \mu_1 - \mu_2 < 0.71$);

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (0.49 - 0.44) \pm 1.729\sqrt{\frac{0.96^2}{20} + \frac{1.4^2}{20}} \text{ (df = 19)}$$

c. Magnets do not appear to be effective in treating back pain. It is valid to argue that the magnets *might* appear to be effective if the sample sizes were larger.

10. a. $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 > \mu_2$; population$_1$ = exposed, population$_2$ = not exposed;
Test statistic: $t = 1.845$; P-value = 0.0351 (Table: P-value < 0.05); Critical value: $t = 1.673$ (Table: $t = 1.685$); Reject H_0. There is sufficient evidence to support the claim that nonsmokers exposed to tobacco smoke have a higher mean cotinine level than nonsmokers not exposed to tobacco smoke.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(60.58 - 16.35) - 0}{\sqrt{\frac{138.08^2}{40} + \frac{62.53^2}{40}}} = 1.845 \text{ (df = 39)}$$

b. 90% CI: $4.12 \text{ ng/mL} < \mu_1 - \mu_2 < 84.34 \text{ ng/mL}$ (Table: $3.85 \text{ ng/mL} < \mu_1 - \mu_2 < 84.61 \text{ ng/mL}$);

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (60.58 - 16.35) \pm 1.685\sqrt{\frac{138.08^2}{40} + \frac{62.53^2}{40}} \text{ (df = 39)}$$

c. Exposure to second-hand smoke appears to have the effect of being associated with greater amounts of nicotine than for those not exposed to secondhand smoke.

11. a. $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 \neq \mu_2$; population$_1$ = female, population$_2$ = male;
Test statistic: $t = 0.674$; P-value = 0.5015 (Table: P-value > 0.20); Critical values: $t = \pm 1.979$ (Table: $t = \pm 1.995$); Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that females and males have the same mean BMI.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(29.10 - 28.38) - 0}{\sqrt{\frac{7.39^2}{70} + \frac{5.37^2}{80}}} = 0.647 \text{ (df = 69)}$$

b. 95% CI: $-1.39 < \mu_1 - \mu_2 < 2.83$ (Table: $-1.41 < \mu_1 - \mu_2 < 2.85$); Because the confidence interval includes 0, there is not sufficient evidence to warrant rejection of the claim that the two samples are from populations with the same mean.

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (29.10 - 28.38) \pm 1.995\sqrt{\frac{7.39^2}{70} + \frac{5.37^2}{80}} \text{ (df = 69)}$$

c. Based on the available sample data, it appears that males and females have the same mean BMI, but we can only conclude that there isn't sufficient evidence to say that they are different.

12. a. $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 > \mu_2$; population$_1$ = low lead, population$_2$ = high lead;
Test statistic: $t = 2.282$; P-value = 0.0132 (Table: P-value < 0.05); Critical value: $t = 1.673$ (Table: $t = 1.725$); Reject H_0. There is sufficient evidence to support the claim that the mean IQ score of people with low blood lead levels is higher than the mean IQ score of people with high blood lead levels.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(92.88462 - 86.90476) - 0}{\sqrt{\frac{15.34451^2}{78} + \frac{8.988352^2}{21}}} = 2.282 \text{ (df = 20)}$$

b. 90% CI: $1.6 < \mu_1 - \mu_2 < 10.4$ (Table: $1.5 < \mu_1 - \mu_2 < 10.5$);

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (92.88462 - 86.90476) \pm 1.725\sqrt{\frac{15.34451^2}{78} + \frac{8.988352^2}{21}} \text{ (df = 20)}$$

12. (continued)

c. Yes, it does appear that exposure to lead has an effect on IQ scores.

13. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = female, population$_2$ = male;
Test statistic: $t = -2.025$; P-value = 0.0460 (Table: P-value < 0.05); Critical values: $t = \pm 1.988$ (Table: $t = \pm 2.023$); Reject H_0. There is sufficient evidence to warrant rejection of the claim that the two samples are from populations with the same mean.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(3.79 - 4.01) - 0}{\sqrt{\frac{0.51^2}{40} + \frac{0.53^2}{53}}} = 0.647 \text{ (df = 39)}$$

b. 95% CI: $-0.44 < \mu_1 - \mu_2 < 0.00$; Because the confidence interval includes negative numbers only and does not include 0, there is sufficient evidence to warrant rejection of the claim that the two samples are from populations with the same mean.

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (3.79 - 4.01) \pm 2.023\sqrt{\frac{0.51^2}{40} + \frac{0.53^2}{53}} \text{ (df = 39)}$$

c. Yes, with the smaller samples of size 12 and 15, there was not sufficient evidence to warrant rejection of the null hypothesis, but there is sufficient evidence with the larger samples.

14. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 < \mu_2$; population$_1$ = wearing seatbelts, population$_2$ = not wearing seatbelts;
Test statistic: $t = -2.330$ P-value = 0.0102 (Table: P-value < 0.025); Critical value: $t = -1.649$; (Table: $t = -1.660$); Reject H_0. There is sufficient evidence to support the claim that children wearing seat belts have a lower mean length of time in an ICU than do children not wearing seat belts.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(0.83 - 1.39) - 0}{\sqrt{\frac{1.77^2}{123} + \frac{3.06^2}{290}}} = -2.330 \text{ (df = 122)}$$

b. 90% CI: -0.96 day $< \mu_1 - \mu_2 < -0.16$ day;

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (0.83 - 1.39) \pm 1.657\sqrt{\frac{1.77^2}{123} + \frac{3.06^2}{290}} \text{ (df = 122)}$$

c. It appears that after motor vehicle crashes, children wearing seat belts spend less time in intensive care units than children who don't wear seat belts. Children should wear seat belts (except for young children who should use properly installed car seats)!

15. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 > \mu_2$; population$_1$ = pre-1964, population$_2$ = post-1964;
Test statistic: $t = 32.771$; P-value = 0.0001 (Table: P-value < 0.005); Critical value: $t = 1.667$ (Table: $t = 1.685$); Reject H_0. There is sufficient evidence to support the claim that pre-1964 quarters have a mean weight that is greater than the mean weight of post-1964 quarters.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(6.19267 - 5.63930) - 0}{\sqrt{\frac{0.08700^2}{40} + \frac{0.06194^2}{40}}} = 32.771 \text{ (df = 39)}$$

b. 90% CI: 0.52522 lb $< \mu_1 - \mu_2 <$ 0.58125 lb (Table: 0.52492 lb $< \mu_1 - \mu_2 <$ 0.58182 lb);

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (6.19267 - 5.63930) \pm 1.685\sqrt{\frac{0.08700^2}{40} + \frac{0.06194^2}{40}} \text{ (df = 39)}$$

c. Yes, vending machines are not affected very much because pre-1964 quarters are mostly out of circulation.

16. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = Disney movies, population$_2$ = other movies;
Test statistic: $t = 0.462$; P-value = 0.6465 (Table: P-value > 0.20); Critical values: $t = \pm 2.012$ (Table: $t = \pm 2.120$); Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that Disney animated children's movies and other animated children's movies have the same mean time showing tobacco use.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(61.6 - 49.3) - 0}{\sqrt{\frac{118.8^2}{33} + \frac{69.3^2}{17}}} = 0.462 \text{ (df} = 16)$$

b. 95% CI: $-41.3 \text{ sec} < \mu_1 - \mu_2 < 65.9 \text{ sec}$ (Table: $-44.2 \text{ sec} < \mu_1 - \mu_2 < 68.8 \text{ sec}$)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (61.6 - 49.3) \pm 2.120\sqrt{\frac{118.8^2}{33} + \frac{69.3^2}{17}} \text{(df} = 16)$$

c. The times appear to be from a population with a distribution that is not normal (the sample is right-skewed), but the methods in this section are robust against departures from normality. (Results obtained by using other methods confirm that the results obtained here are quite good, even though the non-Disney times appear to violate the normality requirement.)

17. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = female, population$_2$ = male;
Test statistic: $t = -0.315$; P-value = 0.7576 (Table: P-value > 0.20); Critical values ($\alpha = 0.05$): $t = \pm 2.159$ (Table: $t = \pm 2.262$); Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that female professors and male professors have the same mean evaluation ratings. There does not appear to be a difference between male and female professor evaluation scores.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(4.02 - 4.10) - 0}{\sqrt{\frac{0.7208^2}{10} + \frac{0.3528^2}{10}}} = -0.315 \text{ (df} = 9)$$

18. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 > \mu_2$; population$_1$ = cars, population$_2$ = taxis;
Test statistic: $t = -0.301$; P-value = 0.6175 (Table: P-value > 0.10); Critical value ($\alpha = 0.05$): $t = 1.679$ (Table: $t = 1.729$); Fail to reject H_0. There is not sufficient evidence to support the claim that in Dublin, cars have a mean age that is greater than the mean age of taxis. The sample data do not support the expectation that taxis are newer.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(5.56 - 5.85) - 0}{\sqrt{\frac{3.876^2}{27} + \frac{2.834^2}{20}}} = -0.301 \text{ (df} = 19)$$

19. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = recent, population$_2$ = past;
Test statistic: $t = -2.385$; P-value = 0.0244 (Table: P-value < 0.05); Critical values ($\alpha = 0.05$): $t = \pm 2.052$ (Table: $t = \pm 2.201$); The conclusion depends on the choice of the significance level. There is a significant difference between the two population means at the 0.05 significance level, but not at the 0.01 significance level.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(78.82 - 89.08) - 0}{\sqrt{\frac{13.965^2}{17} + \frac{9.190^2}{12}}} = -2.385 \text{ (df} = 11)$$

20. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = easy to difficult, population$_2$ = difficult to easy;
Test statistic: $t = -2.657$; P-value = 0.0114 (Table: P-value < 0.02); Critical values ($\alpha = 0.05$): $t = \pm 2.023$ (Table: $t = \pm 2.131$); The conclusion depends on the choice of the significance level. There is a significant difference between the two population means at the 0.05 significance level, but not at the 0.01 significance level.

20. (continued)

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}=\frac{(27.115-31.728)-0}{\sqrt{\frac{6.857^2}{25}+\frac{4.260^2}{16}}}=-2.657 \text{ (df = 15)}$$

21. H_0: $\mu_1=\mu_2$; H_1: $\mu_1<\mu_2$; population$_1$ = men, population$_2$ = women;
Test statistic: $t=-0.132$; P-value = 0.4477 (Table: P-value > 0.10); Critical value: $t=-1.669$ (Table: $t=-1.688$); Fail to reject H_0. There is not sufficient evidence to support the claim that men talk less than women.

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}=\frac{(14{,}060.38-14{,}296.69)-0}{\sqrt{\frac{9065.03^2}{37}+\frac{6440.97^2}{42}}}=-0.132 \text{ (df = 36)}$$

22. H_0: $\mu_1=\mu_2$; H_1: $\mu_1\neq\mu_2$; population$_1$ = female, population$_2$ = male;
Test statistic: $t=-0.863$; P-value = 0.3887 (Table: P-value > 0.20); Critical values: $t=\pm1.968$ (Table: $t=\pm1.984$); Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that women and men have the same mean diastolic blood pressure.

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}=\frac{(70.16-71.32)-0}{\sqrt{\frac{11.22^2}{147}+\frac{11.99^2}{153}}}=-0.863 \text{ (df = 146)}$$

23. H_0: $\mu_1=\mu_2$; H_1: $\mu_1<\mu_2$; population$_1$ = girls, population$_2$ = boys;
Test statistic: $t=-3.450$; P-value = 0.0003 (Table: P-value < 0.005); Critical value ($\alpha=0.05$): $t=-1.649$; (Table: $t=-1.653$); Reject H_0. There is sufficient evidence to support the claim that at birth, girls have a lower mean weight than boys.

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}=\frac{(3037.07-3272.82)-0}{\sqrt{\frac{706.268^2}{205}+\frac{660.154^2}{195}}}=-3.450 \text{ (df = 194)}$$

24. H_0: $\mu_1=\mu_2$; H_1: $\mu_1\neq\mu_2$; population$_1$ = girls, population$_2$ = boys;
Test statistic: $t=0.903$; P-value = 0.3672 (Table: P-value > 0.05); Critical values ($\alpha=0.05$): $t=\pm1.966$ (Table: $t=\pm1.972$); Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that boys and girls have the same mean length of stay.

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}=\frac{(4.92-4.11)-0}{\sqrt{\frac{9.657^2}{205}+\frac{8.248^2}{195}}}=0.903 \text{ (df = 194)}$$

25. a. H_0: $\mu_1=\mu_2$; H_1: $\mu_1>\mu_2$; population$_1$ = low lead, population$_2$ = high lead;
Test statistic: $t=1.705$; P-value = 0.0457 (Table: P-value < 0.05); Critical value: $t=1.661$ (Table: $t=1.987$); df = 78 + 21 − 2 = 977; Reject H_0. There is sufficient evidence to support the claim that the mean IQ score of people with low blood lead levels is higher than the mean IQ score of people with high blood lead levels.

25. (continued)

$$s_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1-1)+(n_2-1)}=\frac{(78-1)15.34451^2+(21-1)8.988352^2}{(78-1)+(21-1)}=203.564;$$

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}}=\frac{(92.88462-86.90476)-0}{\sqrt{\frac{203.564}{78}+\frac{203.564}{21}}}=1.705\ (\text{df}=97)$$

b. 90% CI: $0.15<\mu_1-\mu_2<11.81$;

$$(\bar{x}_1-\bar{x}_2)\pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}=(92.88462-86.90476)\pm 1.661\sqrt{\frac{203.564}{78}+\frac{203.564}{21}}\ (\text{df}=97)$$

c. Yes, it does appear that exposure to lead has an effect on IQ scores.
With pooling, df increases dramatically to 97, but the test statistic decreases from 2.282 to 1.705 (because the estimated standard deviation increases from 2.620268 to 3.507614), the P-value increases to 0.0457, and the 90% confidence interval becomes wider. With pooling, these results do not show greater significance.

26. df = 38.9884; Using " $\text{df}=$ smaller of n_1-1 and n_2-1 " is a more conservative estimate of the number of degrees of freedom (than the estimate obtained with Formula 9-1) in the sense that the confidence interval is wider, so the difference between the sample means needs to be more extreme to be considered a significant difference.

$$\text{df}=\frac{\left(s_1^2/n_1+s_2^2/n_2\right)^2}{\frac{s_1^2/n_1}{n_1-1}+\frac{s_2^2/n_2}{n_2-1}}=\frac{\left(6.857^2/25+4.260^2/16\right)^2}{\frac{6.857^2/25}{25-1}+\frac{4.260^2/16}{16-1}}=38.9884$$

27. H_0: $\mu_1=\mu_2$; H_1: $\mu_1\neq\mu_2$; population$_1$ = treatment, population$_2$ = placebo;
Test statistic: $t=15.322$; P-value = 0.0000 (Table: P-value < 0.01); Critical values: $t=\pm 2.080$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the two populations have the same mean.

$$s_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1-1)+(n_2-1)}=\frac{(22-1)0.015^2+(22-1)0^2}{(22-1)+(22-1)}=0.000125;$$

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}}=\frac{(0.049-0.000)-0}{\sqrt{\frac{0.000125}{22}+\frac{0.000125}{22}}}=15.322$$

Section 9-3: Two Dependent Samples (Matched Pairs)

1. Only parts (a) and (c) are true.
2. $\bar{d}=-0.28$°F and $s_d=0.36$°F; μ_d represents the mean of the differences from the population of paired data.

Temperature (°F) at 8 am	97.8	99.0	97.4	97.4	97.5
Temperature (°F) at 12 am	98.6	99.5	97.5	97.3	97.6
Difference (°F)	−0.8	−0.5	−0.1	0.1	−0.1

3. The results will be the same.
4. $\text{df}=n-1=5-1=4$; $t_{\alpha/2}=4.604$

5. a. H_0: $\mu_d = 0$ year; H_1: $\mu_d < 0$ year; difference = actress − actor;
Test statistic: $t = -2.609$; P-value = 0.0142 (Table: P-value < 0.025); Critical value: $t = -1.833$;
Reject H_0. There is sufficient evidence to support the claim that for the population of ages of Best Actresses and Best Actors, the differences have a mean less than 0. There is sufficient evidence to conclude that Best Actresses are generally younger than Best Actors.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-9.7 - 0}{11.757/\sqrt{10}} = -2.609 \text{ (df = 9)}$$

b. 90% CI: -16.5 years $< \mu_d < -2.9$ years; The confidence interval consists of negative numbers only and does not include 0.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = -9.7 \pm 2.262\frac{11.757}{\sqrt{10}} \text{ (df = 9)}$$

6. a. H_0: $\mu_d = 0$ cm; H_1: $\mu_d > 0$ cm; difference = president − opponent;
Test statistic: $t = 1.304$; P-value = 0.1246 (Table: P-value > 0.10); Critical value: $t = 2.015$; Fail to reject H_0. There is not sufficient evidence to support the claim that for the population of heights of presidents and their main opponents, the differences have a mean greater than 0 cm (or that presidents tend to be taller than their opponents).

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{3.67 - 0}{6.890/\sqrt{6}} = 1.304 \text{ (df = 5)}$$

b. 90% CI: -2.0 cm $< \mu_d < 9.3$ cm; The confidence interval includes 0, so it is possible that $\mu_d = 0$.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = 3.67 \pm 2.015\frac{6.890}{\sqrt{6}} \text{ (df = 5)}$$

7. a. H_0: $\mu_d = 0$°F; H_1: $\mu_d \neq 0$°F; difference = 8 AM − 12 AM;
Test statistic: $t = -7.499$; P-value = 0.0003 (Table: P-value < 0.01); Critical values: $t = \pm 2.447$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that there is no difference between body temperatures measured at 8 AM and at 12 AM. There appears to be a difference.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-1.49 - 0}{0.524/\sqrt{7}} = -7.499 \text{ (df = 6)}$$

b. 95% CI: -1.97°F $< \mu_d < -1.00$°F; The confidence interval consists of negative numbers only and does not include 0.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = -1.49 \pm 2.447\frac{0.524}{\sqrt{7}} \text{ (df = 6)}$$

8. a. H_0: $\mu_d = 0$ words; H_1: $\mu_d < 0$ words; difference = male − female;
Test statistic: $t = -0.472$; P-value = 0.3265 (Table: P-value > 0.10); Critical value: $t = -1.895$; Fail to reject H_0. There is not sufficient evidence to support the claim that among couples, males speak fewer words in a day than females.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-1677.75 - 0}{10{,}052.87/\sqrt{8}} = -0.472 \text{ (df = 7)}$$

b. 90% CI: -8411.5 words $< \mu_d < 5056.0$ words (Table: -8413.0 words $< \mu_d < 5057.5$ words); The confidence interval includes 0 word, so it is possible that $\mu_d = 0$ word.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = -1677.75 \pm 1.895\frac{10{,}052.87}{\sqrt{8}} \text{ (df = 7)}$$

9. H_0: $\mu_d = 0$ in.; H_1: $\mu_d \neq 0$ in.; difference = mother – daughter;
Test statistic: $t = -7.499$; P-value = 0.2013 (Table: P-value > 0.20); Critical values: $t = \pm 2.262$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that there is no difference in heights between mothers and their first daughters.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-0.95 - 0}{2.179/\sqrt{10}} = -1.379 \text{ (df = 9)}$$

10. H_0: $\mu_d = 0$ in.; H_1: $\mu_d \neq 0$ in.; difference = father – son;
Test statistic: $t = 0.034$; P-value = 0.9737 (Table: P-value > 0.20); Critical values: $t = \pm 2.262$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that there is no difference in heights between fathers and their first sons.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{0.02 - 0}{1.863/\sqrt{10}} = -0.034 \text{ (df = 9)}$$

11. H_0: $\mu_d = 0$; H_1: $\mu_d \neq 0$; difference = male by female – female by male;
Test statistic: $t = 0.793$; P-value = 0.4509 (Table: P-value > 0.20); Critical values: $t = \pm 2.306$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a difference between female attribute ratings and male attribute ratings.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{1.89 - 0}{7.149/\sqrt{9}} = 0.793 \text{ (df = 8)}$$

12. H_0: $\mu_d = 0$; H_1: $\mu_d \neq 0$; difference = male by female – female by male;
Test statistic: $t = -0.333$; P-value = 0.7455 (Table: P-value > 0.20); Critical values: $t = \pm 2.201$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a difference between female attractiveness ratings and male attractiveness ratings.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-0.208 - 0}{2.168/\sqrt{12}} = -0.333 \text{ (df = 11)}$$

13. 95% CI: -6.5 admissions < μ_d < -0.2 admissions; Because the confidence interval does not include 0 admission, it appears that there is sufficient evidence to warrant rejection of the claim that when the 13th day of a month falls on a Friday, the numbers of hospital admissions from motor vehicle crashes are not affected. Hospital admissions do appear to be affected.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = -3.333 \pm 2.571\frac{3.011}{\sqrt{6}} \text{ (difference = 6th – 13th, df = 5)}$$

14. 99% CI: $-66.7 \text{ cm}^3 < \mu_d < 49.7 \text{ cm}^3$ (Table: $-66.8 \text{ cm}^3 < \mu_d < 49.8 \text{ cm}^3$); Because the confidence interval includes 0 cm^3, the mean of the differences could be equal to 0 cm^3, so there does not appear to be a significant difference.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = -8.5 \pm 3.249\frac{56.679}{\sqrt{10}} \text{ (difference = first born – second born, df = 9)}$$

15. 95% CI: $0.69 < \mu_d < 5.66$; Because the confidence interval limits do not contain 0 and they consist of positive values only, it appears that the "before" measurements are greater than the "after" measurements, so hypnotism does appear to be effective in reducing pain.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = 3.125 \pm 2.365\frac{2.911}{\sqrt{8}} \text{ (difference = before – after, df = 7)}$$

16. 99% CI: -4.16 in. $< \mu_d < 2.16$ in.; Because the confidence interval limits contain 0, there is not sufficient evidence to support a claim that there is a difference between self-reported heights and measured heights. We might believe that males would tend to exaggerate their heights, but the given data do not provide enough evidence to support that belief.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = -1.0 \pm 3.106\frac{3.520}{\sqrt{12}} \text{ (difference = reported − measured, df = 11)}$$

17. a. H_0: $\mu_d = 0$ year; H_1: $\mu_d < 0$ year; difference = actress − actor;
Test statistic: $t = -5.185$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $t = -1.663$ (Table: $t = -1.662$); Reject H_0. There is sufficient evidence to support the claim that actresses are generally younger than actors.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-7.89 - 0}{14.18/\sqrt{87}} = -5.185 \text{ (df = 86)}$$

b. 90% CI: -10.4 years $< \mu_d < -5.4$ years; The confidence interval consists of negative numbers only and does not include 0.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = -7.89 \pm 1.663\frac{14.18}{\sqrt{87}} \text{ (df = 86)}$$

18. a. H_0: $\mu_d = 0$ cm; H_1: $\mu_d > 0$ cm; difference = president − opponent;
Test statistic: $t = 0.036$; P-value = 0.4859 (Table: P-value > 0.10); Critical value: $t = 1.692$; Fail to reject H_0. There is not sufficient evidence to support the claim that for the population of heights of presidents and their main opponents, the differences have a mean greater than 0 cm. There is not sufficient evidence to support the claim that presidents tend to be taller than their main opponents.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{0.588 - 0}{9.604/\sqrt{34}} = 1.304 \text{ (df = 33)}$$

b. 90% CI: -2.7 cm $< \mu_d < 2.8$ cm; The confidence interval includes 0, so it is possible that $\mu_d = 0$.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = 0.588 \pm 1.692\frac{9.604}{\sqrt{34}} \text{ (df = 33)}$$

19. a. H_0: $\mu_d = 0°F$; H_1: $\mu_d \neq 0°F$; difference = 8 AM − 12 AM;
Test statistic: $t = -8.485$; P-value = 0.0000 (Table: P-value < 0.01); Critical values: $t = \pm 1.996$ (Table: $t = \pm 1.994$); Reject H_0. There is sufficient evidence to warrant rejection of the claim of no difference between body temperatures measured at 8 am and at 12 am. There appears to be a difference.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-0.85 + 0}{0.833/\sqrt{69}} = -8.485 \text{ (df = 68)}$$

b. 95% CI: $-1.05°F < \mu_d < -0.65°F$; The confidence interval consists of negative numbers only and does not include 0.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = -0.85 \pm 1.994\frac{0.833}{\sqrt{69}} \text{ (df = 68)}$$

20. a. H_0: $\mu_d = 0$ year; H_1: $\mu_d < 0$ year; difference = male − female;
Test statistic: $t = -1.560$; P-value = 0.0622 (Table: P-value > 0.05); Critical value: $t = -1.673$ (Table: $-1.671 < t < -1.676$); Fail to reject H_0. There is not sufficient evidence to support the claim that among couples, males speak fewer words in a day than females.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-1867.1 - 0}{8955.2/\sqrt{56}} = -0.472 \text{ (df = 55)}$$

20. (continued)

 b. 90% CI: -3869.2 words $< \mu_d < 135.0$ words (Table: -2872.7 words $< \mu_d < 138.5$ words); The confidence interval includes 0 word, so it is possible that $\mu_d = 0$ word.

$$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}} = -1867.1 \pm 1.673\frac{8955.2}{\sqrt{56}} \text{ (df = 55)}$$

21. H_0: $\mu_d = 0$ in.; H_1: $\mu_d \neq 0$ in.; difference = mother − daughter;
Test statistic: $t = -4.090$; P-value = 0.0001 (Table: P-value < 0.01); Critical values: $t = \pm 1.978$ (Table: $t \approx \pm 1.974$); Reject H_0. There is sufficient evidence to warrant rejection of the claim of no difference in heights between mothers and their first daughters.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-0.93 - 0}{2.636/\sqrt{134}} = -4.090 \text{ (df = 133)}$$

22. H_0: $\mu_d = 0$ in.; H_1: $\mu_d \neq 0$ in.; difference = father − son;
Test statistic: $t = -6.347$; P-value = 0.0000 (Table: P-value < 0.01); Critical values: $t = \pm 1.978$ (Table: $t \approx \pm 1.984$); Reject H_0. There is sufficient evidence to warrant rejection of the claim of no difference in heights between fathers and their first sons.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-1.366 - 0}{2.491/\sqrt{134}} = -0.034 \text{ (df = 133)}$$

23. H_0: $\mu_d = 0$; H_1: $\mu_d \neq 0$; difference = male by female − female by male;
Test statistic: $t = 0.191$; P-value = 0.8485 (Table: P-value > 0.20); Critical values: $t = \pm 1.972$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a difference between female attribute ratings and male attribute ratings.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{0.111 - 0}{8.154/\sqrt{199}} = 0.793 \text{ (df = 198)}$$

24. H_0: $\mu_d = 0$; H_1: $\mu_d \neq 0$; difference = male by female − female by male;
Test statistic: $t = -2.807$; P-value = 0.0055 (Table: P-value < 0.01); Critical values: $t = \pm 1.972$; Reject H_0. There is sufficient evidence to support the claim of a difference between female attractiveness ratings and male attractiveness ratings.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-0.513 - 0}{2.576/\sqrt{199}} = -0.333 \text{ (df = 198)}$$

25. For the temperatures in degrees Fahrenheit and the temperatures in degrees Celsius, the test statistic of $t = 0.124$ is the same, the P-value of 0.9023 is the same, the critical values of $t = \pm 2.028$ are the same, and the conclusions are the same, so the hypothesis test results are the same in both cases. The confidence intervals are $-0.25°\text{F} < \mu_d < 0.28°\text{F}$ and $-0.14°\text{C} < \mu_d < 0.16°\text{C}$. The confidence interval limits of $-0.14°\text{C}$ and $0.16°\text{C}$ have numerical values that are 5/9 of the numerical values of $-0.25°\text{F}$ and $0.28°\text{F}$.

26. a. No, it is possible that values of x are not normally distributed and values of y are not normally distributed, but values of $x - y$ are normally distributed.

 b. Answers vary, but here is a typical result: 95% CI: $-0.24°\text{F} < \mu_d < 0.26°\text{F}$; which is reasonably close to the confidence interval found in Exercise 25.

Section 9-4: Two Variances or Standard Deviations

1. a. No, the numerator will always be larger than the denominator in the fraction.
 b. No, both variances are nonnegative, so their quotient cannot be negative.
 c. The two samples have standard deviations (or variances) that are very close in value.
 d. skewed right

2. a. $s_1^2 = 55.99469 \text{ cm}^2$ and $s_2^2 = 50.42392 \text{ cm}^2$
   b. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 \neq \sigma_2$; population$_1$ = women, population$_2$ = men
   c. $F = s_1^2/s_2^2 = 7.48296^2/7.10098^2 = 1.1105$
   d. Fail to reject H_0. There is not sufficient evidence to support the claim that heights of men and heights of women have different variances.
3. No, unlike some other tests that have a requirement that samples must be from normally distributed populations or the samples must have more than 30 values, the F test has a requirement that the samples must be from normally distributed populations, regardless of how large the samples are.
4. The F test is very sensitive to departures from normality, which means that it works poorly by leading to wrong conclusions when either or both of the populations have a distribution that is not normal.
5. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 \neq \sigma_2$; population$_1$ = red, population$_2$ = blue;
   Test statistic: $F = s_1^2/s_2^2 = 0.97^2/0.63^2 = 2.3706$; P-value = 0.0129; Upper critical value: $F = 1.9678$ (Table: $1.8752 < F < 2.0739$); Reject H_0. There is sufficient evidence to warrant rejection of the claim that creative task scores have the same variation with a red background and a blue background.
6. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 \neq \sigma_2$; population$_1$ = red, population$_2$ = blue;
   Test statistic: $F = s_1^2/s_2^2 = 5.90^2/5.48^2 = 1.1592$; P-value = 0.6656; Upper critical value: $F = 1.9678$ (Table: $1.8752 < F < 2.0739$); Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that variation of scores is the same with the red background and blue background.
7. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 > \sigma_2$; population$_1$ = treatment, population$_2$ = placebo;
   Test statistic: $F = s_1^2/s_2^2 = 2.20^2/0.72^2 = 9.3364$; P-value = 0.0000; Critical value: $F = 2.0842$ (Table: $2.0540 < F < 2.0960$); Reject H_0. There is sufficient evidence to support the claim that the treatment group has errors that vary more than the errors of the placebo group.
8. a. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 > \sigma_2$; population$_1$ = exposed, population$_2$ = not exposed;
      Test statistic: $F = s_1^2/s_2^2 = 119.50^2/62.53^2 = 3.6552$; P-value = 0.0000; Critical value: $F = 1.7045$ (Table: $1.6928 < F < 1.8409$); Reject H_0. There is sufficient evidence to support the claim that the variation of cotinine in smokers is greater than the variation of cotinine in nonsmokers not exposed to tobacco smoke.
   b. The sample is not from a normally distributed population as required, so the results in part (a) are highly questionable.
9. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 \neq \sigma_2$; population$_1$ = regular Coke, population$_2$ = Diet Coke;
   Test statistic: $F = s_1^2/s_2^2 = 0.00751^2/0.00439^2 = 2.9265$; P-value = 0.0020; Upper critical value: $F = 1.9611$ (Table: $1.8752 < F < 2.0739$); Reject H_0. There is sufficient evidence to warrant rejection of the claim that variation is the same for both types of Coke.
10. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 > \sigma_2$; population$_1$ = low lead, population$_2$ = high lead;
    Test statistic: $F = s_1^2/s_2^2 = 15.34451^2/8.988352^2 = 2.9144$; P-value = 0.0045; Critical value: $F = 1.9246$ (Table: $1.8963 < F < 1.9464$); Reject H_0. There is sufficient evidence to support the claim that IQ scores of people with low lead levels vary more than IQ scores of people with high lead levels.
11. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 > \sigma_2$; population$_1$ = sham treatment, population$_2$ = magnet treatment;
    Test statistic: $F = s_1^2/s_2^2 = 1.4^2/0.96^2 = 2.1267$; P-value = 0.0543; Critical value: $F = 2.1682$ (Table: $2.1555 < F < 2.2341$); Fail to reject H_0. There is not sufficient evidence to support the claim that those given a sham treatment have pain reductions that vary more than the pain reductions for those treated with magnets.

12. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 \neq \sigma_2$; population$_1$ = cars, population$_2$ = taxis;
Test statistic: $F = s_1^2/s_2^2 = 3.876^2/2.834^2 = 1.8714$; P-value = 0.1627; Upper critical value: $F = 2.4300$ (Table: $2.3937 < F < 2.4523$); Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that in Dublin, car ages and taxi ages have the same variation.
13. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 \neq \sigma_2$; population$_1$ = female, population$_2$ = male;
Test statistic: $F = s_1^2/s_2^2 = 0.721^2/0.353^2 = 4.1750$; P-value = 0.0447; Upper critical value: $F = 4.0260$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that female professors and male professors have evaluation scores with the same variation.
14. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 > \sigma_2$; population$_1$ = men, population$_2$ = women;
Test statistic: $F = s_1^2/s_2^2 = 0.89^2/0.66^2 = 1.8184$; P-value = 0.0774; Critical value: $F = 1.9983$ (Table: $1.9926 < F < 2.0772$); Fail to reject H_0. There is not sufficient evidence to support the claim that men have body temperatures that vary more than the body temperatures of women.
15. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 \neq \sigma_2$; population$_1$ = recent, population$_2$ = past;
Test statistic: $F = s_1^2/s_2^2 = 13.965^2/9.190^2 = 2.3095$; P-value = 0.1635; Upper critical value ($\alpha = 0.05$): $F = 3.3044$ (Table: $3.2261 < F < 3.3299$); Fail to reject H_0. There is not sufficient evidence to support a claim that the variation of the times between eruptions has changed.
16. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 \neq \sigma_2$; population$_1$ = easy to difficult, population$_2$ = difficult to easy;
Test statistic: $F = s_1^2/s_2^2 = 6.857^2/4.260^2 = 2.5908$; P-value = 0.0599; Upper critical value: $F = 2.7006$; Fail to reject H_0. There is not sufficient evidence to support a claim that the two populations of scores have different amounts of variation.
17. a. Calculations not shown.
    b. $c_1 = 3, c_2 = 0$
    c. Critical value $= \dfrac{\log(0.05/2)}{\log\left(\dfrac{25}{25+16}\right)} = 7.4569$
    d. $c_1 = 3 < 7.4569$; Fail to reject H_0. There is not sufficient evidence to support a claim that the two populations of scores have different amounts of variation.
18. a. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = easy to difficult, population$_2$ = difficult to easy;
Test statistic: $t = 1.403$; P-value = 0.1686 (Table: P-value > 0.10); Critical values: $t = \pm 2.024$ (Table: $t = \pm 2.131$); Fail to reject H_0. There is not sufficient evidence to support a claim that the two populations of scores have different amounts of variation.
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{(4.914 - 3.296) - 0}{\sqrt{\dfrac{22.715^2}{25} + \dfrac{6.759^2}{16}}} = 1.403 \text{ (df} = 15)$$
19. $F_L = \dfrac{1}{2.4374} = 0.4103, F_R = 2.7006$

Quick Quiz

1. H_0: $p_1 = p_2$; H_1: $p_1 \neq p_2$; population$_1$ = women, population$_2$ = men;
2. $x_1 = 258, x_2 = 282, \hat{p}_1 = \dfrac{258}{1121} = 0.230, \hat{p}_2 = \dfrac{282}{1084} = 0.260, \bar{p} = \dfrac{258+282}{1121+1084} = 0.245$
3. P-value = 0.1015 (Table: 0.1010)

4. a. 95% CI: $-0.0659 < p_1 - p_2 < 0.00591$
   b. The confidence interval includes the value of 0, so it is possible that the two proportions are equal. There is not a significant difference.
5. Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that for the people who were aware of the statement, the proportion of women is equal to the proportion of men.
6. True, since $n > 30$.
7. False, the requirements are $np \geq 5$ and $nq \geq 5$.
8. Because the data consist of matched pairs, they are dependent.
9. H_0: $\mu_d = 0$; H_1: $\mu_d \neq 0$; difference = right arm − left arm;
10. a. $t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \dfrac{0.111 - 0}{8.154/\sqrt{199}}$
    b. $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$
    c. $z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}}$
    d. $F = \dfrac{s_1^2}{s_2^2}$

Review Exercises

1. $H_0: p_1 = p_2$; $H_1: p_1 < p_2$; population$_1$ = \$1 bill, population$_2$ = 4 quarters;
   Test statistic: $z = -3.49$; P-value = 0.0002; Critical value: $z = -1.645$; Reject H_0. There is sufficient evidence to support the claim that money in a large denomination is less likely to be spent relative to an equivalent amount in smaller denominations.

$$\bar{p} = \frac{12+27}{46+43} = \frac{39}{89};\ \bar{q} = 1 - \frac{39}{89} = \frac{50}{89};$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}} = \frac{\left(\dfrac{12}{46} - \dfrac{27}{43}\right) - 0}{\sqrt{\dfrac{\left(\dfrac{39}{89}\right)\left(\dfrac{50}{89}\right)}{46} + \dfrac{\left(\dfrac{39}{89}\right)\left(\dfrac{50}{89}\right)}{43}}} = -3.49$$

2. 90% CI: $-0.528 < p_1 - p_2 < -0.206$; The confidence interval limits do not contain 0, so it appears that there is a significant difference between the two proportions. Because the confidence interval consists of negative values only, it appears that p1 is less than p2, so it appears that money in a large denomination is less likely to be spent relative to an equivalent amount in smaller denominations

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{12}{46} - \frac{27}{43}\right) \pm 1.645\sqrt{\frac{\left(\dfrac{12}{46}\right)\left(\dfrac{34}{46}\right)}{46} + \frac{\left(\dfrac{27}{43}\right)\left(\dfrac{16}{43}\right)}{43}}$$

3. 95% CI: $-25.33\text{ cm} < \mu_1 - \mu_2 < -7.51\text{ cm}$ (Table: $-25.70\text{ cm} < \mu_1 - \mu_2 < -7.14\text{ cm}$); With 95% confidence, we conclude that the mean height of women is less than the mean height of men by an amount that is between 7.51 cm and 25.33 cm (Table: 7.14 cm and 25.70 cm).

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (162.35 - 178.77) \pm 2.262\sqrt{\frac{11.847^2}{10} + \frac{5.302^2}{10}}\ (\text{df} = 9);$$

   population$_1$ = women, population$_2$ = men

4. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 < \mu_2$; population$_1$ = women, population$_2$ = men;
Test statistic: $t = -4.001$; P-value = 0.0008 (Table: P-value < 0.005); Critical value: $t = -2.666$ (Table: $t = -2.821$); Reject H_0. There is sufficient evidence to support the claim that women have heights with a mean that is less than the mean height of men.
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(162.35 - 178.77) - 0}{\sqrt{\frac{11.847^2}{10} + \frac{5.302^2}{10}}} = -4.001 \text{ (df = 9)}$$

5. H_0: $\mu_d = 0$ cm; H_1: $\mu_d > 0$ cm; difference = before − after;
Test statistic: $t = 6.371$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $t = 2.718$; Reject H_0. There is sufficient evidence to support the claim that captopril is effective in lowering systolic blood pressure.
$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{18.583 - 0}{10.104/\sqrt{12}} = 6.371 \text{ (df = 11)}$$

6. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 > \mu_2$; population$_1$ = nonstress, population$_2$ = stress;
Test statistic: $t = 2.879$; P-value = 0.0026 (Table: P-value < 0.005); Critical value: $t = 2.376$ (Table: $t = 2.429$); Reject H_0. There is sufficient evidence to support the claim that the mean number of details recalled is lower for the stress group. It appears that "stress decreases the amount recalled," but we should not conclude that stress is the cause of the decrease.
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(53.3 - 45.3) - 0}{\sqrt{\frac{11.6^2}{40} + \frac{13.2^2}{40}}} = 2.429 \text{ (df = 39)}$$

7. H_0: $\mu_d = 0$ cm; H_1: $\mu_d > 0$ cm; difference = 1 day − 30 days;
Test statistic: $t = 14.061$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $t = 3.365$; Reject H_0. There is sufficient evidence to support the claim that flights scheduled 1 day in advance cost more than flights scheduled 30 days in advance. Save money by scheduling flights 30 days in advance.
$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{419.17 - 0}{73.022/\sqrt{6}} = 14.061 \text{ (df = 5)}$$

8. H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 \neq \sigma_2$; population$_1$ = women, population$_2$ = men;
Test statistic: $F = s_1^2/s_2^2 = 11.847^2/5.302^2 = 4.9933$; P-value = 0.0252; Upper critical value : $F = 4.0260$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that women and men have heights with the same variation.

Cumulative Review Exercises

1. a. Because the sample data are matched with each column consisting of heights from the same family, the data are dependent.

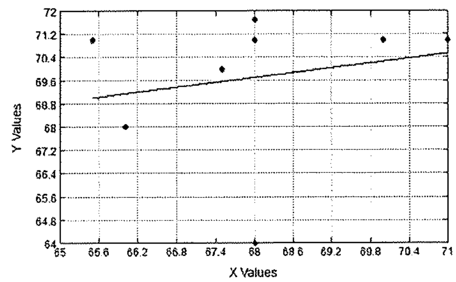
   b. $\bar{x} = \frac{64.0 + 68.0 + 70.0 + 71.0 + 71.0 + 71.0 + 71.0 + 71.7}{8} = 69.7$ in.; $Q_2 = \frac{71.0 + 71.0}{2} = 71.0$ in.;

   range = 71.7 − 64.0 = 7.7 in.; $s = \sqrt{\frac{(64.0 - 69.7)^2 + \cdots + (71.7 - 69.7)^2}{14 - 1}} = 2.6$ in.; $s^2 = 6.6$ in^2

   c. ratio

   d. continuous

2. There does not appear to be a correlation or association between the heights of fathers and the heights of their sons.



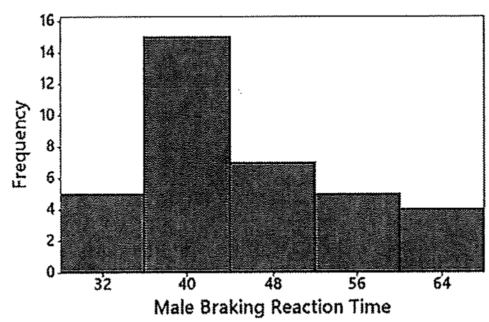
3. 90% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 69.7 \pm 2.306 \cdot \frac{2.570}{\sqrt{8}} \Rightarrow 67.6 \text{ in.} < \mu < 71.9 \text{ in.}$; We have 95% confidence that the limits of 67.6 in. and 71.9 in. actually contain the true value of the mean height of all adult sons.

4. H_0: $\mu_d = 0$ in.; H_1: $\mu_d \neq 0$ in.; difference = father − son;
Test statistic: $t = -1.712$; P-value = 0.1326 (Table: P-value > 0.10); Critical values: $t = \pm 2.365$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that differences between heights of fathers and their sons have a mean of 0. There does not appear to be a difference between heights of fathers and their sons.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{-1.7125 - 0}{2.847/\sqrt{8}} = -1.702 \text{ (df = 7)}$$

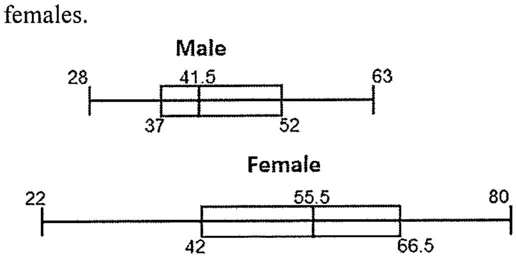
5. Because the points lie reasonably close to a straight-line pattern, and there is no other pattern that is not a straight-line pattern and there are no outliers, the sample data appear to be from a population with a normal distribution.

6. The shape of the histogram indicates that the sample data appear to be from a population with a distribution that is approximately normal.



7. Because the points are reasonably close to a straight-line pattern and there is no other pattern that is not a straight-line pattern, it appears that the braking reaction times of females are from a population with a normal distribution.

8. Because the boxplots overlap, there does not appear to be a significant difference between braking reaction times of males and females, but the braking reaction times for males appear to be generally lower than the braking reaction times of females.



9. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = male, population$_2$ = female;
Test statistic: $t = -3.259$; P-value = 0.0019 (Table: P-value < 0.005); Critical values $t = \pm 2.664$ (Table: $t = \pm 2.724$); Reject H_0. There is sufficient evidence to warrant rejection of the claim that males and females have the same mean braking reaction time. Males appear to have lower reaction times.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(44.361 - 54.278) - 0}{\sqrt{\frac{9.472^2}{36} + \frac{15.611^2}{36}}} = -3.259 \text{ (df = 35)}$$

10. a. The sample sizes are greater than 30 and the data appear to be from a populations that meet the loose requirement of being normally distributed.

$$\text{Males: 99\%\% CI: } \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 44.631 \pm 2.724 \cdot \frac{9.472}{\sqrt{36}} \Rightarrow 40.1 < \mu < 48.7$$

$$\text{Females: 99\%\% CI: } \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 54.278 \pm 2.724 \cdot \frac{15.611}{\sqrt{36}} \Rightarrow 47.2 < \mu < 61.4$$

The confidence intervals overlap, so there does not appear to be significant difference between the mean braking reaction times of males and females.

b. 99% CI: $-18.0 < \mu_1 - \mu_2 < -1.8$ (Table: $-18.2 < \mu_1 - \mu_2 < -1.6$); Because the confidence interval consists of negative numbers and does not include 0, there appears to be a significant difference between the mean braking reaction times of males and females.

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (44.361 - 54.278) \pm 2.724\sqrt{\frac{9.472^2}{36} + \frac{15.611^2}{36}} \text{ (df = 35);}$$

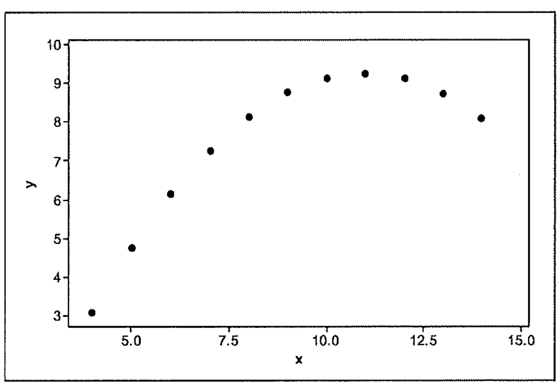
population$_1$ = men, population$_2$ = women

c. The results from part (b) are better.

Chapter 10: Correlation and Regression

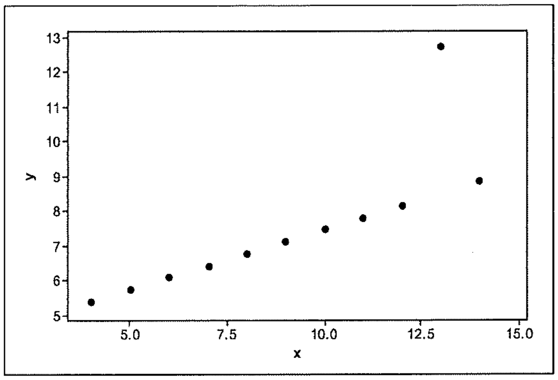
Section 10-1: Correlation

1. a. r is a statistic that represents the value of the linear correlation coefficient computed from the paired sample data, and ρ is a parameter that represents the value of the linear correlation coefficient that would be computed by using all of the paired data in the population of all statistics students.
   b. The value of r is estimated to be 0, because it is likely that there is no correlation between body temperature and head circumference.
   c. The value of r does not change if the body temperatures are converted to Fahrenheit degrees.
2. No, with $r = 0$, there is no *linear* correlation, but there might be some association with a scatterplot showing a distinct pattern that is not a straight-line pattern.
3. No, a correlation between two variables indicates that they are somehow associated, but that association does not necessarily imply that one of the variables has a direct effect on the other variable. Correlation does not imply causality.
4. a. −1
   b. 0.746
   c. 0.268
   d. 0.992
   e. 1
5. $r = 0.963$; P-value $= 0.000$; Critical values: $r = \pm 0.268$ (Table: $r \approx \pm 0.279$); Yes, there is sufficient evidence to support the claim that there is a linear correlation between the weights of bears and their chest sizes. It is easier to measure the chest size of a bear than the weight, which would require lifting the bear onto a scale. It does appear that chest size could be used to predict weight.
6. $r = 0.445$; P-value > 0.05; Critical values: $r = \pm 0.754$; No, there is not sufficient evidence to support the claim that there is a linear correlation between size and revenue. It does not appear that a casino can increase its revenue by expanding its size.
7. $r = 0.117$; P-value > 0.05; Critical values: $r = \pm 0.250$ (Table: $r \approx \pm 0.254$); No, there is not sufficient evidence to support the claim that there is a linear correlation between weights of discarded paper and glass.
8. $r = 0.765$; P-value < 0.01 (Using table A-6); Critical values: $r = \pm 0.497$; Yes, there is sufficient evidence to support the claim that there is a linear correlation between calories and sugar in a gram of cereal.
9. a.



   b. $H_0: \rho = 0$; $H_1: \rho \neq 0$; $r = 0.816$; P-value $= 0.002$ (Table: P-value < 0.01); Critical values ($\alpha = 0.05$): $r = \pm 0.602$; There is sufficient evidence to support the claim of a linear correlation between the two variables.
   c. The scatterplot reveals a distinct pattern that is not a straight line pattern.

10. a.



b. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.816$; P-value $= 0.002$ (Table: P-value < 0.01); Critical values ($\alpha = 0.05$): $r = \pm 0.602$; There is sufficient evidence to support the claim of a linear correlation between the two variables.

c. The scatterplot reveals a perfect straight-line pattern, except for the presence of one outlier.

11. a. Answer will vary, but because there appears to be an upward pattern, it is reasonable to think that there is a linear correlation.

b. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.906$; Critical values ($\alpha = 0.05$): $r = \pm 0.632$; P-value $= 0.000$ (Table: P-value < 0.01); There is sufficient evidence to support the claim of a linear correlation.

c. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0$; Critical values ($\alpha = 0.05$): $r = \pm 0.666$; P-value $= 1.000$ (Table: P-value > 0.05); There is not sufficient evidence to support the claim of a linear correlation.

d. The effect from a single pair of values can be very substantial, and it can change the conclusion.

12. a. There does not appear to be a linear correlation.

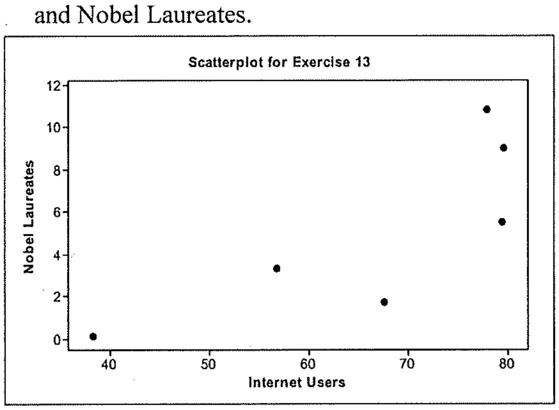
b. There does not appear to be a linear correlation.

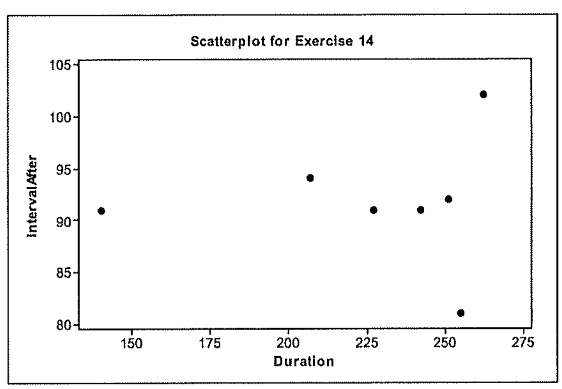
c. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0$; Critical values ($\alpha = 0.05$): $r = \pm 0.950$; P-value $= 1.000$ (Table: P-value > 0.05); There does not appear to be a linear correlation. The same results are obtained with the four points in the upper right corner.

d. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.985$; Critical values ($\alpha = 0.05$): $r = \pm 0.707$; P-value $= 0.000$ (Table: P-value < 0.01); There is sufficient evidence to support the claim of a linear correlation.

e. There are two different populations that should be considered separately. It is misleading to use the combined data from women and men and conclude that there is a relationship between x and y.

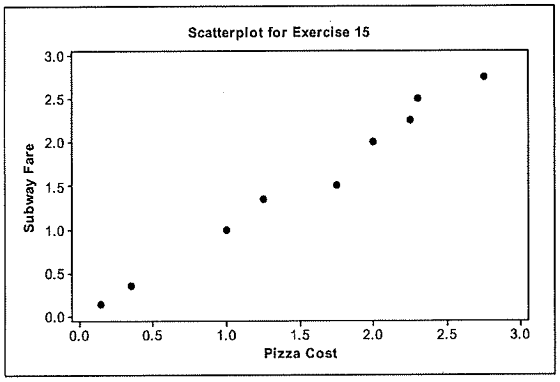
13. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.799$; P-value $= 0.056$ (Table: P-value > 0.05); Critical values: $r = \pm 0.811$; There is not sufficient evidence to support the claim that there is a linear correlation between Internet users and Nobel Laureates.

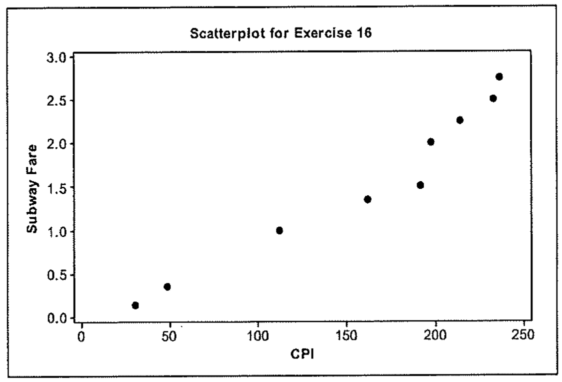




14. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.046$; P-value = 0.921 (Table: P-value > 0.05); Critical values: $r = \pm 0.754$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a linear correlation between duration times and interval after times.

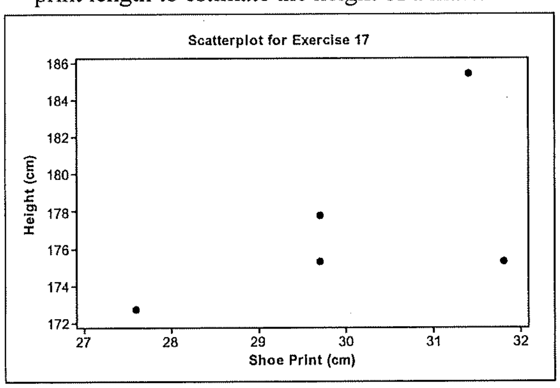
15. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.992$; P-value = 0.000 (Table: P-value < 0.01); Critical values: $r = \pm 0.666$; Reject H_0. There is sufficient evidence to support the claim that there is a significant linear correlation between the cost of a slice of pizza and the subway fare.

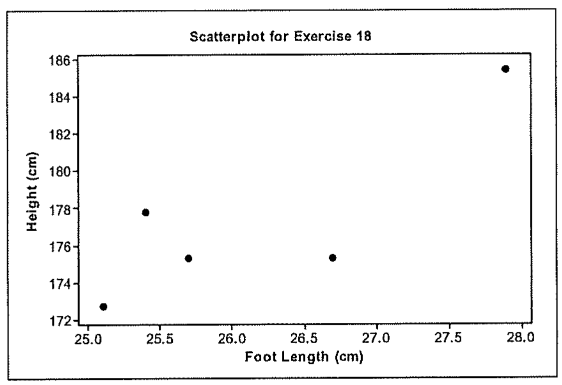




16. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.973$; P-value = 0.000 (Table: P-value < 0.01); Critical values: $r = \pm 0.666$; Reject H_0. There is sufficient evidence to support the claim that there is a significant linear correlation between the CPI (Consumer Price Index) and the subway fare.

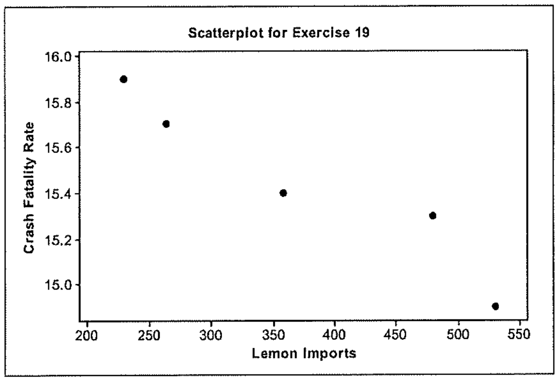
17. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.591$; P-value = 0.294 (Table: P-value > 0.05); Critical values: $r = \pm 0.878$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a linear correlation between shoe print lengths and heights of males. The given results do not suggest that police can use a shoe print length to estimate the height of a male.

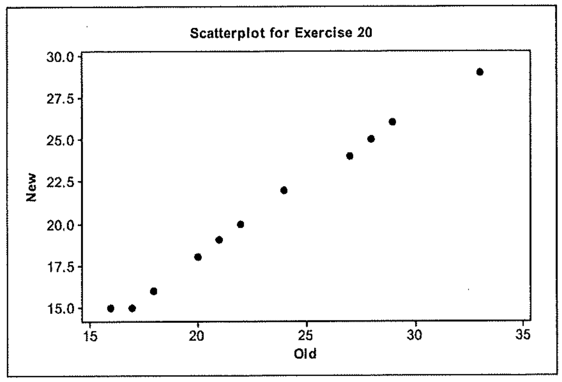




18. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.827$; P-value = 0.084 (Table: P-value > 0.05); Critical values: $r = \pm 0.878$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a linear correlation between foot lengths and heights of males. The given results do not suggest that police can use a foot length to estimate the height of a male.

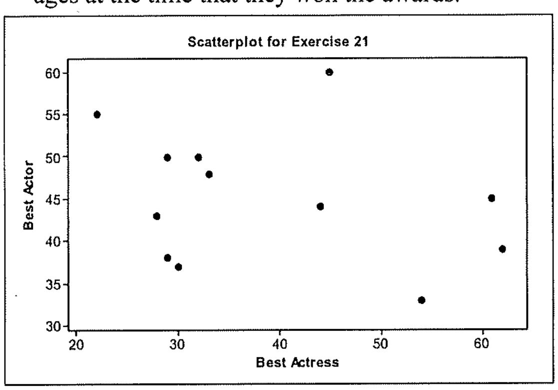
19. $H_0: \rho = 0$; $H_1: \rho \neq 0$; $r = -0.959$; P-value $= 0.010$; Critical values: $r = \pm 0.878$; Reject H_0. There is sufficient evidence to support the claim that there is a linear correlation between weights of lemon imports from Mexico and U.S. car fatality rates. The results do not suggest any cause-effect relationship between the two variables.

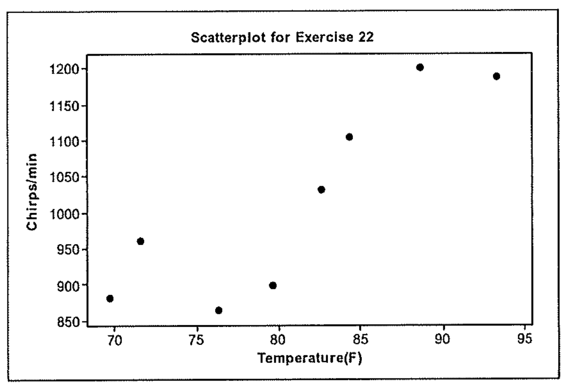




20. $H_0: \rho = 0$; $H_1: \rho \neq 0$; $r = 0.998$; P-value $= 0.000$ (Table: P-value < 0.01); Critical values: $r = \pm 0.602$; Reject H_0. There is sufficient evidence to support the claim of a linear correlation between the old and new fuel economy ratings. There is a linear correlation, but the old ratings appear to be somewhat higher than the new ratings.

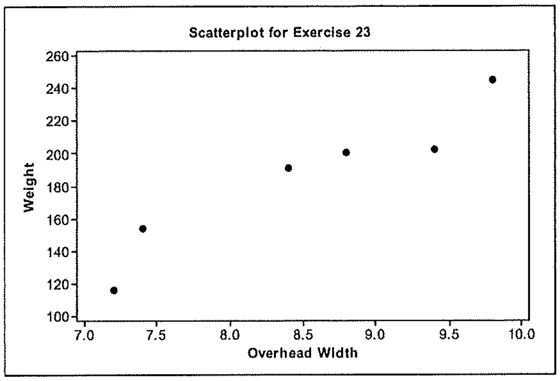
21. $H_0: \rho = 0$; $H_1: \rho \neq 0$; $r = -0.288$; P-value $= 0.365$ (Table: P-value > 0.05); Critical values: $r = \pm 0.576$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a significant linear correlation between the ages of Best Actresses and Best Actors. Because Best Actresses and Best Actors typically appeared in different movies, we should not expect that there would be a correlation between their ages at the time that they won the awards.

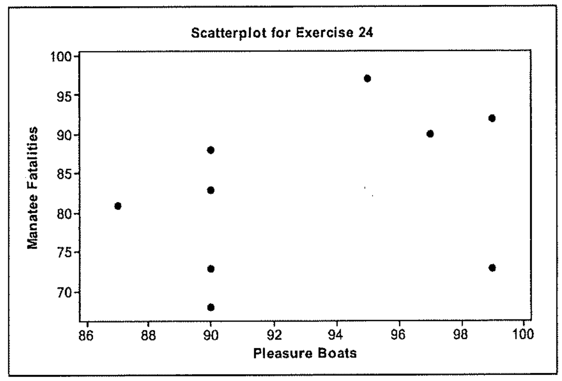




22. $H_0: \rho = 0$; $H_1: \rho \neq 0$; $r = 0.874$; P-value $= 0.005$ (Table: P-value < 0.01); Critical values: $r = \pm 0.707$; Reject H_0. There is sufficient evidence to support the claim of a linear correlation between the number of cricket chirps and the temperature.

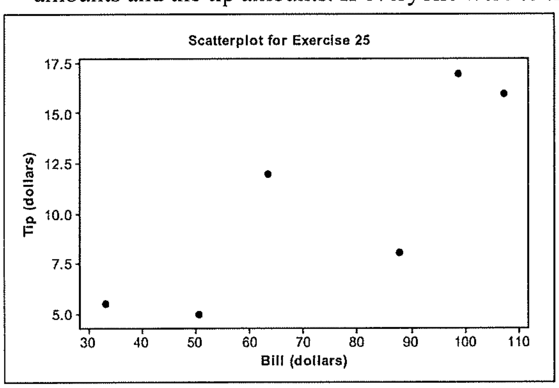
23. $H_0: \rho = 0$; $H_1: \rho \neq 0$; $r = 0.948$; P-value = 0.004 (Table: P-value < 0.01); Critical values: $r = \pm 0.811$; Reject H_0. There is sufficient evidence to support the claim of a linear correlation between the overhead width of a seal in a photograph and the weight of a seal.

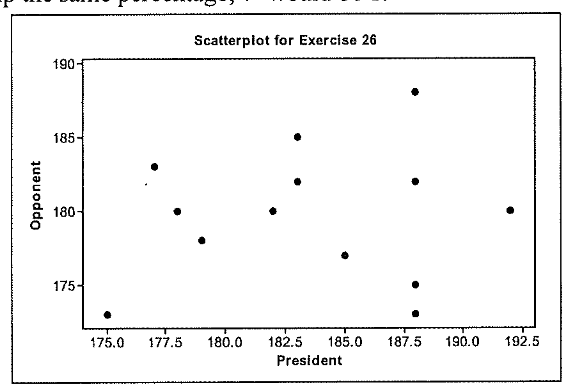




24. $H_0: \rho = 0$; $H_1: \rho \neq 0$; $r = 0.341$; P-value = 0.369 (Table: P-value > 0.05); Critical values: $r = \pm 0.666$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a significant linear correlation between numbers of registered pleasure boats and numbers of manatee boat fatalities.

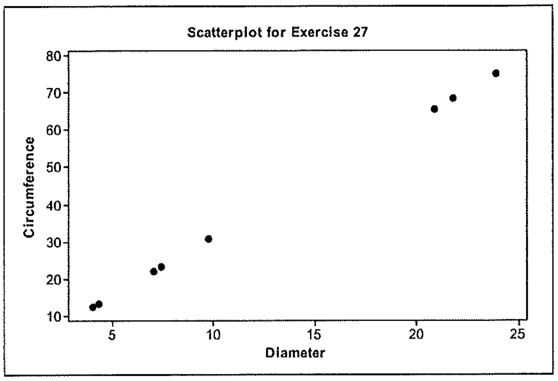
25. $H_0: \rho = 0$; $H_1: \rho \neq 0$; $r = 0.828$; P-value = 0.042 (Table: P-value < 0.05); Critical values: $r = \pm 0.811$; Reject H_0. There is sufficient evidence to support the claim there is a linear correlation between the bill amounts and the tip amounts. If everyone were to tip the same percentage, r would be 1.

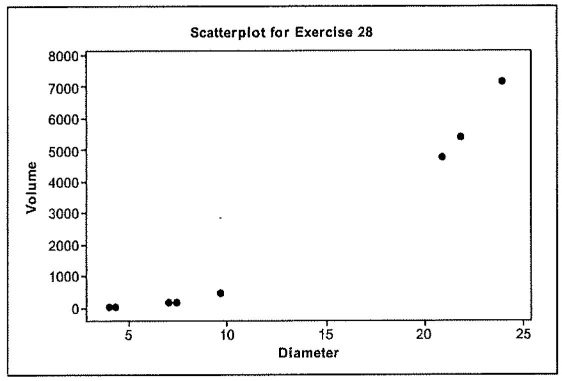




26. $H_0: \rho = 0$; $H_1: \rho \neq 0$; $r = 0.113$; P-value = 0.700 (Table: P-value > 0.05); Critical values: $r = \pm 0.532$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a linear correlation between heights of winning presidential candidates and heights of their main opponents. In an ideal world, voters would focus on important issues and not height or physical appearance of candidates, so there should not be a correlation.

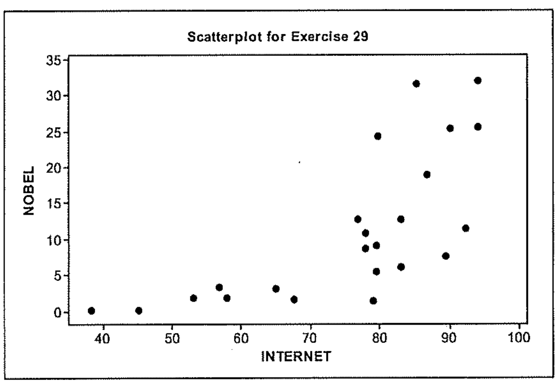
27. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 1.000$; P-value = 0.000 (Table: P-value < 0.01); Critical values: $r = \pm 0.707$; Reject H_0. There is sufficient evidence to support the claim that there is a linear correlation between diameters and circumferences. The scatterplot confirms a linear association.

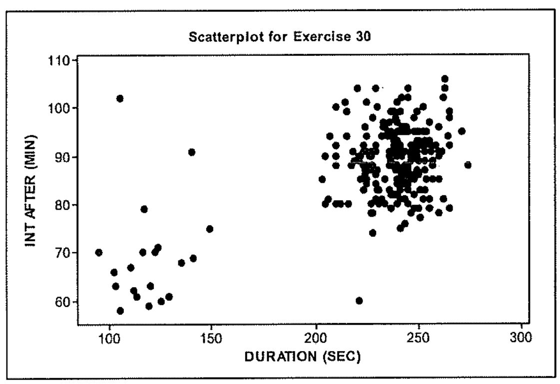




28. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.978$; P-value = 0.000; Critical values: $r = \pm 0.707$; Reject H_0. There is sufficient evidence to support the claim that there is a linear correlation between diameters and volumes. Although the results suggest that there is a linear correlation between diameters and volumes, the scatterplot suggests that there is a very strong correlation that is not linear.

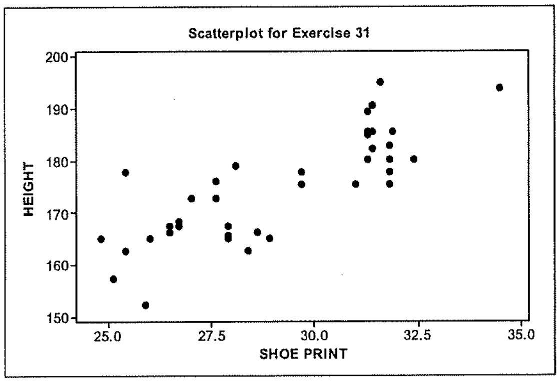
29. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.702$; P-value = 0.000 (Table: P-value < 0.01); Critical values: $r = \pm 0.413$ (Table: $-0.444 < r < -0.396$ or $0.396 < r < 0.444$); Reject H_0. There is sufficient evidence to support the claim that there is a linear correlation between Internet users and Nobel Laureates.

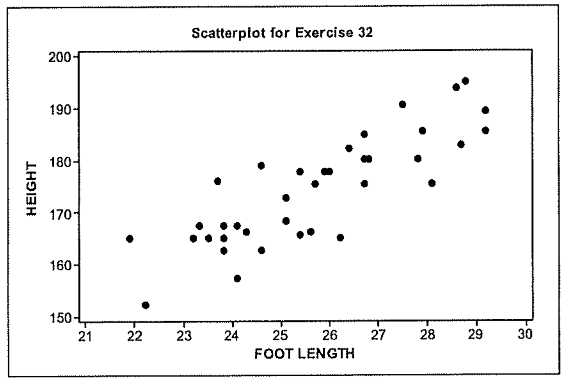




30. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.599$; P-value = 0.000 (Table: P-value < 0.01); Critical values: $r \approx \pm 0.124$ (Table: $r \approx \pm 0.196$); Reject H_0. There is sufficient evidence to support the claim that there is a linear correlation between duration times and interval after times.

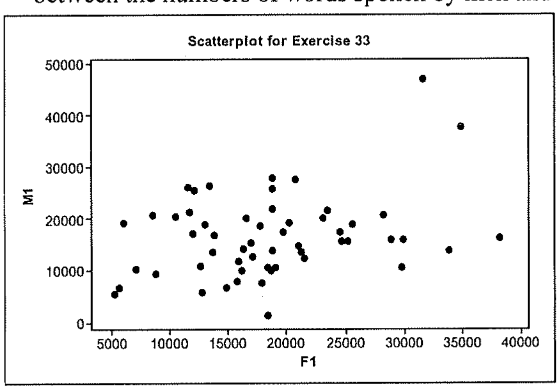
31. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.594$; P-value = 0.007 (Table: P-value < 0.01); Critical values: $r \approx \pm 0.456$; Reject H_0. There is sufficient evidence to support the claim that there is a linear correlation between shoe print lengths and heights of males. The given results do suggest that police can use a shoe print length to estimate the height of a male.

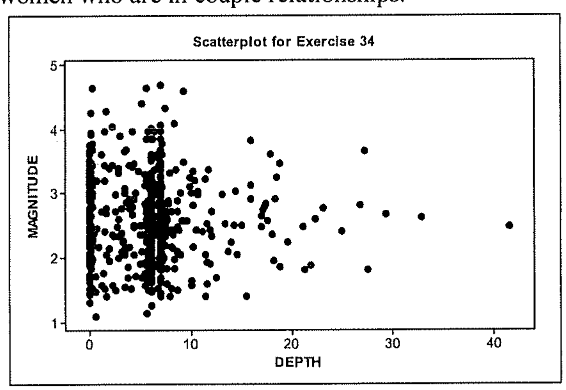




32. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.706$; P-value = 0.001 (Table: P-value < 0.01); Critical values: $r \approx \pm 0.456$; Reject H_0. There is sufficient evidence to support the claim that there is a linear correlation between foot lengths and heights of males. The given results do suggest that police can use a foot length to estimate the height of a male.

33. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.319$; P-value = 0.017 (Table: P-value < 0.05); Critical values: $r \approx \pm 0.263$ (Table: $r \approx \pm 0.254$); Reject H_0. There is sufficient evidence to support the claim of a linear correlation between the numbers of words spoken by men and women who are in couple relationships.





34. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = -0.008$;, P-value = 0.847 (Table: P-value > 0.05); Critical values: $r \approx \pm 0.080$; Fail to reject H_0. There is not sufficient evidence to support the claim of a linear correlation between magnitudes of earthquakes and their depths. It does not appear that depths of earthquakes are associated with their magnitudes.

35. a. 0.911
b. 0.787
c. 0.9999 (largest)
d. 0.976
e. −0.948

y	x	x^2	$\log x$	$\sqrt{x}$	$1/x$
0.3	2	4	0.3010	1.4142	0.5
05	3	9	0.4771	1.7321	0.3333
1.3	20	400	1.3010	4.4721	0.05
1.7	50	2500	1.6990	7.0711	0.02
2.0	95	9025	1.9777	9.7468	0.0105

36. $r=\pm\frac{t}{\sqrt{t^2+n-2}}=\pm\frac{2.485}{\sqrt{2.485^2+27-2}}=\pm 0.445$

Section 10-2: Regression

1. a. $\hat{y}=-368+130x$

   b. $\hat{y}$ represents the predicted value of price from rating.

2. The first equation represents the regression line that best fits *sample* data, and the second equation represents the regression line that best fits all paired data in a population. The values b_0 and b_1 are statistics; the values β_0 and β_1 are parameters.

3. a. A residual is a value of $y-\hat{y}$, which is the difference between an observed value of y and a predicted value of y.

   b. The regression line has the property that the sum of squares of the residuals is the lowest possible sum.

4. The value of r and the value of b_1 have the same sign. They are both positive or they are both negative or they are both 0. If r is positive, the regression line has a positive slope and rises from left to right. If r is negative, the slope of the regression line is negative and it falls from left to right.

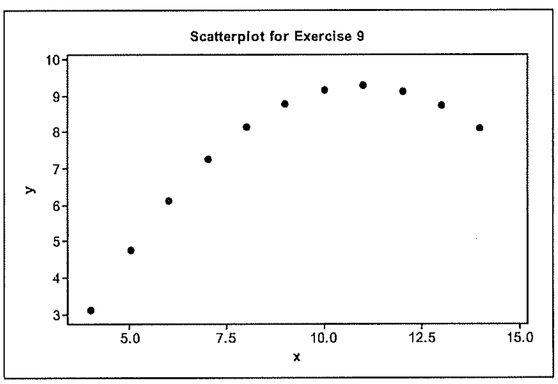
5. With no significant linear correlation, the best predicted value is $\bar{y}=5.9$.

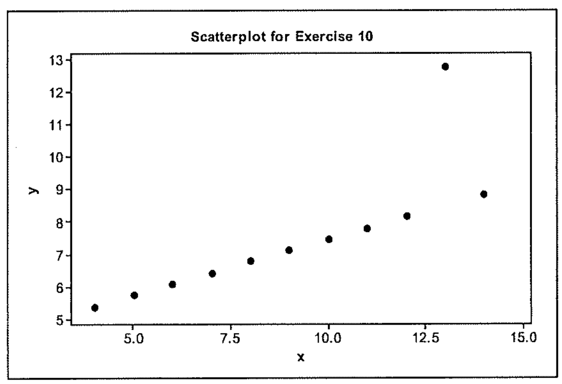
6. With a significant linear correlation, the best predicted value is $\hat{y}=-212+61.9(6.5)=190$ lb.

7. With a significant linear correlation, the best predicted value is $\hat{y}=-106+1.10(180)=92.0$ kg.

8. With no significant linear correlation, the best predicted value is $\bar{y}=37.3$ years.

9. $\hat{y}=3.00+0.500x$; The data have a pattern that is not a straight line.





10. $\hat{y}=3.00+0.500x$; There is an outlier.

11. a. $\hat{y}=0.264+0.906x$

    b. $\hat{y}=2+0x$ (or $\hat{y}=2$)

    c. The results are very different, indicating that one point can dramatically affect the regression equation.

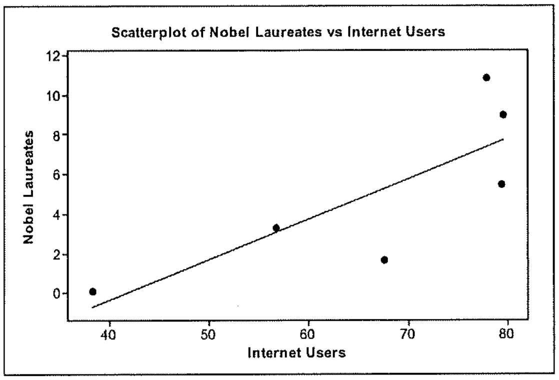
12. a. $\hat{y}=0.0846+0.985x$

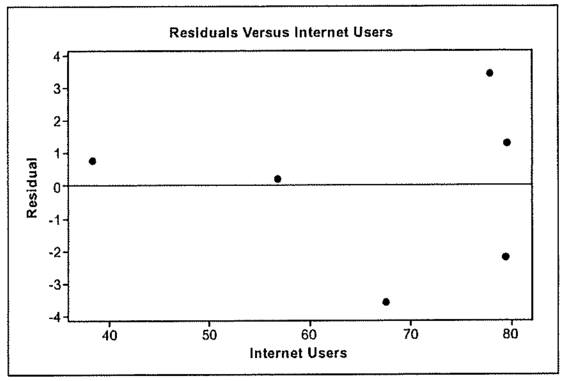
    b. $\hat{y}=1.5+0x$ (or $\hat{y}=1.5$)

    c. $\hat{y}=9.5+0x$ (or $\hat{y}=9.5$)

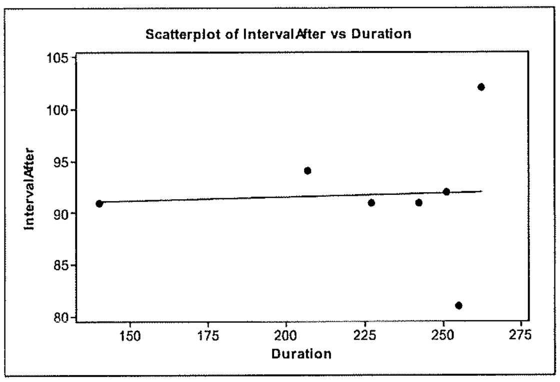
    d. The results are very different, indicating that combinations of clusters can produce results that differ dramatically from results within each cluster alone.

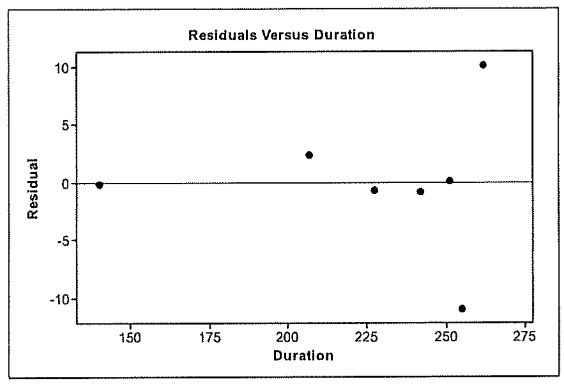
13. $\hat{y} = -8.44 + 0.203x$; $r = 0.799$; P-value $= 0.056$; With no significant linear correlation, the best predicted value is $\bar{y} = 5.1$ per 10 million people. The best predicted value is not at all close to the actual Nobel rate of 1.5 per 10 million people.



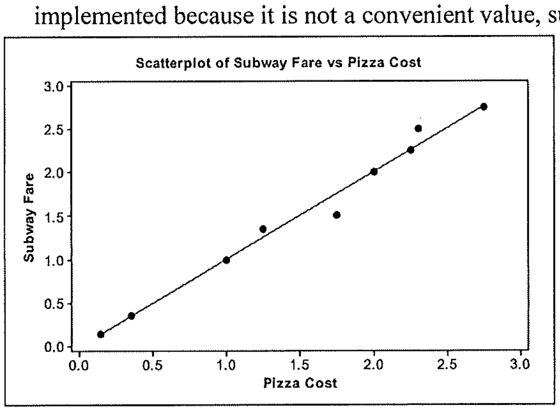


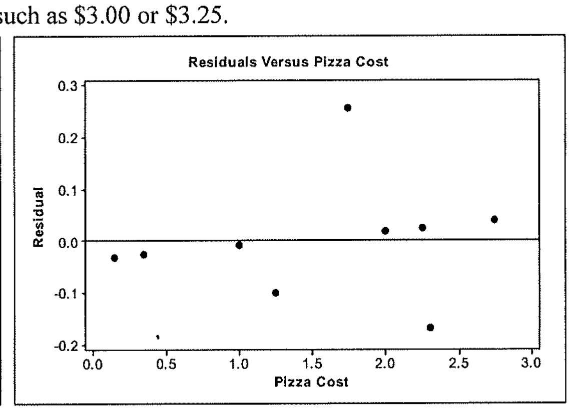
14. $\hat{y} = 90.2 + 0.00673x$; $r = 0.046$; P-value $= 0.921$; With no significant linear correlation, the best predicted value is $\bar{y} = 92$ minutes. The best predicted value is somewhat close to the actual time of 83 minutes.



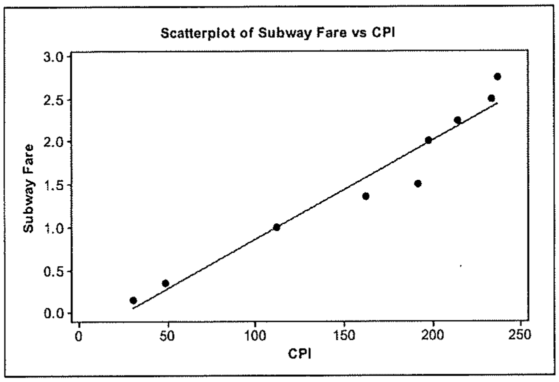


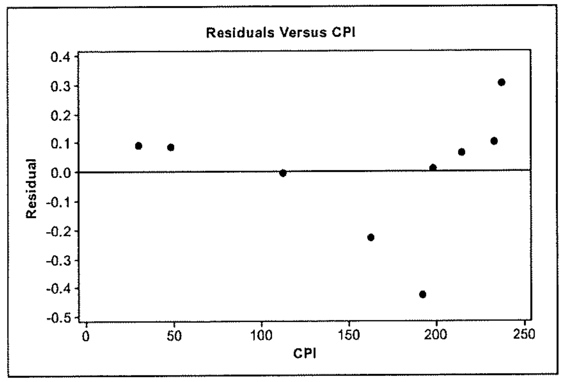
15. $\hat{y} = -0.0111 + 1.01x$; $r = 0.992$; P-value $= 0.000$; With a significant linear correlation, the best predicted value is $\hat{y} = -0.0111 + 1.01(3.00) = \$3.02$. The best predicted subway fare of \$3.02 is not likely to be implemented because it is not a convenient value, such as \$3.00 or \$3.25.



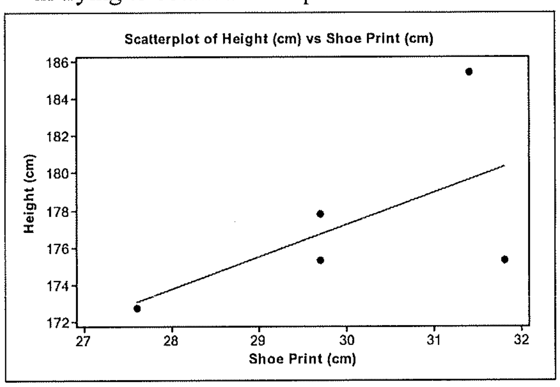


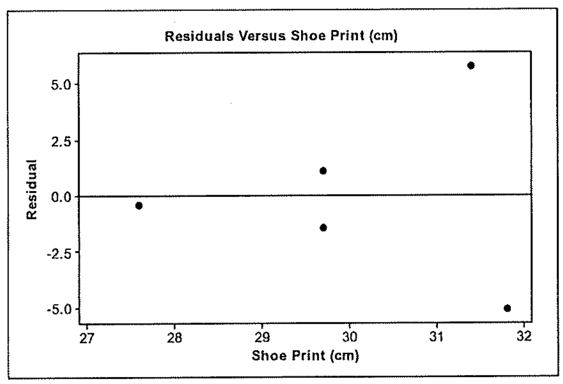
16. $\hat{y} = -0.290 + 0.0115x$; $r = 0.973$; P-value $= 0.000$; With a significant linear correlation, the best predicted value is $\hat{y} = -0.290 + 0.0115(500) = \$5.46$. The CPI of 500 is too far beyond the scope of the data, so it might not be very reliable.



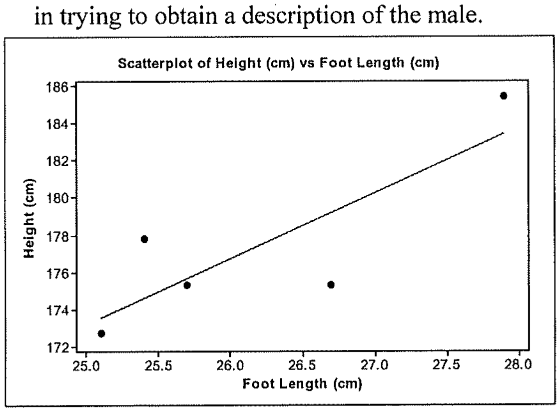


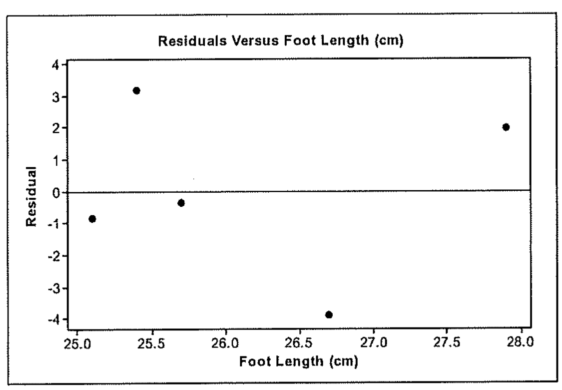
17. $\hat{y} = 125 + 1.73x$; $r = 0.591$; P-value $= 0.294$; With no significant linear correlation, the best predicted value is $\bar{y} = 177$ cm. Because the best predicted value is the mean height, it would not be helpful to police in trying to obtain a description of the male.



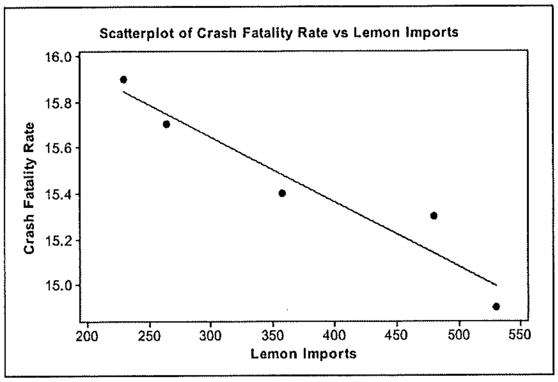


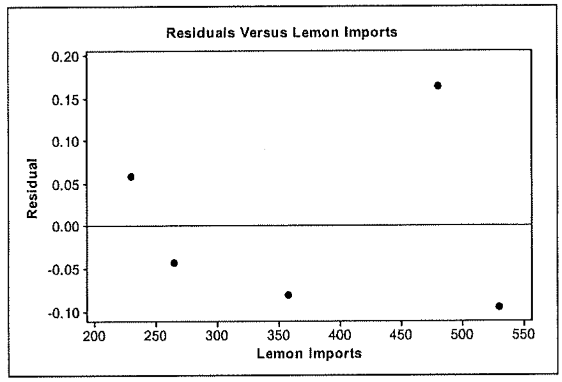
18. $\hat{y} = 85.1 + 3.52x$; $r = 0.827$; P-value $= 0.084$; With no significant linear correlation, the best predicted value is $\bar{y} = 177$ cm. Because the best predicted value is the mean height, it would not be helpful to police in trying to obtain a description of the male.



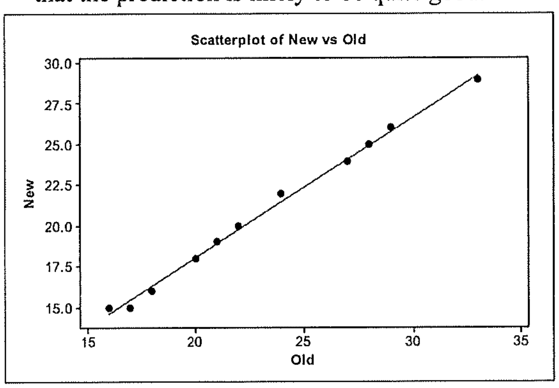


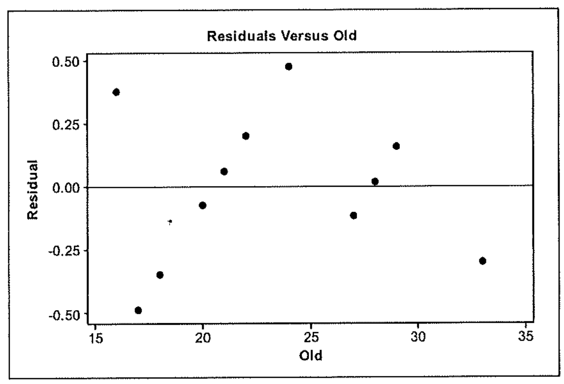
19. $\hat{y} = 16.5 - 0.00282x$; $r = -0.959$; P-value $= 0.010$; With a significant linear correlation, the best predicted value is $\hat{y} = 16.5 - 0.00282(500) = 15.1$ fatalities per 100,000 population. Common sense suggests that the prediction doesn't make much sense.



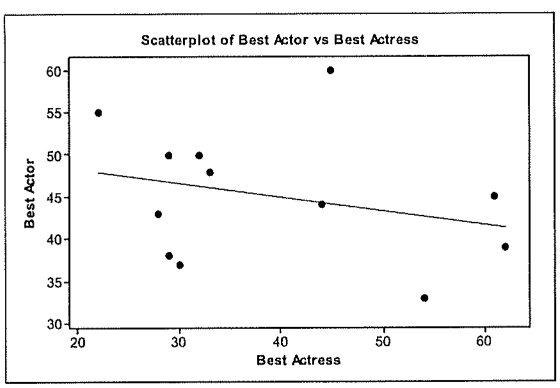


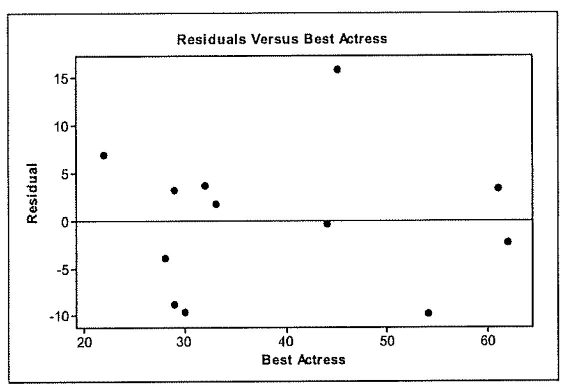
20. $\hat{y} = 0.808 + 0.863x$; $r = 0.998$; P-value $= 0.000$; With a significant linear correlation, the best predicted value is $\hat{y} = 0.808 + 0.863(30) = 27$ mpg. Because the linear correlation coefficient is so high, it appears that the prediction is likely to be quite good.



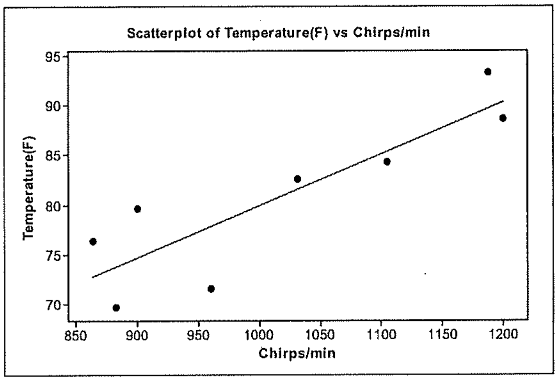


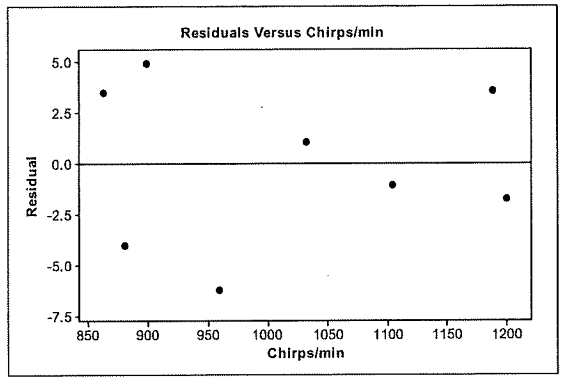
21. $\hat{y} = 51.6 - 0.165x$; $r = -0.288$; P-value $= 0.365$; With no significant linear correlation, the best predicted value is $\bar{y} = 45$ years, which is not close to the actual age of 33 years.



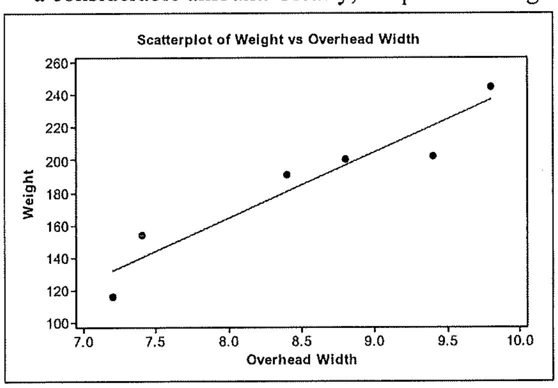


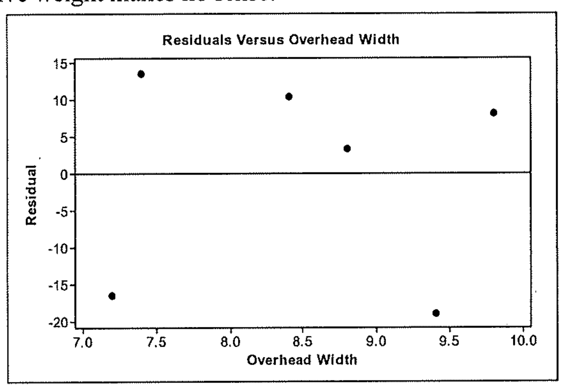
22. $\hat{y}=27.6+0.0523x$; $r=0.874$; P-value $=0.005$; With a significant linear correlation, the best predicted value is $\hat{y}=27.6+0.0523(3000)=185°\text{F}$. The value of 3000 chirps in one minute is well beyond the scope of the listed sample data, so the extrapolation might be off by a considerable amount, especially if the cricket is dead from such a high temperature.



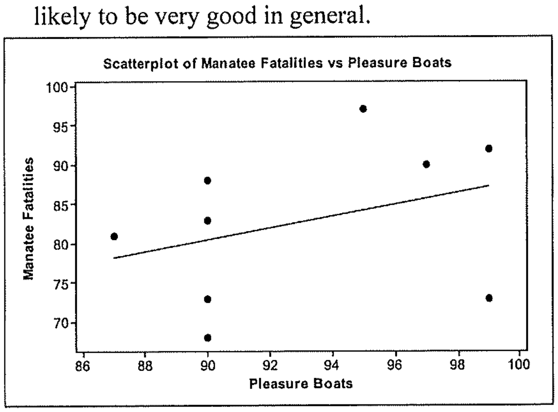


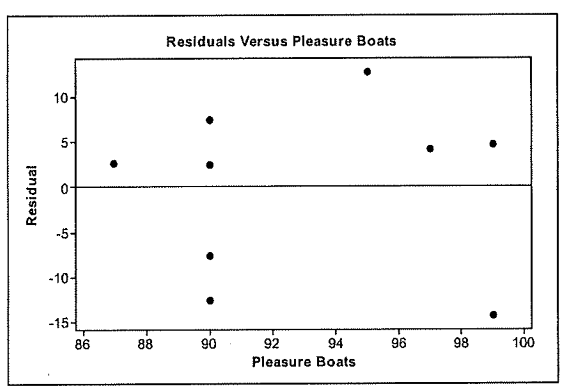
23. $\hat{y}=-157+40.2x$; $r=0.948$; P-value $=0.004$; With a significant linear correlation, the best predicted value is $\hat{y}=-157+40.2(2)=-76.6$ kg; The prediction is a negative weight that cannot be correct. The overhead width of 2 cm is well beyond the scope of the sample widths, so the extrapolation might be off by a considerable amount. Clearly, the predicted negative weight makes no sense.



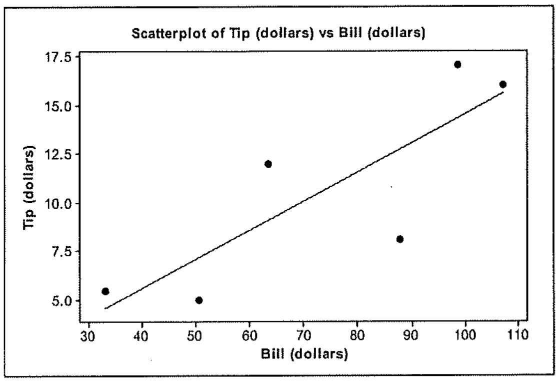


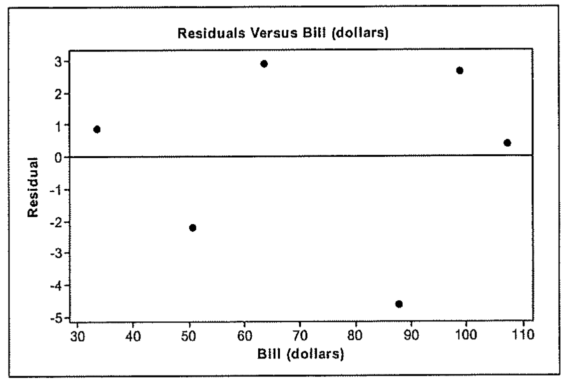
24. $\hat{y}=13.6+0.744x$; $r=0.341$; P-value $=0.369$; With no significant linear correlation, the best predicted value is $\bar{y}=83$ fatalities, which happens to be somewhat close to the predicted value of 79 fatalities. Because in this case, the best predicted number of fatalities is always the mean, the predicted values are not likely to be very good in general.



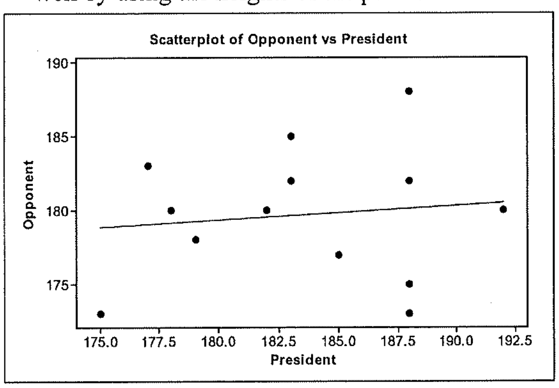


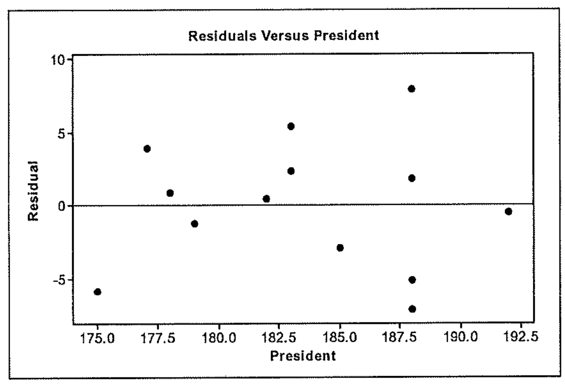
25. $\hat{y} = -0.347 + 0.149x$; $r = 0.828$; P-value $= 0.042$; With a significant linear correlation, the best predicted value is $\hat{y} = -0.347 + 0.149(100) = \$14.55$. Tipping rule: Multiply the bill by 0.149 (or 14.9%) and subtract 35 cents. A more approximate but easier rule is this: Leave a tip of 15%.



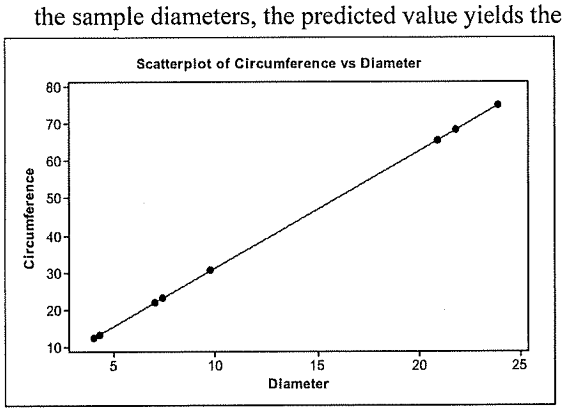


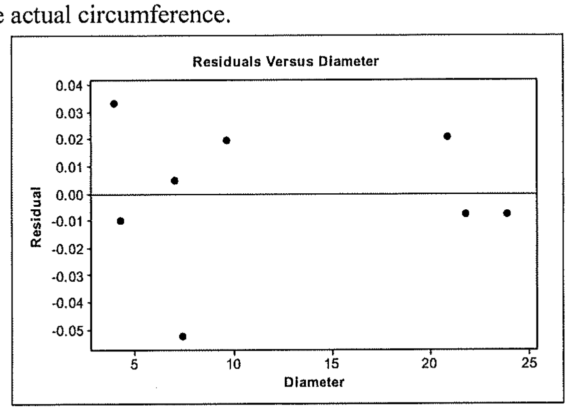
26. $\hat{y} = 162 + 0.0975x$; $r = 0.113$; P-value $= 0.700$ (Table: P-value > 0.05); With no significant linear correlation, the best predicted value is $\bar{y} = 179.7$ cm. Heights of opponents do not appear to be predicted well by using the heights of the presidents.



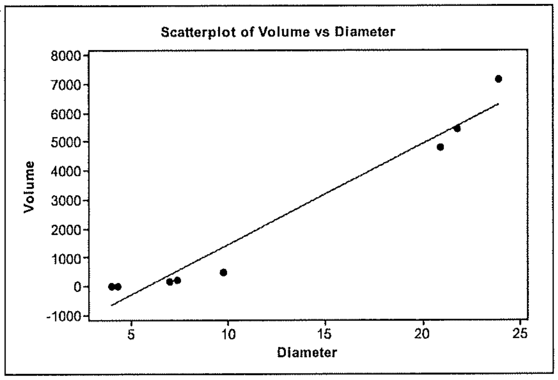


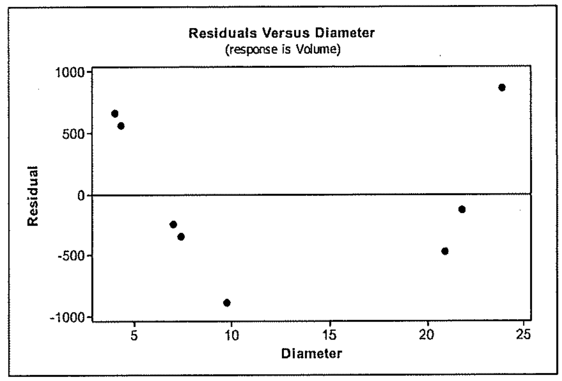
27. $\hat{y} = -0.00396 + 3.14x$; $r = 1.000$; P-value $= 0.000$; With a significant linear correlation, the best predicted value is $\hat{y} = -0.00396 + 3.14(1.50) = 4.7$ cm. Even though the diameter of 1.50 cm is beyond the scope of the sample diameters, the predicted value yields the actual circumference.



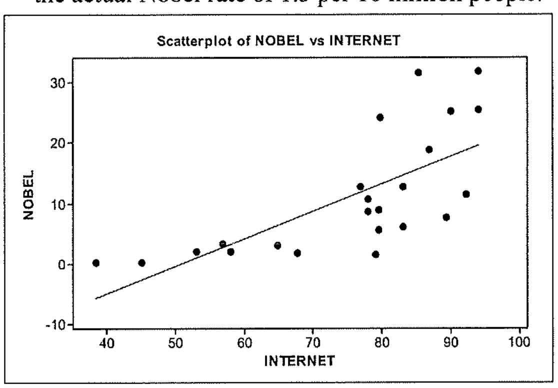


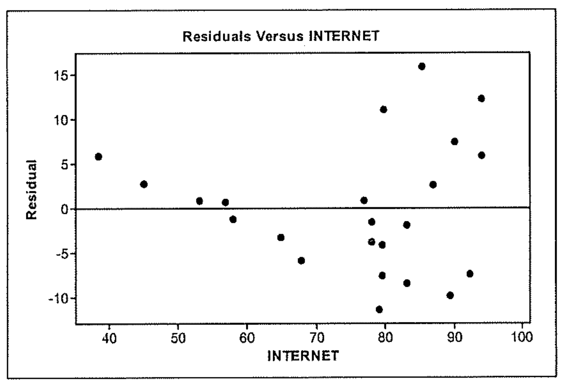
28. $\hat{y} = -2011 + 347x$; $r = 0.978$; P-value $= 0.000$; With a significant linear correlation, the best predicted value is $\hat{y} = -2011 + 347(1.50) = -1490.5\ \text{cm}^3$. The predicted value is negative and is far from the actual volume of $1.8\ \text{cm}^3$. The diameter of 1.50 cm is beyond the scope of the sample diameters, and the predicted value is way wrong. The scatterplot suggests that a nonlinear model would yield better results.



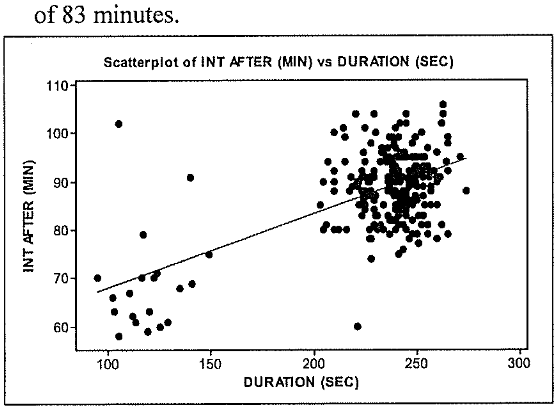


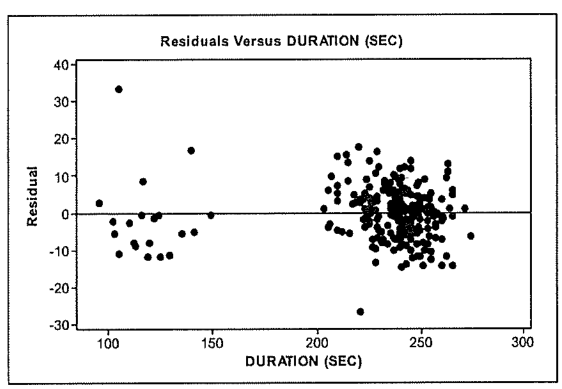
29. $\hat{y} = -23.2 + 0.456x$; $r = 0.702$; P-value $= 0.000$; With a significant linear correlation, the best predicted value is $\hat{y} = -23.2 + 0.456(79.1) = 12.9$ per 10 million people. The best predicted value is not at all close to the actual Nobel rate of 1.5 per 10 million people.



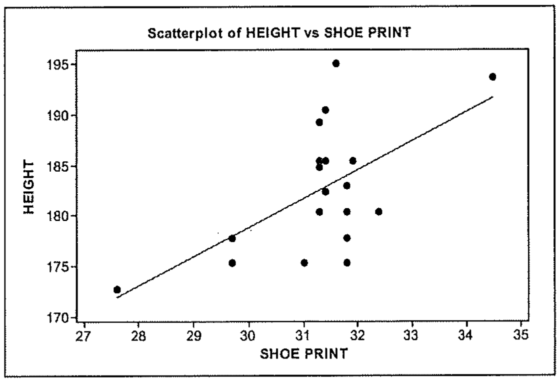


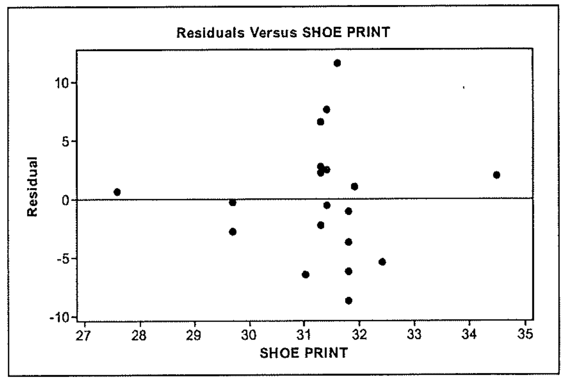
30. $\hat{y} = 52.8 + 0.152x$; $r = 0.599$; P-value $= 0.000$; With a significant linear correlation, the best predicted value is $\hat{y} = 52.8 + 0.152(253) = 91$ minutes. The best predicted value is somewhat close to the actual time of 83 minutes.



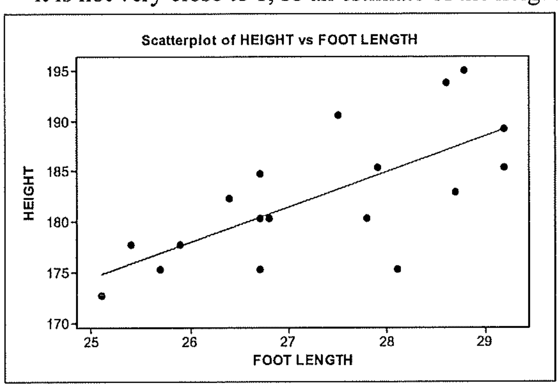


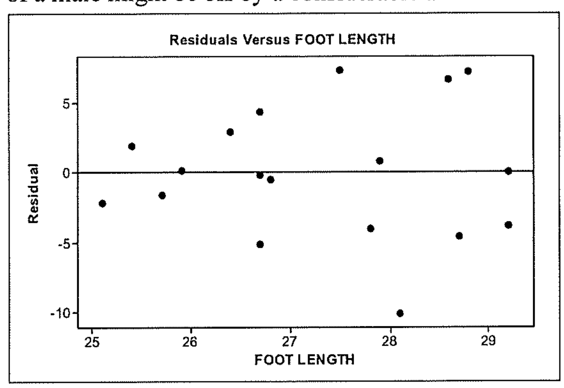
31. $\hat{y} = 93.5 + 2.85x$; $r = 0.594$; P-value $= 0.007$; With a significant linear correlation, the best predicted value is $\hat{y} = 93.5 + 2.85(31.3) = 183$ cm. Although there is a linear correlation, with $r = 0.594$, we see that it is not very strong, so an estimate of the height of a male might be off by a considerable amount.



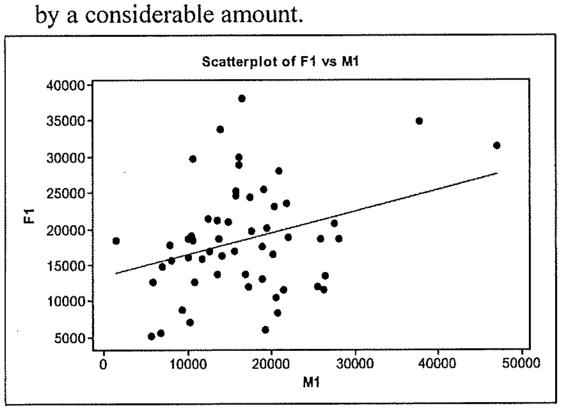


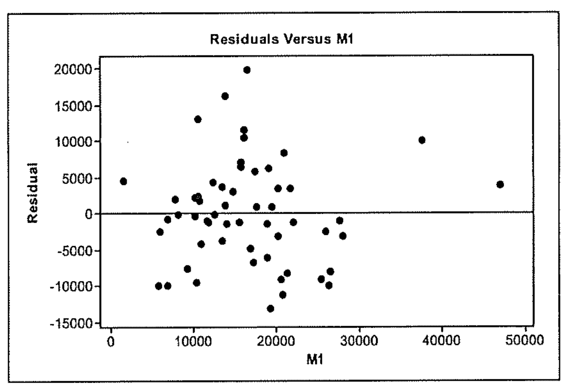
32. $\hat{y} = 87.1 + 3.50x$; $r = 0.706$; P-value $= 0.001$; With no significant linear correlation, the best predicted value is $\hat{y} = 87.1 + 3.50(28.0) = 185$ cm. Although there is a linear correlation, with $r = 0.706$, we see that it is not very close to 1, so an estimate of the height of a male might be off by a considerable amount.



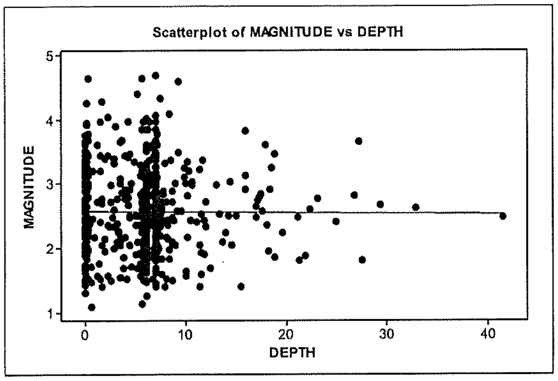


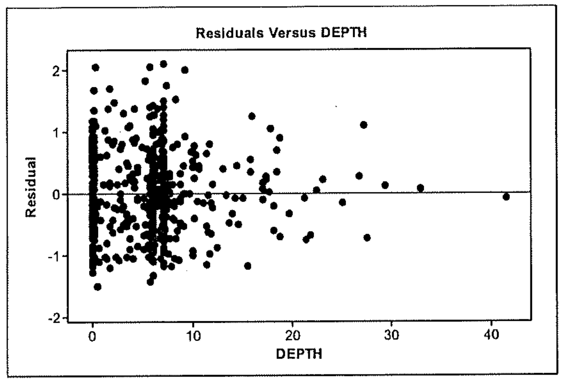
33. $\hat{y} = 13{,}400 + 0.302x$; $r = 0.319$; P-value $= 0.017$; With a significant linear correlation, the best predicted value is $\hat{y} = 13{,}400 + 0.302(16{,}000) = 18{,}200$ words. Although there is a linear correlation, with $r = 0.319$, we see that it is not very close to 1, so an estimate of the number of words spoken by a female might be off by a considerable amount.





34. $\hat{y} = 2.58 - 0.00104x$; $r = -0.008$; P-value $= 0.847$; With no significant linear correlation, the best predicted value is $\bar{y} = 2.57$.





35. a. Using $\hat{y} = -3.37 + 2.49x$, the sum of squares of the residuals is 823.64.

b. Using $\hat{y} = -3 + 2.5x$, the sum of squares of the residuals is 827.45, which is larger than 823.64, which is the sum of squares of the residuals for the regression line.

x	$\hat{y} = -3.37 + 2.49x$	$(\hat{y} - \bar{y})^2$
4.5	7.8350	5.4522
10.2	22.0280	5.1620
4.4	7.5860	1.0282
2.9	3.8510	14.0700
3.9	6.3410	0.0581
0.7	−1.6270	2.9825
8.5	17.7950	56.3250
7.3	14.8070	51.9408
6.3	12.3170	11.0025
11.6	25.5140	164.1986
2.5	2.8550	0.9120
8.8	18.5420	34.1290
3.7	5.8430	6.4668
1.8	1.1120	0.1505
4.5	7.8350	12.7092
9.4	20.0360	29.8553
3.6	5.5940	6.2200
2.0	1.6100	0.0841
3.6	5.5940	15.1632
6.4	12.5660	373.8036
11.9	26.2610	27.4471
9.7	20.7830	3.5457
5.3	9.8270	0.9467
	Sum:	823.65

x	$\hat{y} = -3 + 2.5x$	$(\hat{y} - \bar{y})^2$
4.5	8.2500	7.5625
10.2	22.5000	3.2400
4.4	8.0000	0.3600
2.9	4.2500	17.2225
3.9	6.7500	0.4225
0.7	−1.2500	1.8225
8.5	18.2500	49.7025
7.3	15.2500	58.5225
6.3	12.7500	14.0625
11.6	26.0000	176.8900
2.5	3.2500	1.8225
8.8	19.0000	39.6900
3.7	6.2500	8.7025
1.8	1.5000	0.0000
4.5	8.2500	9.9225
9.4	20.5000	25.0000
3.6	6.0000	8.4100
2.0	2.0000	0.0100
3.6	6.0000	18.4900
6.4	13.0000	357.2100
11.9	26.7500	22.5625
9.7	21.2500	5.5225
5.3	10.2500	0.3025
	Sum:	827.45

Section 10-3: Prediction Intervals and Variation

1. The value of $s_e = 16.27555$ cm is the standard error of estimate, which is a measure of the differences between the observed weights and the weights predicted from the regression equation. It is a measure of the variation of the sample points about the regression line.
2. We have 95% confidence that the limits of 59.0 kg and 123.6 kg contain the value of the weight for a male with a height of 180 cm. The major advantage of using a prediction interval is that it provides us with a range of likely weights, so we have a sense of how accurate the predicted weight is likely to be. The terminology of *prediction interval* is used for an interval estimate of a variable, whereas the terminology of *confidence interval* is used for an interval estimate of a population parameter.
3. The coefficient of determination is $r^2 = 0.155$. We know that 15.5% of the variation in weight is explained by the linear correlation between height and weight, and 84.5% of the variation in weight is explained by other factors and/or random variation.
4. For the paired weights, $s_e = 0$ because there is an exact conversion formula. For a student who weighs 100 lb, the predicted weight is 45.4 kg, and there is no prediction interval because the conversion yields an exact result.
5. $r^2 = (0.874)^2 = 0.764$; 76.4% of the variation in temperature is explained by the linear correlation between chirps and temperature, and 23.6% of the variation in temperature is explained by other factors and/or random variation.
6. $r^2 = (0.992)^2 = 0.984$; 98.4% of the variation in subway fares is explained by the linear correlation between costs of a slice of pizza and subway fares, and 1.6% of the variation in subway fares is explained by other factors and/or random variation.
7. $r^2 = (0.885)^2 = 0.783$; 78.3% of the variation in waist size is explained by the linear correlation between weight and waist size, and 21.7% of the variation in waist size is explained by other factors and/or random variation.
8. $r^2 = (0.783)^2 = 0.613$; 61.3% of the variation in weight is explained by the linear correlation between head width and weight, and 38.7% of the variation in weight is explained by other factors and/or random variation.
9. $r = 0.850$; Critical values $(\alpha = 0.05)$: $r = \pm 0.404$ (Table: $r \approx \pm 0.396$); Yes, there is sufficient evidence to support a claim of a linear correlation between registered boats and manatee fatalities.
10. $r^2 = (0.850)^2 = 0.723$, or 72.3%
11. The best predicted value is 70.5 manatees.
12. The 95% prediction interval estimate is 50.0 manatees $< y <$ 90.9 manatees. We have 95% confidence that the limits of 50.0 and 90.9 contain the number of manatee fatalities in a year with 850,000 registered boats.
13. 99% CI: 42.7 manatees $< y <$ 98.3 manatees

$$\hat{y} = -49.049 + 1.406(85) = 70.46$$

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}} = 2.819(9.6605)\sqrt{1 + \frac{1}{24} + \frac{24(85 - 85.25)^2}{24(127822) - (2046)^2}} = 27.79$$

14. 95% CI: 67.7 manatees $< y <$ 109.8 manatees

$$\hat{y} = -49.049 + 1.406(98) = 88.74$$

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}} = 2.074(9.6605)\sqrt{1 + \frac{1}{24} + \frac{24(98 - 85.25)^2}{24(127822) - (2046)^2}} = 20.41$$

15. 95% CI: 65.1 manatees $< y <$ 106.8 manatees

$$\hat{y} = -49.049 + 1.406(96) = 85.93$$

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}} = 2.074(9.6605)\sqrt{1 + \frac{1}{24} + \frac{24(96 - 85.25)^2}{24(127822) - (2046)^2}} = 20.42$$

16. 95% CI: 45.5 manatees $< y <$ 101.1 manatees

$$\hat{y} = -49.049 + 1.406(87) = 73.27$$

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}} = 2.819(9.6605)\sqrt{1 + \frac{1}{24} + \frac{24(87 - 85.25)^2}{24(127822) - (2046)^2}} = 27.79$$

17. a. 10,626.59
    b. 68.83577
    c. 95% CI: 38.0°F $< y <$ 60.4°F

$$\hat{y} = 72.5 - 3.68(6.327) = 49.2$$

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}} = 2.5706(3.710411)\sqrt{1 + \frac{1}{7} + \frac{7(6.327 - 20.143)^2}{7(3623) - (141)^2}} = 11.2$$

18. a. 3210.364
    b. 1087.191
    c. 99% CI: \$10,400 $< y <$ \$105,000

$$\hat{y} = 27.7 + 0.0373(800) = 57.5$$

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}} = 3.4995(12.462466)\sqrt{1 + \frac{1}{9} + \frac{9(800 - 443.1)^2}{9(4{,}076{,}640) - (3988)^2}} = 47.1$$

19. a. 352.7278
    b. 109.3722
    c. 90% CI: 71.09°F $< y <$ 88.71°F

$$\hat{y} = 27.628 + 0.05227(1000) = 79.90$$

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}} = 1.943(4.2695)\sqrt{1 + \frac{1}{8} + \frac{8(1000 - 1016.25)^2}{8(8391204) - (8130)^2}} = 8.81.$$

20. a. 8880.12
    b. 991.1515
    c. 99% CI: 125.0 kg $< y <$ 284.5 kg

$$\hat{y} = -156.879 + 40.182(9.0) = 204.76$$

$$E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}} = 4.604(15.7413)\sqrt{1 + \frac{1}{6} + \frac{6(9.0 - 8.5)^2}{6(439) - (51)^2}} = 79.79$$

21. 99% CI: $17.1 < \bar{y} < 26.0$ (values are Nobel Laureates per 10 million people

$$\hat{y} = -3.367 + 2.493(10) = 21.565$$

$$E = t_{\alpha/2} s_e \sqrt{\frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}} = 2.080(6.26267)\sqrt{\frac{1}{23} + \frac{23(10.0 - 5.8)^2}{23(1011.45) - (133.5)^2}} = 4.476$$

Section 10-4: Multiple Regression

1. The response variable is weight and the predictor variables are length and chest size.
2. No, it is not better to use the regression equation with the three predictor variables of length, chest size, and neck size. The adjusted R^2 value of 0.925 is just a little less than 0.933, so in this case it is better to use two predictor variables instead of three.
3. The unadjusted R^2 increases (or remains the same) as more variables are included, but the adjusted R^2 is adjusted for the number of variables and sample size. The unadjusted R^2 incorrectly suggests that the best multiple regression equation is obtained by including all of the available variables, but by taking into account the sample size and number of predictor variables, the adjusted R^2 is much more helpful in weeding out variables that should not be included.
4. 92.8% of the variation in weights of bears can be explained by the variables of length and chest size, so 7.2% of the variation in weights can be explained by other factors and/or random variation.
5. $\text{Son} = 18.0 + 0.504\,\text{Father} + 0.277\,\text{Mother}$
6. a. $P\text{-value} < 0.0001$

   b. $R^2 = 0.3649$

   c. adjusted $R^2 = 0.3552$
7. A P-value less than 0.0001 is low, but the values of R^2 (0.3649) and adjusted R^2 (0.3552) are not high. Although the multiple regression equation fits the sample data best, it is not a good fit, so it should not be used for predicting the height of a son based on the height of his father and the height of his mother.
8. Predicted height: $\text{Son} = 18.0 + 0.504(70) + 0.277(60) = 70$ in.; This result is not likely to be a good predicted value because the multiple regression equation is not a good model (based on the results from Exercise 7).
9. HWY (highway fuel consumption) because it has the best combination of small P-value (0.000) and highest adjusted R^2 (0.920).
10. WT (weight) and HWY (highway fuel consumption) because they have the best combination of small P-value (0.000) and highest adjusted R^2 (0.935).
11. $\text{CITY} = -3.15 + 0.819\,\text{HWY}$; That equation has a low P-value of 0.0000 and its adjusted R^2 value of 0.920 isn't very much less than the values of 0.928 and 0.935 that use two predictor variables, so in this case it is better to use the one predictor variable instead of two.
12. Predicted city fuel consumption is $\text{CITY} = -3.15 + 0.819(36) = 26.3$ mi/gal; (Based on the result from Exercise 11.) The predicted value is a good estimate, but it might not be very accurate because the sample consists of only 21 cars.
13. The best regression equation is $\hat{y} = 0.127 + 0.0878x_1 - 0.0250x_2$, where x_1 represents tar and x_2 represents carbon monoxide. It is best because it has the highest adjusted R^2 value of 0.927 and the lowest P-value of 0.000. It is a good regression equation for predicting nicotine content because it has a high value of adjusted R^2 and a low P-value. Possible models:

    $\hat{y} = 0.080 + 0.0633x_1$, adjusted $R^2 = 0.877$

    $\hat{y} = 0.328 + 0.0397x_2$, adjusted $R^2 = 0.437$

    $\hat{y} = 0.127 + 0.0878x_1 - 0.0250x_2$, adjusted $R^2 = 0.927$

14. The best regression equation is $\hat{y} = 0.251 + 0.101x_1 - 0.0454x_2$, where x_1 represents tar and x_2 represents carbon monoxide. It is best because it has the highest adjusted R^2 value of 0.908 and the lowest P-value of 0.000. It is a good regression equation for predicting nicotine content because it has a high value of adjusted R^2 and a low P-value. Possible models:

    $\hat{y} = 0.139 + 0.0567x_1$, adjusted $R^2 = 0.752$

    $\hat{y} = 0.385 + 0.0325x_2$, adjusted $R^2 = 0.283$

    $\hat{y} = 0.251 + 0.101x_1 - 0.0454x_2$, adjusted $R^2 = 0.908$

15. The best regression equation is $\hat{y} = 109 - 0.00670x_1$, where x_1 represents volume. It is best because it has the highest adjusted R^2 value of -0.0513 and the lowest P-value of 0.791. The three regression equations all have adjusted values of R^2 that are very close to 0, so none of them are good for predicting IQ. It does not appear that people with larger brains have higher IQ scores. Possible models:

    $\hat{y} = 109 - 0.00670x_1$, adjusted $R^2 = -0.0513$

    $\hat{y} = 101 - 0.00178x_2$, adjusted $R^2 = -0.0555$

    $\hat{y} = 108 - 0.00694x_1 + 0.00722x_2$, adjusted $R^2 = -0.113$

16. The best regression equation is $\hat{y} = -10.0 + 0.567x_1 + 0.532x_2$, where x_1 represents verbal IQ score and x_2 represents performance IQ score. It is best because it has the highest adjusted R^2 value of 0.999 and the lowest P-value of 0.000. Because the adjusted R^2 is so close to 1, it is likely that predicted values will be very accurate. Possible models:

    $\hat{y} = 11.5 + 0.940x_1$, adjusted $R^2 = 0.762$

    $\hat{y} = 10.5 + 0.806x_2$, adjusted $R^2 = 0.814$

    $\hat{y} = -10.0 + 0.567x_1 + 0.532x_2$, adjusted $R^2 = 0.999$

17. For H_0: $\beta_1 = 0$, Test statistic: $t = \dfrac{0.769317 - 0}{0.0711414} = 10.813917$; P-value < 0.0001; Reject H_0 and conclude that the regression coefficient of $b_1 = 0.769$ should be kept.

    For H_0: $\beta_2 = 0$, Test statistic: $t = \dfrac{1.009510 - 0}{0.0338123} = 29.856$; P-value < 0.0001; Reject H_0 and conclude that the regression coefficient of $b_2 = 1.01$ should be kept.

    It appears that the regression equation should include both independent variables of height and waist circumference.

18. $0.629 < \beta_1 < 0.910$; $0.943 < \beta_2 < 1.08$; Neither confidence interval includes 0, so neither of the two variables should be eliminated from the regression equation.

$$\text{CI for } \beta_1$$
$$b_1 - E < \beta_1 < b_1 + E$$
$$b_1 - t_{\alpha/2}s_{b_1} < \beta_1 < b_1 + t_{\alpha/2}s_{b_1}$$
$$0.7693 - 1.976(0.0711414) < \beta_1 < 0.7693 + 1.976(0.0711414)$$
$$0.629 < \beta_1 < 0.910$$

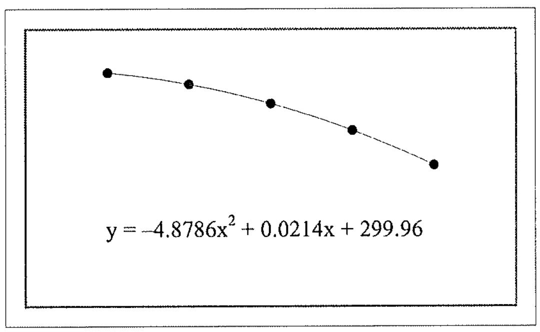
$$\text{CI for } \beta_2$$
$$b_2 - E < \beta_2 < b_2 + E$$
$$b_2 - t_{\alpha/2}s_{b_2} < \beta_2 < b_2 + t_{\alpha/2}s_{b_2}$$
$$1.0095 - 1.976(0.033812) < \beta_2 < 1.0095 - 1.976(0.033812)$$
$$0.943 < \beta_2 < 1.08$$

19. $\hat{y} = 3.06 + 82.4x_1 + 2.91x_2$, where x_1 represents sex and x_2 represents age.
Female: $\hat{y} = 3.06 + 82.4(0) + 2.91(20) = 61$ lb; Male: $\hat{y} = 3.06 + 82.4(1) + 2.91(20) = 144$ lb; The sex of the bear does appear to have an effect on its weight. The regression equation indicates that the predicted weight of a male bear is about 82 lb more than the predicted weight of a female bear with other characteristics being the same.

Section 10-5: Nonlinear Regression

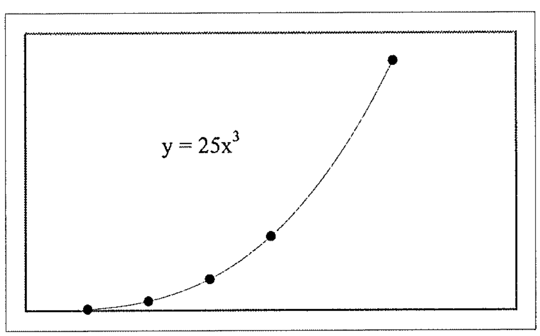
1. Since the area of a square is the square of its side, the best model quadratic: $y = x^2$; $R^2 = 1$.
2. The quadratic is the best option because it has the highest R^2 value, but this is not a good model because the value of R^2 is so low. Using the models discussed in this section, it appears that we cannot make accurate predictions of the numbers of points scored in future Super Bowl games. Common sense suggests that no such model could be found.
3. 25.5% of the variation in Super Bowl points can be explained by the quadratic model that relates the variable of year and the variable of points scored. Because such a small percentage of the variation is explained by the model, the model is not very useful.
4. Instead of showing a pattern that approximates the graph of the quadratic equation, the points are scattered about with no obvious pattern. The points do not fit the graph of the quadratic equation well, so the value of $R^2 = 0.255$ is very low.
5. Quadratic: $d = -4.88t^2 + 0.0214t + 300$

Model	R^2
Linear	0.962
Quadratic	1.000
Logarithmic	0.831
Exponential	0.933
Power	0.783



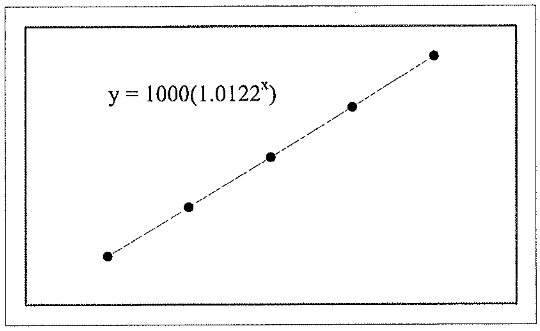
6. Power: $y = 25x^3$

Model	R^2
Linear	0.934
Quadratic	0.999
Logarithmic	0.653
Exponential	0.962
Power	1.000



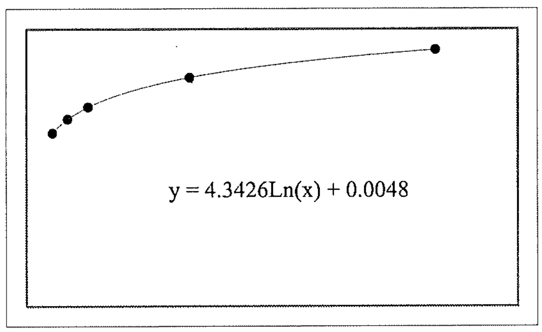
7. Exponential: $y = 1000\left(1.0122^{x}\right)$; The value of R^2 is slightly higher for the exponential model.

Model	R^2
Linear	0.999
Quadratic	0.999
Logarithmic	0.944
Exponential	0.999
Power	0.973



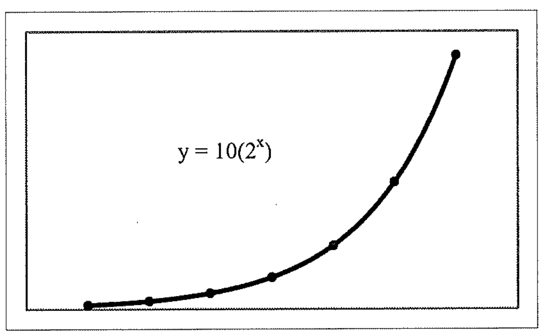
8. Logarithmic: $y = 0.00476 + 4.34 \ln x$

Model	R^2
Linear	0.895
Quadratic	0.988
Logarithmic	1.000
Exponential	0.861
Power	0.997



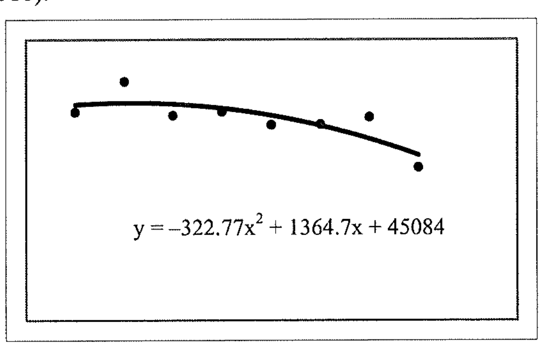
9. Exponential: $y = 10\left(2^{x}\right)$; with end of first day coded as 1.

Model	R^2
Linear	0.771
Quadratic	0.975
Logarithmic	0.549
Exponential	1.000
Power	0.927



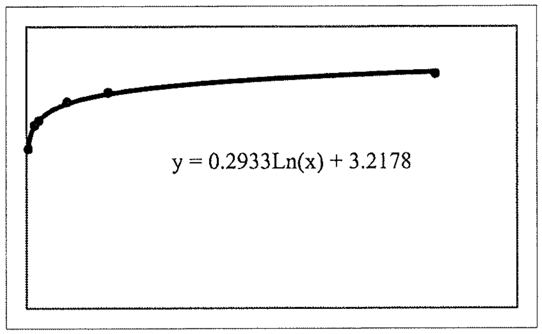
10. Quadratic: $\hat{y} = -323x^2 + 1365x + 45,084$ (with 1975 coded as 1); Projected value for 2025: 21,040 (Using rounded coefficients: 21,016).

Model	R^2
Linear	0.555
Quadratic	0.652
Logarithmic	0.388
Exponential	0.549
Power	0.377



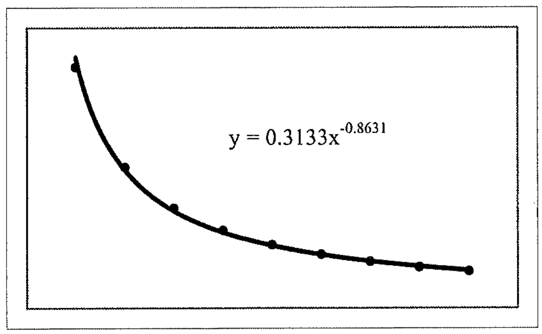
11. Logarithmic: $y = 3.22 + 0.293 \ln x$

Model	R^2
Linear	0.620
Quadratic	0.901
Logarithmic	0.997
Exponential	0.566
Power	0.989



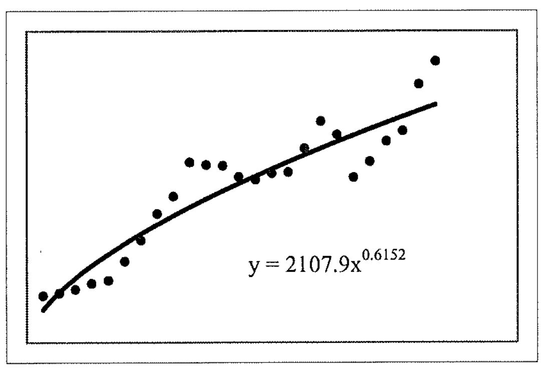
12. Power: $y = 0.313x^{-0.863}$

Model	R^2
Linear	0.746
Quadratic	0.945
Logarithmic	0.947
Exponential	0.931
Power	0.999



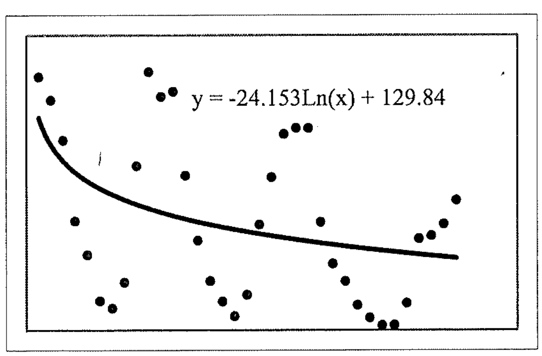
13. Power: $y = 2107.9x^{0.615}$; (Result is based on 1990 coded as 1.) The projected value for 2014 is 15,271 (Using rounded coefficients: 15,261), which is considerably less than the actual value of 18,054.

Model	R^2
Linear	0.856
Quadratic	0.876
Logarithmic	0.820
Exponential	0.804
Power	0.896



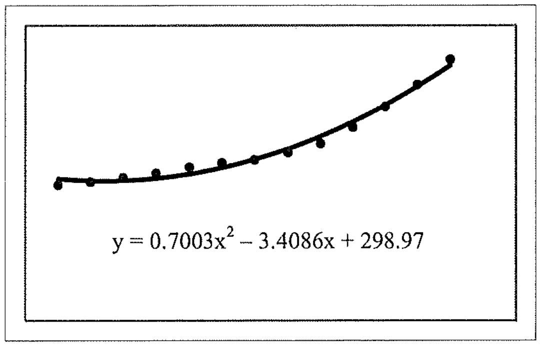
14. Logarithmic: $y = 130 - 24.2 \ln x$; (1980 coded as 1.) With $R^2 = 0.177$, this best model is not a good model. There is a cyclical pattern that does not fit any of the five models included in this section.

Model	R^2
Linear	0.142
Quadratic	0.150
Logarithmic	0.177
Exponential	0.118
Power	0.128



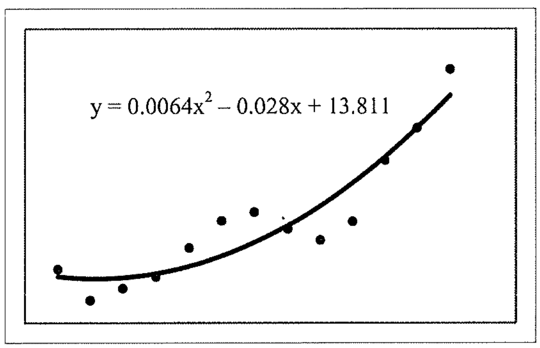
15. Quadratic: $y = 0.700x^2 - 3.41x + 299$; The projected value is $y = 0.700(22)^2 - 3.41(22) + 299 = 563$. (1880–1889 coded as 1.) The decade of 2090–2099 is too far beyond the scope of the available data, so the predicted value is questionable.

Model	R^2
Linear	0.870
Quadratic	0.985
Logarithmic	0.627
Exponential	0.891
Power	0.655



16. Quadratic: $y = 0.00641x^2 - 0.0280x + 13.8$; (1880–1889 coded as 1.) The projected value is $y = 0.00641(22)^2 - 0.0280(22) + 13.8 = 16.286°C$ (16.295°C using unrounded coefficients); The decade of 2090–2099 is too far beyond the scope of the available data, so the predicted value is questionable.

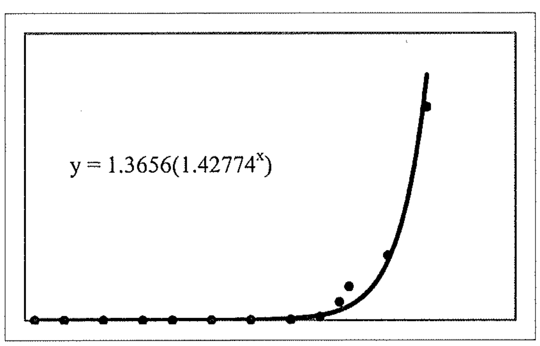
Model	R^2
Linear	0.785
Quadratic	0.878
Logarithmic	0.564
Exponential	0.789
Power	0.570



17. a. Exponential: $y = 2^{\frac{2}{3}(x-1)}$ [or $y = 0.629961(1.587401)^x$ for an initial value of 1 that doubles every 1.5 years].

b. Exponential: $y = 1.36558(1.42774)^x$, where x is year after 1970.

Model	R^2
Linear	0.380
Quadratic	0.55
Logarithmic	0.158
Exponential	0.990
Power	0.790



c. Moore's law does appear to be working reasonably well. With $R^2 = 0.990$, the model appears to be very good.

18. a. 6641.8

b. 73.2

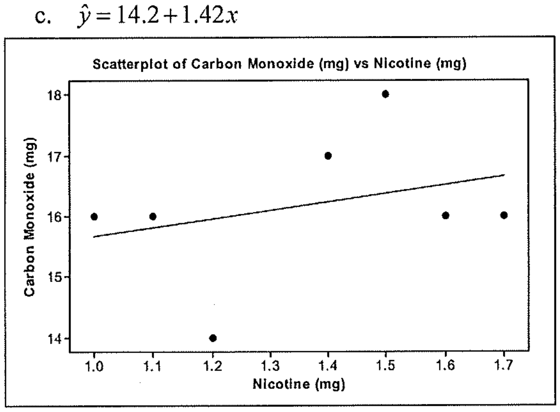
c. The quadratic sum of squares of residuals (73.2) is less than the sum of squares of residuals from the linear model (6641.8).

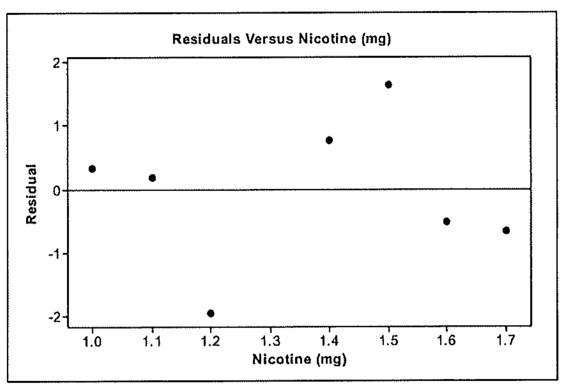
Quick Quiz

1. Conclude that there is not sufficient evidence to support the claim of a linear correlation between enrollment and burglaries.
2. None of the given values change when the variables are switched.
3. No, the value of r does not change if all values of one of the variables are multiplied by the same constant.
4. Because r must be between -1 and 1 inclusive, the value of 1.500 is the result of an error in the calculations.
5. The best predicted number of burglaries is 92.6, which is the mean of the five sample burglary counts.
6. The best predicted number of burglaries would be 123.3, which is found by substituting 50 for x in the regression equation.
7. $r^2 = 0.249$
8. false
9. false
10. $r = -1$

Review Exercises

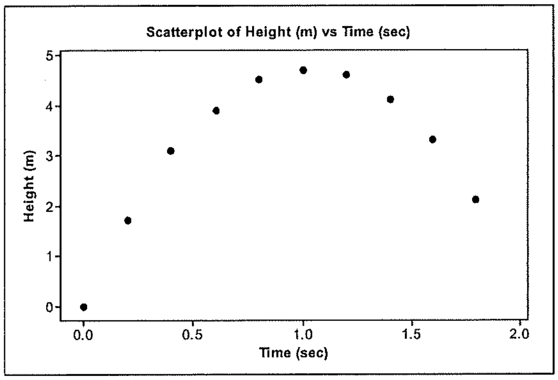
1. a. $r = 0.962$; P-value $= 0.000$ (Table: P-value < 0.01); Critical values ($\alpha = 0.05$): $r = \pm 0.707$; There is sufficient evidence to support the claim that there is a linear correlation between the amount of tar and the amount of nicotine.
   b. $(0.962)^2 = 0.925$, or 92.5%
   c. $\hat{y} = -0.758 + 0.0920x$
   d. The predicted value is $\hat{y} = -0.758 + 0.0920(23) = 1.358$ mg or 1.4 mg rounded, which is close to the actual amount of 1.3 mg.
2. a. The scatterplot (see part c) shows a pattern with nicotine and carbon monoxide both increasing from left to right, but it is a very weak pattern and the points are not very close to a straight-line pattern, so it appears that there is not sufficient sample evidence to support the claim of a linear correlation between amounts of nicotine and carbon monoxide.
   b. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.329$; P-value $= 0.427$ (Table: P-value > 0.05); Critical values ($\alpha = 0.05$): $r = \pm 0.707$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a linear correlation between amount of nicotine and amount of carbon monoxide.
   c. $\hat{y} = 14.2 + 1.42x$





   d. The predicted value is $\bar{y} = 16.1$ mg, which is close to the actual amount of 15 mg.

3. H_0: $\rho = 0$; H_1: $\rho \neq 0$; $r = 0.450$; P-value $= 0.192$ P-value > 0.05); Critical values ($\alpha = 0.05$): $r = \pm 0.632$; Fail to reject H_0. There is not sufficient evidence to support the claim that there is a linear correlation between time and height. Although there is no *linear* correlation between time and height, the scatterplot shows a very distinct pattern revealing that time and height are associated by some function that is not linear.



4. a. NICOTINE $= -0.443 + 0.0968\,\text{TAR} - 0.0262\,\text{CO}$, or $\hat{y} = -0.443 + 0.0968x_1 - 0.0262x_2$, where x_1 represents tar and x_2 represents carbon monoxide.
   b. $R^2 = 0.936$; adjusted $R^2 = 0.910$; P-value $= 0.001$
   c. With high values of R^2 and adjusted R^2 and a small P-value of 0.001, it appears that the regression equation can be used to predict the amount of nicotine given the amounts of tar and carbon monoxide.
   d. The predicted value is $\hat{y} = -0.443 + 0.0968(23) - 0.0262(15) = 1.39$ mg or 1.4 mg rounded, which is close to the actual value of 1.3 mg of nicotine.

Cumulative Review Exercises

1. a. $\bar{x} = 35.91$, $Q_2 = 36.10$, range $= 76.40$, $s = 31.45$, $s^2 = 989.10$
   b. quantitative data
   c. ratio
2. $r = 0.731$; P-value $= 0.039$ (Table: P-value < 0.05); Critical values: $r = \pm 0.707$; There is sufficient evidence to support the claim of a linear correlation between the DJIA values and sunspot numbers. Because it would be reasonable to think that there is no correlation between stocks and sunspot numbers, the result is not as expected. Although there appears to be a linear correlation, a reasonable investor would be wise to ignore sunspot numbers when investing in stocks.
3. The highest sunspot number is 79.3, which converts to $z = (79.3 - 35.91)/31.45 = 1.38$. The highest sunspot number is not significantly high because its z score of 1.38 shows that it is within 2 standard deviations of the mean.
4. The data do not appear to fit the loose definition of a normal distribution and $n < 30$, so proceed with caution. H_0: $\mu = 49.7$; H_1: $\mu \neq 49.7$;

   Test statistic: $t = \dfrac{35.91 - 49.7}{31.45/\sqrt{8}} = -1.240$; Critical values: $t = \pm 2.365$; P-value $= 0.255$

   (Table: P-value > 0.20); Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the sample is from a population with a mean equal to 49.7.
5. 95% CI: $\bar{x} \pm t_{\alpha/2}\dfrac{s}{\sqrt{n}} = 35.91 \pm 2.365 \cdot \dfrac{31.45}{\sqrt{8}} \Rightarrow 9.62 < \mu < 62.21$; We have 95% confidence that the interval limits of 9.62 and 62.21 contain the true value of the mean of the population of sunspot numbers.

6. $H_0: p = 0.10$; $H_1: p < 0.10$; Test statistic: $z = \dfrac{0.07 - 0.10}{\sqrt{\frac{(0.10)(0.90)}{2733}}} = -5.23$ ($z = -5.25$ using $x = 191$);

P-value $= P(z < -5.23) = 0.0000$ (Table: 0.0001); Critical value: $z = -1.645$;

Reject H_0. There is sufficient evidence to support the claim that fewer than 10% of police traffic stops are attributable to improper cell phone use.

7. $$\bar{x} = \frac{7 \cdot 6.5 + 15 \cdot 14.5 + 19 \cdot 21.0 + 19 \cdot 31.0 + 15 \cdot 44.5 + 11 \cdot 54.5 + 14 \cdot 70}{7 + 15 + 19 + 19 + 15 + 11 + 14} = 35.2 \text{ years}$$

$$s = \sqrt{\frac{100\left(7 \cdot 6.5^2 + \cdots + 14 \cdot 70^2\right) - \left(7 \cdot 6.5 + \cdots + 14 \cdot 70\right)^2}{100(100-1)}} = 19.7 \text{ years}$$

$s^2 = (19.7 \text{ years})^2 = 389.6 \text{ years}^2$

8. a. $z_{x=30} = \dfrac{30 - 35}{20} = -0.25$; which has a probability of 0.4013, or 40.13%, to the left.

b. The z score for the bottom 25% is –0.67, which correspond to the length $-0.67 \cdot 20 + 35 = 21.6$ years (Tech: 21.5 years).

c. $z_{x=30} = \dfrac{30 - 35}{20/\sqrt{25}} = -1.25$; which has a probability of 0.1056 to the left.

d. 0+ or 0.0000 (from $0.4013^{25} = 0.0000$); The audience for a particular movie and showtime is not a simple random sample. Some movies and showtimes attract very young audiences.

Chapter 11: Goodness-of-Fit and Contingency Tables

Section 11-1: Goodness-of-Fit

1. a. Observed values are represented by O and expected values are represented by E.
   b. For the leading digit of 2, $O = 62$ and $E = (317)(0.176) = 55.792$.
   c. For the leading digit of 2, $(O-E)^2/E = 0.691$.
2. H_0: $p_1 = 0.301, p_2 = 0.176, p_3 = 0.125, \ldots, p_9 = 0.046$;
   H_1: At least one of the proportions is not equal to the given claimed value.
3. There is sufficient evidence to warrant rejection of the claim that the leading digits have a distribution that fits well with Benford's law.
4. Because the leading digits of inter-arrival traffic times do not fit the distribution described by Benford's law, it appears that those times are not typical. The anomaly could be due to other factors, but there is a good chance that the computer has been hacked. Computer experts should be used to correct the incursion and secure the computer against further incursions.
5. H_0: The frequency counts agree with the claimed distribution.
   H_1: The frequency counts do not agree with the claimed distribution.

   Test statistic: $\chi^2 = 8.185$; P-value $= 0.516$ (Table: P-value > 0.10); Critical value: $\chi^2 = 16.919$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the observed outcomes agree with the expected frequencies. The slot machine appears to be functioning as expected.
6. H_0: $p_1 = p_2 = p_3 = p_4 = 0.25$;
   H_1: At least one of the proportions is not equal to the given claimed value.

   Test statistic: $\chi^2 = 4.600$; P-value $= 0.204$ (Table: P-value > 0.10); Critical value: $\chi^2 = 7.815$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the tires selected by the students are equally likely. It appears that when four students identify a tire, it is not likely that they would all select the same tire.

   $$\chi^2 = \frac{(11-10.25)^2}{0.25\cdot 41} + \frac{(15-10.25)^2}{0.25\cdot 41} + \frac{(18-10.25)^2}{0.25\cdot 41} + \frac{(6-10.25)^2}{0.25\cdot 41} = 4.600 \text{ (df} = 3)$$
7. H_0: $p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = 1/6$;
   H_1: At least one of the proportions is not equal to the given claimed value.

   Test statistic: $\chi^2 = 5.860$; P-value $= 0.320$ (Table: P-value > 0.10); Critical value: $\chi^2 = 11.071$; Fail to reject H_0. There is not sufficient evidence to support the claim that the outcomes are not equally likely. The outcomes appear to be equally likely, so the loaded die does not appear to behave differently from a fair die.

   $$\chi^2 = \frac{(27-28.571)^2}{200/7} + \frac{(31-28.571)^2}{200/7} + \cdots + \frac{(28-28.571)^2}{200/7} + \frac{(32-28.571)^2}{200/7} = 5.860 \text{ (df} = 5)$$
8. H_0: $p_1 = 0.756, p_2 = 0.091, p_3 = 0.108, p_4 = 0.038, p_5 = 0.007$;
   H_1: At least one of the proportions is not equal to the given claimed value.

   Test statistic: $\chi^2 = 524.713$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 13.277$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the distribution of clinical trial participants fits well with the population distribution. Hispanics have an observed frequency of 60 and an expected frequency of 391.027, so they are very underrepresented. Also, the Asian>Pacific Islander subjects have an observed frequency of 54 and an expected frequency of 163.286, so they are also underrepresented.

8. (continued)

$$\chi^2 = \frac{(3855-3248.532)^2}{0.756\cdot 4297} + \frac{(60-391.027)^2}{0.091\cdot 4297} + \cdots + \frac{(54-163.286)^2}{0.038\cdot 4297} + \frac{(12-30.079)^2}{0.007\cdot 4297} = 13.277 \text{ (df = 4)}$$

9. H_0: $p_{\text{YS}} = 9/16, p_{\text{GS}} = 3/16, p_{\text{YW}} = 3/16, p_{\text{GW}} = 1/16$;

   H_1: At least one of the proportions is not equal to the given claimed value.

   Test statistic: $\chi^2 = 11.161$; P-value $= 0.011$ (Table: P-value < 0.025); Critical value: $\chi^2 = 7.815$; Reject H_0. There is sufficient evidence to support the claim that the results contradict Mendel's theory.

$$\chi^2 = \frac{(307-281.25)^2}{0.5625\cdot 500} + \frac{(77-93.75)^2}{0.1875\cdot 500} + \frac{(98-93.75)^2}{0.1875\cdot 500} + \frac{(18-32.25)^2}{0.0625\cdot 500} = 11.161 \text{ (df = 3)}$$

10. H_0: The frequency counts agree with the claimed distribution.

    H_1: The frequency counts do not agree with the claimed distribution.

    Test statistic: $\chi^2 = 0.976$; P-value $= 0.913$ (Table: P-value > 0.10); Critical value: $\chi^2 = 9.488$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the actual frequencies fit a Poisson distribution. There is no way to prove that the frequencies fit a Poisson distribution even though it appears that they do.

$$\chi^2 = \frac{(229-227.5)^2}{227.5} + \frac{(211-211.4)^2}{211.4} + \frac{(93-97.9)^2}{97.9} + \frac{(35-30.5)^2}{30.5} + \frac{(8-8.7)^2}{8.7} = 0.976 \text{ (df = 4)}$$

11. H_0: $p_{\text{Sun}} = p_{\text{Mon}} = p_{\text{Tue}} = p_{\text{Wed}} = p_{\text{Thu}} = p_{\text{Fri}} = p_{\text{Sat}} = 1/7$;

    H_1: At least one of the proportions is not equal to the given claimed value.

    Test statistic: $\chi^2 = 29.814$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 16.812$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the different days of the week have the same frequencies of police calls. The highest numbers of calls appear to fall on Friday and Saturday, and these are weekend days with disproportionately more partying and drinking.

$$\chi^2 = \frac{(130-156.43)^2}{1095/7} + \frac{(114-156.43)^2}{1095/7} + \cdots + \frac{(179-156.43)^2}{1095/7} + \frac{(196-156.43)^2}{1095/7} = 29.814 \text{ (df = 6)}$$

12. H_0: $p_{\text{Sun}} = p_{\text{Mon}} = p_{\text{Tue}} = p_{\text{Wed}} = p_{\text{Thu}} = p_{\text{Fri}} = p_{\text{Sat}} = 1/7$;

    H_1: At least one of the proportions is not equal to the given claimed value.

    Test statistic: $\chi^2 = 31.963$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 16.812$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the different days of the week have the same frequencies of police calls. Because March has 31 days, three of the days of the week occur more often than the other days of the week, so the comparison does not make sense with the given data.

$$\chi^2 = \frac{(154-207.29)^2}{1451/7} + \frac{(208-207.29)^2}{1451/7} + \cdots + \frac{(210-207.29)^2}{1451/7} + \frac{(236-207.29)^2}{1451/7} = 31.963 \text{ (df = 6)}$$

13. H_0: $p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = p_9 = p_{10} = 0.1$;

    H_1: At least one of the proportions is not equal to the given claimed value.

    Test statistic: $\chi^2 = 13.855$; P-value $= 0.128$ (Table: P-value > 0.10); Critical value: $\chi^2 = 16.919$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the likelihood of winning is the same for the different post positions. Based on these results, post position should not be considered when betting on the Kentucky Derby race.

$$\chi^2 = \frac{(19-11.7)^2}{0.1\cdot 117} + \frac{(14-11.7)^2}{0.1\cdot 117} + \cdots + \frac{(5-11.7)^2}{0.1\cdot 117} + \frac{(11-11.7)^2}{0.1\cdot 117} = 13.855 \text{ (df = 9)}$$

14. $H_0: p_0 = p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = p_9 = 0.1$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 11.583$; P-value = 0.238 (Table: P-value > 0.10); Critical value: $\chi^2 = 16.919$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the digits are selected in a way that they are equally likely.

$$\chi^2 = \frac{(21-24)^2}{0.1\cdot 240} + \frac{(30-24)^2}{0.1\cdot 240} + \cdots + \frac{(24-24)^2}{0.1\cdot 240} + \frac{(22-24)^2}{0.1\cdot 240} = 11.583 \text{ (df = 9)}$$

15. $H_0: p_4 = 0.125, p_5 = 0.25, p_6 = 0.3125, p_7 = 0.3125$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 8.882$; P-value = 0.031 (Table: P-value < 0.05); Critical value: $\chi^2 = 7.815$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the actual numbers of games fit the distribution indicated by the proportions listed in the given table.

$$\chi^2 = \frac{(21-13.125)^2}{0.125\cdot 105} + \frac{(23-26.25)^2}{0.25\cdot 105} + \frac{(23-32.813)^2}{0.3125\cdot 105} + \frac{(38-32.813)^2}{0.3125\cdot 105} = 8.882 \text{ (df = 3)}$$

16. $H_0: p_{\text{Jan}} = p_{\text{Feb}} = p_{\text{Mar}} = p_{\text{Apr}} = p_{\text{May}} = p_{\text{Jun}} = p_{\text{Jul}} = p_{\text{Aug}} = p_{\text{Sep}} = p_{\text{Oct}} = p_{\text{Nov}} = p_{\text{Dec}} = 1/12$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 93.072$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 19.675$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that American born Major League Baseball players are born in different months with the same frequency. The sample data appear to support Gladwell's claim.

$$\chi^2 = \frac{(387-376.25)^2}{4515/12} + \frac{(329-376.25)^2}{4515/12} + \cdots + \frac{(398-376.25)^2}{4515/12} + \frac{(371-376.25)^2}{4515/12} = 93.072 \text{ (df = 11)}$$

17. $H_0: p_{\text{Sun}} = p_{\text{Mon}} = p_{\text{Tue}} = p_{\text{Wed}} = p_{\text{Thu}} = p_{\text{Fri}} = p_{\text{Sat}} = 1/7$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 9.500$; P-value = 0.147 (Table: P-value > 0.10); Critical value: $\chi^2 = 16.812$; Fail to reject H_0. There is not sufficient evidence to support the claim that births do not occur on the seven different days of the week with equal frequency.

Day	**Sun**	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
Expected	57.14	57.14	57.14	57.14	57.14	57.14	57.14
Observed	53	52	66	72	57	57	43

$$\chi^2 = \frac{(53-57.14)^2}{400/7} + \frac{(52-57.14)^2}{400/7} + \cdots + \frac{(57-57.14)^2}{400/7} + \frac{(43-57.14)^2}{400/7} = 9.500 \text{ (df = 6)}$$

18. $H_0: p_{\text{Sun}} = p_{\text{Mon}} = p_{\text{Tue}} = p_{\text{Wed}} = p_{\text{Thu}} = p_{\text{Fri}} = p_{\text{Sat}} = 1/7$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 10.760$; P-value = 0.096 (Table: P-value > 0.05); Critical value: $\chi^2 = 16.812$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that discharges occur on the seven different days of the week with equal frequency.

Day	**Sun**	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
Expected	57.14	57.14	57.14	57.14	57.14	57.14	57.14
Observed	65	50	47	48	73	64	53

18. (continued)

$$\chi^2 = \frac{(65-57.14)^2}{400/7} + \frac{(50-57.14)^2}{400/7} + \cdots + \frac{(64-57.14)^2}{400/7} + \frac{(53-57.14)^2}{400/7} = 10.760 \text{ (df = 6)}$$

19. H_0: $p_{\text{red}} = 0.13, p_{\text{orange}} = 0.20, p_{\text{yellow}} = 0.14, p_{\text{brown}} = 0.13, p_{\text{blue}} = 0.24, p_{\text{green}} = 0.16$;

H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 6.682$; P-value = 0.245 (Table: P-value > 0.10); Critical value: $\chi^2 = 11.071$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the color distribution is as claimed.

Color	Red	Orange	Yellow	Brown	Blue	Green
Expected	13	20	14	13	24	16
Observed	13	25	8	8	27	19

$$\chi^2 = \frac{(13-13)^2}{13} + \frac{(20-25)^2}{20} + \frac{(14-8)^2}{8} + \frac{(24-27)^2}{24} + \frac{(16-19)^2}{16} = 6.682 \text{ (df = 5)}$$

20. H_0: $p_0 = p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = p_9 = 0.1$;

H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 11.400$; P-value = 0.249 (Table: P-value > 0.10); Critical value: $\chi^2 = 16.919$; Fail to reject H_0. There is not sufficient evidence to support the claim that the sample is from a population of weights in which the last digits do *not* occur with the same frequency. That is, the last digits appear to occur with approximately the same frequencies. The results suggest that the weights were measured.

Digit	0	1	2	3	4	5	6	7	8	9
Expected	30	30	30	30	30	30	30	30	30	30
Observed	20	31	30	32	31	39	29	27	39	22

$$\chi^2 = \frac{(20-30)^2}{30} + \frac{(31-30)^2}{30} + \cdots + \frac{(39-30)^2}{30} + \frac{(22-30)^2}{30} = 11.400 \text{ (df = 9)}$$

21. H_0: $p_1 = 0.301, p_2 = 0.176, p_3 = 0.125, \ldots, p_7 = 0.058, p_8 = 0.051, p_9 = 0.046$;

H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 3650.251$; P-value = 0.000 (Table: P-value < 0.005); Critical value: $\chi^2 = 20.090$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the leading digits are from a population with a distribution that conforms to Benford's law. It does appear that the checks are the result of fraud (although the results cannot confirm that fraud is the cause of the discrepancy between the observed results and the expected results).

$$\chi^2 = \frac{(0-235.98)^2}{0.301\cdot 784} + \frac{(15-137.98)^2}{0.176\cdot 784} + \cdots + \frac{(23-39.98)^2}{0.051\cdot 784} + \frac{(0-36.06)^2}{0.046\cdot 784} = 3650.251 \text{ (df = 8)}$$

22. H_0: $p_1 = 0.301, p_2 = 0.176, p_3 = 0.125, \ldots, p_7 = 0.058, p_8 = 0.051, p_9 = 0.046$;

H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 18.955$; P-value = 0.015 (Table: P-value > 0.01); Critical value: $\chi^2 = 20.090$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the leading digits are from a population with a distribution that conforms to Benford's law. The author's check amounts appear to be legitimate. For a 0.05 significance level, the conclusion changes to the statement that there is sufficient evidence to warrant rejection of the claim that the leading digits are from a population with a distribution that conforms to Benford's law.

$$\chi^2 = \frac{(83-75.25)^2}{0.301\cdot 250} + \frac{(58-44.00)^2}{0.176\cdot 250} + \cdots + \frac{(4-12.75)^2}{0.051\cdot 250} + \frac{(9-11.50)^2}{0.046\cdot 250} = 18.955 \text{ (df = 8)}$$

23. H_0: $p_1 = 0.301, p_2 = 0.176, p_3 = 0.125, \ldots, p_7 = 0.058, p_8 = 0.051, p_9 = 0.046$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 1.762$; P-value $= 0.988$ (Table: P-value > 0.10); Critical value: $\chi^2 = 15.507$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the leading digits are from a population with a distribution that conforms to Benford's law. The tax entries do appear to be legitimate.

$$\chi^2 = \frac{(152-153.8)^2}{0.301\cdot 511} + \frac{(89-89.9)^2}{0.176\cdot 511} + \cdots + \frac{(25-26.1)^2}{0.051\cdot 511} + \frac{(27-23.5)^2}{0.046\cdot 511} = 1.762 \text{ (df = 8)}$$

24. H_0: $p_1 = 0.301, p_2 = 0.176, p_3 = 0.125, \ldots, p_7 = 0.058, p_8 = 0.051, p_9 = 0.046$;
H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 12.922$; P-value $= 0.112$ (Table: P-value > 0.10); Critical value: $\chi^2 = 15.507$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the leading digits are from a population with a distribution that conforms to Benford's law.

$$\chi^2 = \frac{(55-51.2)^2}{0.301\cdot 170} + \frac{(25-9.9)^2}{0.176\cdot 170} + \cdots + \frac{(3-8.7)^2}{0.051\cdot 170} + \frac{(4-7.8)^2}{0.046\cdot 170} = 12.922 \text{ (df = 8)}$$

25. H_0: Heights selected from a normal distribution.
H_1: Heights not selected from a normal distribution.

	Height (cm)	Less than 155.45	155.45 – 162.05	162.05–168.65	Greater than 168.65
a.	**Frequency**	26	46	49	26
b.	**Tech:**	0.2023	0.3171	0.3046	0.1761
	Table:	0.2033	0.3166	0.3039	0.1762
c.	**Tech:**	0.2023(247) = 29.7381	0.3171(247) = 46.6137	0.3046(247) = 44.7762	0.1761(247) = 25.8867
	Table:	0.2033(247) = 29.8851	0.3166(247) = 46.5402	0.3039(247) = 44.6733	0.1762(247) = 25.9014

d. Test statistic: $\chi^2 = 0.877$ (Table: $\chi^2 = 0.831$); P-value $= 0.831$ (Table: P-value > 0.10); Critical value: $\chi^2 = 11.345$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that heights were randomly selected from a normally distributed population. The test suggests that we cannot rule out the possibility that the data are from a normally distributed population.

$$\chi^2 = \frac{(26-29.7381)^2}{29.7381} + \frac{(46-46.6137)^2}{46.6137} + \frac{(49-44.7762)^2}{44.7762} + \frac{(26-25.8867)^2}{25.8867} = 0.877 \text{ (df = 3)}$$

Section 11-2: Contingency Tables

1. a. $E = \dfrac{(16+50+3)(40+3)}{436+166+40+16+50+3} = \dfrac{(69)(43)}{711} = 4.173$

b. Because the expected frequency of a cell is less than 5, the requirements for the hypothesis test are not satisfied.

2. H_0: Whether a subject is right-handed or left-handed is independent of ear preference for cell phone use.
H_1: Right-left-handedness and ear preference for cell phone use are dependent.

3. Test statistic: $\chi^2 = 64.517$; P-value $= 0.0000$; Reject the null hypothesis of independence between handedness and cell phone ear preference.

4. The test is right-tailed. The test statistic is based on differences between observed frequencies and the frequencies expected with the assumption of independence between the row and column variables. Only large values of the test statistic correspond to substantial differences between the observed and expected values, and such large values are located in the right tail of the distribution.

5. H_0: Whether a subject lies is independent of polygraph indication.

   H_1: Subject lies depends on polygraph indication.

   Test statistic: $\chi^2 = 25.571$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; df = (2 − 1)(2 − 1) = 1; Reject H_0. There is sufficient evidence to warrant rejection of the claim that whether a subject lies is independent of the polygraph test indication. The results suggest that polygraphs are effective in distinguishing between truths and lies, but there are many false positives and false negatives, so they are not highly reliable.

$$\chi^2 = \frac{(15-27.3)^2}{27.3} + \frac{(42-29.7)^2}{29.7} + \frac{(32-19.7)^2}{19.7} + \frac{(9-21.3)^2}{21.3} = 25.571$$

6. H_0: Whether success is independent of type of treatment.

   H_1: Success depends on type of treatment.

   Test statistic: $\chi^2 = 9.750$; P-value = 0.002 (Table: P-value < 0.005); Critical value: $\chi^2 = 6.635$; df = (2 − 1)(2 − 1) = 1; Reject H_0. There is sufficient evidence to warrant rejection of the claim that success is independent of the type of treatment. The results suggest that the surgery treatment is better.

$$\chi^2 = \frac{(60-67.6)^2}{67.6} + \frac{(23-15.4)^2}{15.4} + \frac{(67-59.4)^2}{59.4} + \frac{(6-13.6)^2}{13.6} = 9.750$$

7. H_0: Whether texting while driving is independent of driving when drinking alcohol.

   H_1: Texting while driving depends on driving when drinking alcohol.

   Test statistic: $\chi^2 = 576.224$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; df = (2 − 1)(2 − 1) = 1; Reject H_0. There is sufficient evidence to warrant rejection of the claim that success is independent of the type of treatment. The results suggest that the surgery treatment is better.

$$\chi^2 = \frac{(731-394.7)^2}{394.7} + \frac{(3054-3390.3)^2}{3390.3} + \frac{(156-492.3)^2}{492.3} + \frac{(4564-4227.7)^2}{4227.7} = 576.224$$

8. H_0: Whether texting while driving is independent of irregular seat belt use.

   H_1: Texting while driving depends on irregular seat belt use.

   Test statistic: $\chi^2 = 18.773$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; df = (2 − 1)(2 − 1) = 1; Reject H_0. There is sufficient evidence to warrant rejection of the claim of independence between texting while driving and irregular seat belt use. Those two risky behaviors appear to be somehow related.

$$\chi^2 = \frac{(1737-1638.6)^2}{1638.6} + \frac{(2048-2146.4)^2}{2146.4} + \frac{(1945-2043.4)^2}{2043.4} + \frac{(2775-2676.6)^2}{2676.6} = 18.773$$

9. H_0: Whether students spent or kept the money is independent of form of \$1.

   H_1: Students spending or keeping the money depends on form of \$1.

   Test statistic: $\chi^2 = 12.162$; P-value = 0.001 (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; df = (2 − 1)(2 − 1) = 1; Reject H_0. There is sufficient evidence to warrant rejection of the claim that whether students purchased gum or kept the money is independent of whether they were given four quarters or a \$1 bill. It appears that there is a denomination effect.

$$\chi^2 = \frac{(27-18.8)^2}{18.8} + \frac{(16-24.2)^2}{24.2} + \frac{(12-20.2)^2}{20.2} + \frac{(34-25.8)^2}{25.8} = 12.162$$

10. H_0: Whether winning an overtime game is independent of playing under the old rule or the new rule.
H_1: Winning an overtime game depends on playing under the old rule or the new rule.
Test statistic: $\chi^2 = 0.238$; P-value $= 0.626$ (Table: P-value > 0.10); Critical value: $\chi^2 = 3.841$; $\text{df} = (2-1)(2-1) = 1$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim of independence between winning an overtime game and whether playing under the old rule or the new rule. It appears that the change in the rule has no effect.

$$\chi^2 = \frac{(252-250.4)^2}{250.4} + \frac{(24-25.6)^2}{25.6} + \frac{(208-209.6)^2}{209.6} + \frac{(23-21.4)^2}{21.4} = 0.238$$

11. H_0: Whether gender is independent of whether call is overturned.
H_1: Call is overturned depends on gender.
Test statistic: $\chi^2 = 0.064$; P-value $= 0.801$ (Table: P-value > 0.10); Critical value: $\chi^2 = 3.841$; $\text{df} = (2-1)(2-1) = 1$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the gender of the tennis player is independent of whether the call is overturned. Neither men nor women appear to be better at challenging calls.

$$\chi^2 = \frac{(161-162.5)^2}{162.5} + \frac{(376-374.6)^2}{374.6} + \frac{(69-66.6)^2}{66.6} + \frac{(152-135.5)^2}{135.5} = 0.064\,11.$$

12. H_0: Whether deaths on a shift is independent of whether Gilbert was working.
H_1: Deaths on a shift depends on whether Gilbert was working.
Test statistic: $\chi^2 = 86.481$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 6.635$; $\text{df} = (2-1)(2-1) = 1$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that deaths on shifts are independent of whether Gilbert was working. The results favor the guilt of Gilbert.

$$\chi^2 = \frac{(40-11.6)^2}{11.6} + \frac{(217-245.4)^2}{245.4} + \frac{(34-62.4)^2}{62.4} + \frac{(1350-1321.6)^2}{1321.6} = 86.481$$

13. H_0: Whether direction of kick is independent of direction of the goalkeeper jump.
H_1: Direction of kick depends on direction of the goalkeeper jump.
Test statistic: $\chi^2 = 14.589$; P-value $= 0.0056$ (Table: P-value < 0.01); Critical value: $\chi^2 = 9.488$; $\text{df} = (3-1)(3-1) = 4$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the direction of the kick is independent of the direction of the goalkeeper jump. The results do not support the theory that because the kicks are so fast, goalkeepers have no time to react. It appears that goalkeepers can choose directions based on the directions of the kicks.

$$\chi^2 = \frac{(54-45.4)^2}{45.4} + \frac{(1-5.8)^2}{5.8} + \frac{(37-40.9)^2}{40.9} + \frac{(41-40.4)^2}{40.4} + \frac{(10-5.2)^2}{5.2} + \frac{(31-36.4)^2}{36.4} + \frac{(46-55.2)^2}{55.2} + \frac{(7-7.1)^2}{7.1} + \frac{(59-49.7)^2}{49.7} = 14.589$$

14. H_0: Whether amount of smoking is independent of seat belt use.
H_1: Amount of smoking depends on seat belt use.
Test statistic: $\chi^2 = 1.358$; P-value $= 0.715$ (Table: P-value > 0.10); Critical value $(\alpha = 0.05)$: $\chi^2 = 7.815$; $\text{df} = (2-1)(4-1) = 3$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the amount of smoking is independent of seat belt use. The theory is not supported by the given data.

$$\chi^2 = \frac{(175-171.5)^2}{171.5} + \cdots + \frac{(6-7.9)^2}{7.9} + \frac{(149-152.5)^2}{152.5} + \cdots + \frac{(9-7.1)^2}{7.1} = 1.358$$

15. H_0: Whether getting a cold is independent of treatment.

H_1: Getting a cold depends on treatment.

Test statistic: $\chi^2 = 2.925$; P-value $= 0.232$ (Table: P-value > 0.10); Critical value: $\chi^2 = 5.991$; df $= (2-1)(3-1) = 2$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that getting a cold is independent of the treatment group. The results suggest that echinacea is not effective for preventing colds.

$$\chi^2 = \frac{(88-88.6)^2}{88.6} + \frac{(48-44.7)^2}{44.7} + \frac{(42-44.7)^2}{44.7} + \frac{(15-14.4)^2}{14.4} + \frac{(4-7.3)^2}{7.3} + \frac{(10-7.3)^2}{7.3} = 2.925$$

16. H_0: Whether injuries are independent of helmet color.

H_1: Injuries depend on helmet color.

Test statistic: $\chi^2 = 9.971$; P-value $= 0.041$ (Table: P-value < 0.05); Critical value ($\alpha = 0.05$): $\chi^2 = 9.488$; df $= (2-1)(5-1) = 4$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that injuries are independent of helmet color. It appears that motorcycle drivers should use yellow or orange helmets.

$$\chi^2 = \frac{(491-509.5)^2}{509.5} + \cdots + \frac{(55-58.6)^2}{58.6} + \frac{(213-194.5)^2}{194.5} + \cdots + \frac{(26-22.4)^2}{22.4} = 9.971$$

17. H_0: Whether cooperation of the subject is independent of age category.

H_1: Cooperation of the subject depends on age category.

Test statistic: $\chi^2 = 20.271$; P-value $= 0.0011$ (Table: P-value < 0.005); Critical value: $\chi^2 = 15.086$; df $= (2-1)(6-1) = 5$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that cooperation of the subject is independent of the age category. The age group of 60 and over appears to be particularly uncooperative.

$$\chi^2 = \frac{(73-73.1)^2}{73.1} + \cdots + \frac{(202-218.5)^2}{218.5} + \frac{(11-10.9)^2}{10.9} + \cdots + \frac{(49-32.5)^2}{32.5} = 20.271$$

18. H_0: Whether months of birth of baseball players are independent of being born in America.

H_1: Months of birth of baseball players depends on being born in America.

Test statistic: $\chi^2 = 20.054$; P-value $= 0.0446$ (Table: P-value < 0.05); Critical value: $\chi^2 = 19.675$; df $= (2-1)(12-1) = 11$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that months of births of baseball players are independent of whether they are born in America. The data do appear to support Gladwell's claim.

$$\chi^2 = \frac{(387-397.2)^2}{397.2} + \cdots + \frac{(371-368.7)^2}{368.7} + \frac{(101-90.8)^2}{90.8} + \cdots + \frac{(82-84.3)^2}{84.3} = 20.054$$

19. H_0: Whether state is independent of car having front and rear license plates.

H_1: State depends on car having front and rear license plates.

Test statistic: $\chi^2 = 50.446$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 5.991$; df $= (2-1)(3-1) = 2$; Reject H_0. There is sufficient evidence to warrant rejection of the claim of independence between the state and whether a car has front and rear license plates. It does not appear that the license plate laws are followed at the same rates in the three states.

$$\chi^2 = \frac{(35-34.6)^2}{34.6} + \cdots + \frac{(9-33.8)^2}{33.8} + \frac{(528-528.4)^2}{528.4} + \cdots + \frac{(541-516.2)^2}{516.2} = 50.446$$

20. H_0: Whether home/visitor wins is independent of the sport.

H_1: Home/visitor wins depends on the sport.

Test statistic: $\chi^2 = 4.737$; P-value $= 0.192$ (Table: P-value > 0.10); Critical value: $\chi^2 = 6.251$; df $= (2-1)(4-1) = 3$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that home / visitor wins are independent of the sport. Based on the given sample data, baseball teams are not effective in modifying field dimensions for their own players.

$$\chi^2 = \frac{(127-116.0)^2}{116.0} + \cdots + \frac{(57-58.0)^2}{58.0} + \frac{(71-82.0)^2}{82.0} + \cdots + \frac{(42-41.0)^2}{41.0} = 4.737$$

21. From Exercise 9, $\chi^2 = 12.1619258$ and from Review Exercise 1 in Chapter 9, $z = 3.487395274$, so $z^2 = \chi^2$. The critical values are : $\chi^2 = 3.841$ and $z^2 = \pm 1.96$, so $z^2 = \chi^2$ (approximately).

22. Without Yates's correction, the test statistic is $\chi^2 = 12.162$. With Yates's correction, the test statistic is $\chi^2 = 10.717$. Yates's correction decreases the test statistic so that sample data must be more extreme in order to reject the null hypothesis of independence.

Without Yates's correction:

$$\chi^2 = \frac{(27-18.84)^2}{18.84} + \frac{(16-24.16)^2}{24.16} + \frac{(12-20.16)^2}{20.16} + \frac{(34-25.84)^2}{25.84} = 12.162$$

With Yates's Correction:

$$\chi^2 = \frac{(|27-18.84|-0.5)^2}{18.84} + \frac{(|16-24.16|-0.5)^2}{24.16} + \frac{(|12-20.16|-0.5)^2}{20.16} + \frac{(|34-25.84|-0.5)^2}{25.84} = 10.717$$

Quick Quiz

1. $H_0: p_0 = p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = p_9 = 0.1$;

   H_1: At least one of the probabilities is different from the others.

2. $O = 27$ and $E = 0.1 \cdot 300 = 30$

3. right-tailed

4. df $= n - 1 = 10 - 1 = 9$

5. There is not sufficient evidence to warrant rejection of the claim that the last digits are equally likely. Because reported heights would likely include more last digits of 0 and 5, it appears that the heights were measured instead of reported. (Also, most U.S. residents would have difficulty reporting heights in centimeters, because the United States, Liberia, and Myanmar are the only countries that continue to use the Imperial system of measurement.)

6. H_0: Surviving the sinking is independent of whether the person is a man, woman, boy, or girl.

   H_1: Surviving the sinking and whether the person is a man, woman, boy, or girl are somehow related.

7. chi-squared distribution

8. right-tailed

9. df $= (r-1)(c-1) = (2-1)(4-1) = 3$

10. There is sufficient evidence to warrant rejection of the claim that surviving the sinking is independent of whether the person is a man, woman, boy, or girl. Most of the women survived, 45% of the boys survived, and most girls survived, but only about 20% of the men survived, so it appears that the rule was followed quite well.

Review Exercises

1. H_0: $p_{\text{Sun}} = p_{\text{Mon}} = p_{\text{Tue}} = p_{\text{Wed}} = p_{\text{Thu}} = p_{\text{Fri}} = p_{\text{Sat}} = 1/7$;
   H_1: At least one of the proportions is not equal to the given claimed value.
   Test statistic: $\chi^2 = 787.018$; P-value = 0.0000 (Table: P-value < 0.005); Critical value: $\chi^2 = 16.812$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that auto fatalities occur on the different days of the week with the same frequency. Because people generally have more free time on weekends and more drinking occurs on weekends, the days of Friday, Saturday, and Sunday appear to have disproportionately more fatalities.
   $$\chi^2 = \frac{(5304-4674.14)^2}{32,719/7} + \cdots + \frac{(5985-4674.14)^2}{32,719/7} = 787.018 \text{ (df} = 6)$$
2. H_0: Whether health condition is independent of type of filling.
   H_1: Health condition depends on type of filling.
   Test statistic: $\chi^2 = 0.751$; P-value = 0.356 (Table: P-value > 0.10); Critical value: $\chi^2 = 3.841$; $\text{df} = (2-1)(2-1) = 1$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim of independence between the type of filling and adverse health conditions. Fillings that contain mercury do not appear to affect health conditions.
   $$\chi^2 = \frac{(135-140)^2}{140} + \frac{(145-140)^2}{140} + \frac{(132-127)^2}{127} + \frac{(122-127)^2}{127} = 0.751$$
3. H_0: The frequency counts agree with the claimed distribution.
   H_1: The frequency counts do not agree with the claimed distribution.
   Test statistic: $\chi^2 = 5.624$; P-value = 0.467 (Table: P-value > 0.10); Critical value: $\chi^2 = 12.592$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the actual eliminations agree with the expected numbers. The leadoff singers do appear to be at a disadvantage because 20 of them were eliminated compared to the expected value of 12.9 eliminations, but that result does not appear to be *significantly* high.
   $$\chi^2 = \frac{(20-12.9)^2}{12.9} + \frac{(12-12.9)^2}{12.9} + \frac{(9-9.9)^2}{9.9} + \frac{(6-6.4)^2}{6.4} + \frac{(5-5.5)^2}{5.5} + \frac{(9-13.5)^2}{13.5} = 5.624 \text{ (df} = 6)$$
4. H_0: Whether getting an infection is independent of type of treatment.
   H_1: Getting an infection depends on type of treatment.
   Test statistic: $\chi^2 = 0.773$; P-value = 0.856 (Table: P-value > 0.10); Critical value: $\chi^2 = 11.345$; $\text{df} = (2-1)(4-1) = 3$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that getting an infection is independent of the treatment. The atorvastatin (Lipitor) treatment does not appear to have an effect on infections.
   $$\chi^2 = \frac{(27-27.08)^2}{27.08} + \cdots + \frac{(7-9.43)^2}{9.43} + \frac{(234-242.92)^2}{242.92} + \cdots + \frac{(87-84.57)^2}{84.57} = 0.773$$

5. H_0: $p_{\text{Jan}} = p_{\text{Feb}} = p_{\text{Mar}} = p_{\text{Apr}} = p_{\text{May}} = p_{\text{Jun}} = p_{\text{Jul}} = p_{\text{Aug}} = p_{\text{Sep}} = p_{\text{Oct}} = p_{\text{Nov}} = p_{\text{Dec}} = 1/12$;
   H_1: At least one of the proportions is not equal to the given claimed value.
   Test statistic: $\chi^2 = 269.147$; P-value $= 0.0000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 24.725$;
   Reject H_0. There is sufficient evidence to warrant rejection of the claim that weather related deaths occur in the different months with the same frequency. The months of May, June, and July appear to have disproportionately more weather-related deaths, and that is probably due to the fact that vacations and outdoor activities are much greater during those months.
   $$\chi^2 = \frac{(28-37.5)^2}{450/12} + \frac{(17-37.5)^2}{450/12} + \cdots + \frac{(26-37.5)^2}{450/12} + \frac{(25-37.5)^2}{450/12} = 269.147 \text{ (df} = 11)$$

Cumulative Review Exercises

1. H_0: $p = 0.5$; H_1: $p \neq 0.5$; Test statistic: $z = \dfrac{\frac{320}{450} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{450}}} = 8.96$;
   P-value $= 2 \cdot P(z > 8.96) = 0.0000$ (Table: 0.0002); Critical values: $z = \pm 1.96$;
   Reject H_0. There is sufficient evidence to warrant rejection of the claim that among those who die in weather-related deaths, the percentage of males is equal to 50%. One possible explanation is that more men participate in some outdoor activities, such as golf, fishing, and boating.
2. a. There is a possibility that the results were affected because the sponsor of the survey produces chocolate and therefore has an interest in the results.
   b. $0.85 \cdot 1708 = 1452$
3. 99% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}} = 0.85 \pm 2.756\sqrt{\dfrac{(0.85)(0.15)}{1708}} \Rightarrow 0.828 < p < 0.872$, or $82.8\% < p < 87.2\%$; We have 99% confidence that the limits of 82.8% and 87.2% contain the value of the true percentage of the population of women saying that chocolate makes them happier.
4. H_0: $p = 0.80$; H_1: $p > 0.80$; Test statistic: $z = \dfrac{\frac{1452}{1708} - 0.80}{\sqrt{\frac{(0.80)(0.20)}{1708}}} = 5.18$;
   P-value $= P(z > 5.18) = 0.0000$ (Table: 0.0001); Critical value: $z = 2.33$;
   Reject H_0. There is sufficient evidence to support the claim that when asked, more than 80% of women say that chocolate makes them happier.
5. H_0: Whether money was spent is independent of form of 100 Yuan.
   H_1: Whether money was spent depends on form of 100 Yuan.
   Test statistic: $\chi^2 = 3.409$; P-value $= 0.0648$ (Table: P-value > 0.05); Critical value: $\chi^2 = 3.841$;
   $\text{df} = (2-1)(2-1) = 1$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the form of the 100-yuan gift is independent of whether the money was spent. There is not sufficient evidence to support the claim of a denomination effect. Women in China do not appear to be affected by whether 100 Yuan are in the form of a single bill or several smaller bills.
   $$\chi^2 = \frac{(60-64)^2}{64} + \frac{(15-11)^2}{11} + \frac{(68-64)^2}{64} + \frac{(7-11)^2}{11} = 3.409$$

6. a. $\dfrac{60+68}{150}=\dfrac{128}{150}=\dfrac{64}{75}=0.853$

   b. $\dfrac{60+68}{150}+\dfrac{60+15}{150}-\dfrac{60}{150}=\dfrac{143}{150}=0.953$

   c. $\dfrac{128}{150}\cdot\dfrac{127}{149}=\dfrac{8128}{11{,}175}=0.727,\ \dfrac{128}{150}\cdot\dfrac{128}{150}=0.728$

7. $r=-0.283$; P-value $=0.539$; Critical values $(\alpha=0.05)$: $r=\pm0.754$; There is not sufficient evidence to support the claim of a linear correlation between the repair costs from full-front crashes and full-rear crashes.

8. a. The z score for the bottom 5% is –1.645, which correspond a forward grip reach of $-1.645\cdot34+686$ $=630$ mm.

   b. $z_{x=650}=\dfrac{650-686}{34}=-1.06$; which has a probability of 0.1448, or 14.48% (Table: 14.46%) to the left.
   That percentage is too high, because too many women would not be accommodated.

   c. $z_{x=680}=\dfrac{680-686}{34/\sqrt{16}}=-0.71$; which has a probability of $1-0.2401=0.7599$ (Table: 0.7611) to the right.
   Groups of 16 women do not occupy a driver's seat or cockpit; because individual women occupy the driver's seat/cockpit, this result has no effect on the design.

Chapter 12: Analysis of Variance

Section 12-1: One-Way ANOVA

1. a. The arrival delay times are categorized according to the one characteristic of the flight number.
   b. The terminology of *analysis of variance* refers to the method used to test for equality of the three population means. That method is based on two different estimates of a common population variance.
2. As we increase the number of individual tests of significance, we increase the risk of finding a difference by chance alone (instead of a real difference in the means). The risk of a type I error, finding a difference in one of the pairs when no such difference actually exists, is too high. The method of analysis of variance helps us avoid that particular pitfall (rejecting a true null hypothesis) by using one test for equality of several means, instead of several tests that each compare two means at a time.
3. The test statistic is $F = 1.334$, and the F distribution applies.
4. The P-value is 0.285. Because the P-value is greater than the significance level of 0.05, we fail to reject the null hypothesis of equal means. There is not sufficient evidence to warrant rejection of the claim that Flights 1, Flights 19, and Flights 21 have the same mean arrival delay time. With no apparent significant difference in arrival delay times, it does not appear that a passenger can be helped by selecting one of the flights.
5. Test statistic: $F = 0.39$; P-value: 0.677; Fail to reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is not sufficient evidence to warrant rejection of the claim that the three categories of blood lead level have the same mean verbal IQ score. Exposure to lead does not appear to have an effect on verbal IQ scores.
6. Test statistic: $F = 2.3034$; P-value: 0.1044; Fail to reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is not sufficient evidence to warrant rejection of the claim that the three categories of blood lead level have the same mean full IQ score. There is not sufficient evidence to conclude that exposure to lead has an effect on full IQ scores.
7. Test statistic: $F = 5.5963$; P-value: 0.0045; Reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is sufficient evidence to warrant rejection of the claim that the three samples are from populations with the same mean. It appears that at least one of the mean service times is different from the others.
8. Test statistic: $F = 1.1810$; P-value: 0.3167; Fail to reject H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$. There is not sufficient evidence to support the claim that the different hospitals have different mean birth weights. It appears that birth weights are about the same in urban and rural areas.
9. Test statistic: $F = 7.9338$; P-value: 0.0005; Reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is sufficient evidence to warrant rejection of the claim that females from the three age brackets have the same mean pulse rate. It appears that pulse rates of females are affected by age bracket.
10. Test statistic: $F = 1.304$; P-value: 0.275; Fail to reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is not sufficient evidence to warrant rejection of the claim that males from the three age brackets have the same mean pulse rate. It appears that pulse rates of males are not affected by age bracket.
11. Test statistic: $F = 27.2488$; P-value: 0.000; Reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is sufficient evidence to warrant rejection of the claim that the three different miles have the same mean time. These data suggest that the third mile appears to take longer, and a reasonable explanation is that the third lap has a hill.

    EXCEL
    ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.103444	2	0.051722	27.24878	3.45E-05	3.885294
Within Groups	0.022778	12	0.001898			
Total	0.126222	14				

12. Test statistic: $F = 22.9477$; P-value: 0.000; Reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is sufficient evidence to warrant rejection of the claim that the three different states have the same mean arsenic content in brown rice. Even though Texas has the highest mean, the results from ANOVA do not allow us to conclude that any one specific population mean is different from the others, so we cannot conclude that brown rice from Texas poses the greatest health problem.

EXCEL
ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	31.65167	2	15.82583	22.94775	5.68E-07	3.284918
Within Groups	22.75833	33	0.689646			
Total	54.41	35				

13. Test statistic: $F = 2.3163$; P-value: 0.123; Fail to reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is not sufficient evidence to warrant rejection of the claim that the three different flights have the same mean departure delay time. The departure delay times from Flight 1 have very little variation, and departures of Flight 1 appear to be on time or slightly early. Departure delay times from Flight 21 appear to have considerable variation. With variances of 2.5 min^2, 709.8 min^2, and 2525.4 min^2. the ANOVA requirement of the same variance appears to be violated even for this loose requirement.

EXCEL
ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	5028.074	2	2514.037	2.63723	0.092187	3.402826
Within Groups	22878.89	24	953.287			
Total	27906.96	26				

14. Test statistic: $F = 0.2322$; P-value: 0.794; Fail to reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is not sufficient evidence to warrant rejection of the claim that females in the different age brackets give attribute ratings with the same mean. Age does not appear to be a factor in the female attribute ratings.

EXCEL
ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	23.11667	2	11.55833	0.232246	0.794318	3.354131
Within Groups	1343.725	27	49.76759			
Total	1366.842	29				

15. Test statistic: $F = 28.1666$; P-value: 0.000; Reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is sufficient evidence to warrant rejection of the claim that the three different types of Chips Ahoy cookies have the same mean number of chocolate chips. The reduced fat cookies have a mean of 19.6 chocolate chips, which is slightly more than the mean of 19.1 chocolate chips for the chewy cookies, so the reduced fat does not appear to be the result of including fewer chocolate chips. Perhaps the fat content in the chocolate chips is different and/or the fat content in the cookie material is different.

EXCEL
ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	542.772321	2	271.386161	28.1666001	1.3776E-10	3.07959586
Within Groups	1050.21875	109	9.6350344			
Total	1592.99107	111				

16. Test statistic: $F = 20.8562$; P-value: 0.000; Reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is sufficient evidence to warrant rejection of the claim that the three samples are from populations with the same mean. It appears that cotinine levels are greater with more exposure to tobacco smoke. (The samples do not appear to be from normally distributed populations, but ANOVA is robust against departures from normality. The samples appear to have very different variances, but the largest variance is roughly five times that of the smallest, and given that the sample sizes are all 40, the requirement of equal variances appears to be satisfied.)

EXCEL

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	518033.017	2	259016.508	20.8562144	1.7912E-08	3.0737629
Within Groups	1453040.85	117	12419.1526			
Total	1971073.87	119				

17. The Tukey test results show different P-values, but they are not dramatically different. The Tukey results suggest the same conclusions as the Bonferroni test.

18. a. Test statistic: $F = 6.1413$; P-value: 0.0056; Reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is sufficient evidence to warrant rejection of the claim that the four treatment categories yield poplar trees with the same mean weight.

    b. The displayed Bonferroni results show that with a P-value of 0.039, there is a significant difference between the mean of the no treatment group (group 1) and the mean of the group treated with both fertilizer and irrigation (group 4). (Also, when comparing group 1 and group 2, there is no significant difference between means, and when comparing group 1 and group 3, there is no significant difference between means.)

    c. Test statistic : $t = -4.007$; P-value: $6(0.001018) = 0.00611$; Reject the null hypothesis that the mean weight from the irrigation treatment group is equal to the mean from the group treated with both fertilizer and irrigation.

Section 12-2: Two-Way ANOVA

1. The pulse rates are categorized using two different factors of (1) age bracket and (2) gender.
2. No, to use two individual tests of one-way analysis of variance is to totally ignore the very important feature of the possible effect from an interaction between age bracket and gender. If there is an interaction, it doesn't make sense to consider the effects of one factor without the other.
3. a. An interaction between two factors or variables occurs if the effect of one of the factors changes for different categories of the other factor.

   b. If there is an interaction effect, we should not proceed with individual tests for effects from the row factor and column factor. If there is an interaction, we should not consider the effects of one factor without considering the effects of the other factor.

   c. Because the lines are far from parallel, the two genders have very different effects for the different age brackets, so there does appear to be an interaction between gender and age bracket.
4. Yes, the result is a balanced design because each cell has the same number (10) of values.
5. For interaction, the test statistic is $F = 9.58$ and the P-value is 0.0003, so there is sufficient evidence to warrant rejection of the null hypothesis of no interaction effect. Because there appears to be an interaction between age bracket and gender, we should not proceed with a test for an effect from age bracket and a test for an effect from gender. It appears an interaction between age bracket and gender has an effect on pulse rates. (Remember, these results are based on fabricated data used in one of the cells, so this conclusion does not necessarily correspond to real data.)
6. For interaction, the test statistic is $F = 3.6653$ and the P-value is 0.0322, so there is sufficient evidence to warrant rejection of no interaction effect. Because there appears to be an interaction effect, the tests for effects from age bracket and gender are not conducted. There appears to be sufficient evidence to support a claim that weight appears to be affected by an interaction between age bracket and gender.

7. For interaction, the test statistic is $F = 1.7970$ and the P-value is 0.1756, so there is not sufficient evidence to conclude that there is an interaction effect. For the row variable of age bracket, the test statistic is $F = 2.0403$ and the P-value is 0.1399, so there is not sufficient evidence to conclude that age bracket has an effect on height. For the column variable of gender, the test statistic is $F = 43.4607$ and the P-value is less than 0.0001, so there is sufficient evidence to support the claim that gender has an effect on height.
8. For interaction, the test statistic is $F = 41.38$ and the P-value is 0.000, so there is sufficient evidence to conclude that there is an interaction effect. The ratings appear to be affected by an interaction between the use of a supplement and the amount of whey. Because there appears to be an interaction effect, we should not proceed with individual tests of the row factor (supplement) and the column factor (amount of whey).
9. For interaction, the test statistic is $F = 1.1653$ and the P-value is 0.3289, so there is no significant interaction effect. For gender, the test statistic is 1.6864 and the P-value is 0.2064, so there is no significant effect from gender. For age, the test statistic is $F = 5.0998$ and the P-value is 0.0143, so there is a significant effect from age.

   EXCEL

   ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Rows	15225413	1	15225413	1.686377	0.206419	4.259677
Columns	92086979	2	46043490	5.099807	0.014265	3.402826
Interaction	21042069	2	10521034	1.165317	0.328851	3.402826
Total	3.45E+08	29				

10. For interaction, the test statistic is $F = 0.3328$ and the P-value is 0.5747, so there is not sufficient evidence to warrant rejection of no interaction effect. There does not appear to be an interaction between gender and smoking. For the row variable of gender, the test statistic is $F = 1.3313$ and the P-value is 0.2710, so there is not sufficient evidence to warrant rejection of the claim of no effect from gender. For the column variable of smoking, the test statistic is $F = 1.1186$ and the P-value is 0.3110, so there is not sufficient evidence to conclude that smoking has an effect on body temperatures. It appears that body temperatures are not affected by an interaction between gender and smoking, they are not affected by gender, and they are not affected by smoking.

    ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Sample	0.36	1	0.36	1.331279	0.271041	4.747225
Columns	0.3025	1	0.3025	1.118644	0.311038	4.747225
Interaction	0.09	1	0.09	0.33282	0.574667	4.747225
Total	3.9975	15				

11. a. Test statistics and P-values do not change.
    b. Test statistics and P-values do not change.
    c. Test statistics and P-values do not change.
    d. An outlier can dramatically affect and change test statistics and P-values.

Quick Quiz

1. H_0: $\mu_1 = \mu_2 = \mu_3$; Because the displayed P-value of 0.000 is small, reject H_0. There is sufficient evidence to warrant rejection of the claim that the four samples have the same mean weight.
2. No, it appears that mean weights of Diet Coke and Diet Pepsi are lower than the mean weights of regular Coke and regular Pepsi, but the method of analysis of variance does not justify a conclusion that any particular means are significantly different from the others.
3. right-tailed.
4. Test statistic: $F = 503.06$; Larger test statistics result in smaller P-values.
5. The four samples are categorized using only one factor: the type of cola (regular Coke, Diet Coke, regular Pepsi, Diet Pepsi).

6. One-way analysis of variance is used to test a null hypothesis that three or more samples are from populations with equal means.
7. With one-way analysis of variance, data from the different samples are categorized using only one factor, but with two-way analysis of variance, the sample data are categorized into different cells determined by two different factors.
8. Fail to reject the null hypothesis of no interaction. There does not appear to be an effect due to an interaction between sex and major.
9. There is not sufficient evidence to support a claim that the length estimates are affected by the sex of the subject.
10. There is not sufficient evidence to support a claim that the length estimates are affected by the subject's major.

Review Exercises

1. Test statistic: $F = 2.7347$; P-value: 0.0829; Fail to reject H_0: $\mu_1 = \mu_2 = \mu_3$. There is not sufficient evidence to warrant rejection of the claim that males in the different age brackets give attribute ratings with the same mean. Age does not appear to be a factor in the male attribute ratings.
2. Test statistic: $F = 9.4695$; P-value: 0.0006; Reject H_0: $\mu_1 = \mu_2 = \mu_3$ There is sufficient evidence to warrant rejection of the claim that the three books have the same mean Flesch Reading Ease score. The data suggest that the books appear to have mean scores that are not all the same, so the authors do not appear to have the same level of readability.

EXCEL

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1338.002	2	669.0011	9.469487	0.000562	3.284918
Within Groups	2331.387	33	70.64808			
Total	3669.389	35				

3. For interaction, the test statistic is $F = 1.7171$ and the P-value is 0.1940, so there is not sufficient evidence to warrant rejection of no interaction effect. There does not appear to be an interaction between femur and car size. For the row variable of femur, the test statistic is $F = 1.3896$ and the P-value is 0.2462, so there is not sufficient evidence to conclude that whether the femur is right or left has an effect on load. For the column variable of car size, the test statistic is $F = 2.2296$ and the P-value is 0.1222, so there is not sufficient evidence to warrant rejection of the claim of no effect from car size. It appears that the crash test loads are not affected by an interaction between femur and car size, they are not affected by femur, and they are not affected by car size.
4. For interaction, the test statistic is $F = 0.4784$ and the P-value is 0.7513, so there is not sufficient evidence to conclude that there is an interaction effect. For the row variable of age bracket of females, the test statistic is $F = 0.3149$ and the P-value is 0.7318, so there is not sufficient evidence to conclude that the age bracket of females has an effect on the ratings. For the column variable of age bracket of males, the test statistic is $F = 1.1939$ and the P-value is 0.3148, so there is not sufficient evidence to conclude that the age bracket of males has an effect on the ratings.

EXCEL

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Rows	30.57778	2	15.28889	0.314946	0.731819	3.259446
Columns	115.9111	2	57.95556	1.193866	0.314761	3.259446
Interaction	92.88889	4	23.22222	0.47837	0.751347	2.633532
Total	1986.978	44				

Cumulative Review Exercises

1. a. Flight 3: $\bar{x} = 2.0$ min; Flight 19: $\bar{x} = 9.9$ min; Flight 21: $\bar{x} = 33.4$ min
   b. Flight 3: $s = 10.6$ min; Flight 19: $s = 26.6$ min; Flight 21: $s = 50.3$ min
   c. Flight 3: $s^2 = 112.0\text{ min}^2$; Flight 19: $s^2 = 709.8\text{ min}^2$; Flight 21: $s^2 = 2524.4\text{ min}^2$
   d. The departure delay time of 142 min is an outlier.
   e. ratio
2. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; population$_1$ = Flight 3, population$_2$ = Flight 21;
   Test statistic: $t = -1.728$; P-value = 0.1241 (Table: P-value > 0.10); Critical values ($\alpha = 0.05$): $t = \pm 2.326$ (Table: $t = \pm 2.365$); Fail to reject H_0. There is not sufficient evidence to support the claim that there is a difference between the departure delay times for the two flights.
   $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(2.0 - 33.4) - 0}{\sqrt{\frac{10.6^2}{8} + \frac{50.3^2}{8}}} = -1.728 \text{ (df = 7)}$$
3. Because the pattern of the points is far from a straight-line pattern, the departure delay times for Flight 19 do not appear to be from a population with a normal distribution.
4. The data appear to fit the loose definition of a normally distribution.
   95% CI: $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 2.0 \pm 2.365 \cdot \frac{26.6}{\sqrt{8}} \Rightarrow -6.8 \text{ min} < \mu < 10.8 \text{ min}$; We have 95% confidence that the limits of −6.8 min and 10.8 min contain the value of the population mean for all Flight 3 departure delays.
5. a. H_0: $\mu_1 = \mu_2 = \mu_3$
   b. Because the P-value of 0.1729 is greater than the significance level of 0.05, fail to reject the null hypothesis of equal means. There is not sufficient evidence to warrant rejection of the claim that the three means are equal. The three populations do not appear to have means that are significantly different.
6. a. $z_{x=5.600} = \frac{5.600 - 5.670}{0.062} = -1.13$ and $z_{x=5.700} = \frac{5.700 - 5.670}{0.062} = 0.48$ which have a probability of $0.6844 - 0.1292 = 0.5552$ (Tech: 0.5552) between them.
   b. $z_{x=5.675} = \frac{5.675 - 5.670}{0.062/\sqrt{25}} = 0.40$; which has a probability of $1 - 0.6554 = 0.3446$ (Tech: 0.3434) to the right.
   c. Half the quarters will weigh less than the mean of 5.670 g, so the probability that eight quarters selected randomly all weighing less than 5.670 g is $(1/2)^8 = 1/256$, or about 0.00391.
   d. The z score for the bottom 10% is −1.28, which correspond to a weight of $-1.28 \cdot 0.062 + 5.760 = 5.591$ g.
7. a. $0.20(1000) = 200$
   b. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.20 \pm 1.96\sqrt{\frac{(0.20)(0.80)}{1000}} \Rightarrow 0.175 < p < 0.225$
   c. Yes, the confidence interval shows us that we have 95% confidence that the true population proportion is contained within the limits of 0.175 and 0.225, and $1/4$ is not included within that range.
8. a. Because the vertical scale begins at 15 instead of 0, the graph is deceptive by exaggerating the differences among the frequencies.
   b. No, a normal distribution is approximately bell-shaped, but the given histogram is far from being bell-shaped. Because the digits are supposed to be equally likely, the histogram should be flat with all bars having approximately the same height.
   c. The frequencies are 19, 21, 22, 21, 18, 23, 16, 16, 22, and 22.

8. (continued)

d. H_0: $p_0 = p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = p_9 = 0.10$;

H_1: At least one of the proportions is not equal to the given claimed value.

Test statistic: $\chi^2 = 3.000$; P-value = 0.964 (Table: P-value > 0.95); Critical value ($\alpha = 0.05$):

$\chi^2 = 16.919$; Fail to reject H_0. There is not sufficient evidence to warrant rejection of the claim that the digits are selected from a population in which the digits are all equally likely. There does not appear to be a problem with the lottery.

$$\chi^2 = \frac{(19-20)^2}{0.10 \cdot 200} + \frac{(21-20)^2}{0.10 \cdot 200} + \cdots + \frac{(22-20)^2}{0.10 \cdot 200} + \frac{(22-20)^2}{0.10 \cdot 200} = 3.000 \text{ (df = 9)}$$

Chapter 13: Nonparametric Tests

13-2: Sign Test

1. The only requirement for the matched pairs is that they constitute a simple random sample. There is no requirement of a normal distribution or any other specific distribution. The sign test is "distribution free" in the sense that it does not require a normal distribution or any other specific distribution.
2. There are 2 positive signs, 7 negative signs, 1 tie, $n = 9$, and the test statistic is $x = 2$ (the smaller of 2 and 7).
3. H_0: There is no difference between the populations of September weights and the matching April weights.
   H_1: There is a difference between the populations of September weights and the matching April weights.
   The sample data do not contradict H_1 because the numbers of positive signs (2) and negative signs (7) are not exactly the same.
4. The efficiency of 0.63 indicates that with all other things being equal, the sign test requires 100 sample observations to achieve the same results as 63 sample observations analyzed through a parametric test.
5. The test statistic of $x = 3$ is not less than or equal to the critical value of 1 (from Table A-7). There is not sufficient evidence to warrant rejection of the claim of no difference. There is not sufficient evidence to support the claim that there is a difference between female attribute ratings and male attribute ratings.
6. The test statistic of $x = 4$ is not less than or equal to the critical value of 1 (from Table A-7). There is not sufficient evidence to warrant rejection of the claim of no difference. There is not sufficient evidence to support the claim that there is a difference between female attractiveness ratings and male attractiveness ratings.
7. The test statistic of $z = \frac{(82 + 0.5) - \frac{187}{2}}{\sqrt{187}/2} = -1.61$ results in a P-value of 0.1074, and it does not fall in the critical region bounded by $z = -1.96$ and 1.96. There is not sufficient evidence to warrant rejection of the claim of no difference. There is not sufficient evidence to support the claim that there is a difference between female attribute ratings and male attribute ratings.
8. The test statistic of $z = \frac{(65 + 0.5) - \frac{157}{2}}{\sqrt{157}/2} = -2.08$ results in a P-value of 0.0375, and it falls in the critical region bounded by $z = -1.96$ and 1.96. There is sufficient evidence to warrant rejection of the claim of no difference. There is sufficient evidence to support the claim that there is a difference between female attractiveness ratings and male attractiveness ratings.
9. The test statistic of $z = \frac{(401 + 0.5) - \frac{882}{2}}{\sqrt{882}/2} = -2.66$ results in a P-value of 0.0078, and it is in the critical region bounded by $z = -2.575$ and 2.575. There is sufficient evidence to warrant rejection of the claim that there is no difference between the proportions of those opposed and those in favor.
10. The test statistic of $z = \frac{(372 + 0.5) - \frac{1228}{2}}{\sqrt{1228}/2} = -13.78$ results in a P-value of 0.0000, and it is in the critical region bounded by $z = -2.575$ and 2.575. There is sufficient evidence to support the claim that there is a difference between the rate of medical malpractice lawsuits that go to trial and the rate of such lawsuits that are dropped or dismissed.
11. The test statistic of $z = \frac{(426 + 0.5) - \frac{860}{2}}{\sqrt{860}/2} = -0.24$ results in a P-value of 0.8103, and it is not in the critical region bounded by $z = -1.96$ and 1.96. There is not sufficient evidence to reject the claim that boys and girls are equally likely.

12. The test statistic of $z=\frac{(208+0.5)-\frac{460}{2}}{\sqrt{460}/2}=-2.00$ results in a P-value of 0.0455, and it is in the critical region bounded by $z=-1.96$ and 1.96. There is sufficient evidence to warrant rejection of the claim that that the coin toss is fair in the sense that neither team has an advantage by winning it. The coin toss does not appear to be fair.

13. The test statistic of $z=\frac{(116+0.5)-\frac{598}{2}}{\sqrt{598}/2}=-14.93$ results in a P-value of 0.0000, and it is in the critical region bounded by $z=-2.575$ and 2.575. There is sufficient evidence to warrant rejection of the claim that the median is equal to 2.00.

14. The test statistic of $z=\frac{(193+0.5)-\frac{597}{2}}{\sqrt{597}/2}=-8.59$ results in a P-value of 0.0000, and it is in the critical region bounded by $z=-2.575$ and 2.575. There is sufficient evidence to warrant rejection of the claim that the median earthquake depth is equal to 5.0 km.

15. The test statistic of $z=\frac{(12+0.5)-\frac{40}{2}}{\sqrt{40}/2}=-2.37$ results in a P-value of 0.0178 and it is not in the critical region bounded by $z=-2.575$ and 2.575. There is not sufficient evidence to warrant rejection of the claim that the median is equal to 5.670 g. The quarters appear to be minted according to specifications.

16. The test statistic of $z=\frac{(107+0.5)-\frac{234}{2}}{\sqrt{234}/2}=-1.24$ results in a P-value of 0.2150, and it is not in the critical region bounded by $z=-1.96$ and 1.96. There is not sufficient evidence to warrant rejection of the claim that the median is equal to 90 minutes.

17. Second approach: The test statistic of $z=\frac{(30+0.5)-\frac{105}{2}}{\sqrt{105}/2}=-4.29$ is in the critical region bounded by $z=-1.645$, so the conclusions are the same as in Example 4.

    Third approach: The test statistic of $z=\frac{(38+0.5)-\frac{106}{2}}{\sqrt{106}/2}=-2.82$ is in the critical region bounded by $z=-1.645$, so the conclusions are the same as in Example 4. The different approaches can lead to very different results; as seen in the test statistics of –4.21, –4.29, and –2.82. The conclusions are the same in this case, but they could be different in other cases.

18. The column entries are *, *, *, *, *, 0, 0, 0.

13-3: Wilcoxon Signed-Ranks Test for Matched Pairs

1. a. The only requirements are that the matched pairs be a simple random sample and the population of differences be approximately symmetric.
   b. There is no requirement of a normal distribution or any other specific distribution.
   c. The Wilcoxon signed-ranks test is "distribution free" in the sense that it does not require a normal distribution or any other specific distribution.

2. a. 1.0, –1.2, –0.4, 0, –0.7, –0.8, 0.6
   b. 5, 6, 1, 3, 4, 2
   c. 5, –6, –1, –3, –4, 2
   d. 7, 14
   e. $T=7$
   f. Critical value of T is 1.

3. The sign test uses only the signs of the differences, but the Wilcoxon signed-ranks test uses ranks that are affected by the magnitudes of the differences.

4. The efficiency of 0.95 indicates that with all other things being equal, the Wilcoxon signed-ranks test requires 100 sample observations to achieve the same results as 95 sample observations analyzed through a parametric test.

5. Test statistic: $T = 16.5$; Critical value: $T = 6$; Fail to reject the null hypothesis that the population of differences has a median of 0. There is not sufficient evidence to support the claim that there is a difference between female attribute ratings and male attribute ratings.
6. Test statistic: $T = 19$; Critical value: $T = 6$; Fail to reject the null hypothesis that the population of differences has a median of 0. There is not sufficient evidence to support the claim that there is a difference between female attractiveness ratings and male attractiveness ratings.
7. Convert $T = 8323.5$ to the test statistic $z = -0.63$. P-value: 0.5287; Critical values: $z = \pm 1.96$; There is not sufficient evidence to warrant rejection of the claim of no difference. There is not sufficient evidence to support the claim that there is a difference between female attribute ratings and male attribute ratings.
$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} = \frac{8323.5 - \frac{187(187+1)}{4}}{\sqrt{\frac{187(187+1)(2 \cdot 187+1)}{24}}} = -0.63$$
8. Convert $T = 4628$ to the test statistic $z = -2.76$. P-value: 0.0058. Critical values: $z = \pm 1.96$; There is sufficient evidence to warrant rejection of the claim of no difference. There does appear to be a difference between female attractiveness ratings and male attractiveness ratings.
$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} = \frac{4628 - \frac{157(157+1)}{4}}{\sqrt{\frac{157(157+1)(2 \cdot 157+1)}{24}}} = -2.76$$
9. Convert $T = 18,014$ to the test statistic $z = -16.92$. P-value: 0.0000. Critical values: $z = \pm 2.575$; There is sufficient evidence to warrant rejection of the claim that the median is equal to 2.00.
$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} = \frac{18,014 - \frac{598(598+1)}{4}}{\sqrt{\frac{598(598+1)(2 \cdot 598+1)}{24}}} = -16.92$$
10. Convert $T = 76,235$ to the test statistic $z = -3.09$. P-value: 0.0020. Critical values: $z = \pm 2.575$; There is sufficient evidence to warrant rejection of the claim that the median earthquake depth is equal to 5.0 km.
$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} = \frac{876,235 - \frac{597(597+1)}{4}}{\sqrt{\frac{597(597+1)(2 \cdot 597+1)}{24}}} = -3.09$$
11. Convert $T = 196$ to the test statistic $z = -2.88$. P-value: 0.0040. Critical values: $z = \pm 2.575$; There is sufficient evidence to warrant rejection of the claim that the median is equal to 5.670 g. The quarters do not appear to be minted according to specifications.
$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} = \frac{196 - \frac{40(40+1)}{4}}{\sqrt{\frac{40(40+1)(2 \cdot 40+1)}{24}}} = -1.42$$
12. Convert $T = 12,271.5$ to the test statistic $z = -1.42$. P-value: 0.1556. Critical values: $z = \pm 1.96$; There is not sufficient evidence to warrant rejection of the claim that the times are from a population with a median of 90 minutes.
$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} = \frac{12,271.5 - \frac{234(234+1)}{4}}{\sqrt{\frac{234(234+1)(2 \cdot 234+1)}{24}}} = -1.42$$

13. a. Smallest: $T = 0$; Largest: $T = 1+2+3+\cdots+248+249+250 = \frac{250(250+1)}{2} = 31{,}375$

b. $\frac{31{,}375}{2} = 15687.5$

c. $31{,}375 - 1234 = 30{,}141$

d. $\frac{n(n+1)}{2} - k$

13-4: Wilcoxon Rank-Sum Test for Two Independent Samples

1. Yes, the two samples are independent. The evaluations of female professors and male professors are not matched in any way. The samples are simple random samples. Each sample has more than 10 values.
2. $R = 11.5+4.0+8.5+14.5+8.5+6.0+21.0+4.0+24.0+14.5+1.0+17.5+7.0+21.0 = 163$

Female	3.9	3.4	3.7	4.1	3.7	3.5	4.4	3.4	4.8	4.1	2.3	4.2	3.6	4.4
Rank	11.5	4.0	8.5	14.5	8.5	6.0	21.0	4.0	24.0	14.5	1.0	17.5	7.0	21.0
Male	3.8	3.4	4.9	4.1	3.2	4.2	3.9	4.9	4.7	4.4	4.3	4.1		
Rank	10.0	4.0	25.5	14.5	2.0	17.5	11.5	25.5	23.0	21.0	19.0	14.5		

3. H_0: Evaluations of female professors and male professors have the same median. There are three different possible alternative hypotheses: H_1: Evaluations of female professors and male professors have different medians. H_1: Evaluations of female professors have a median greater than the median of male professor evaluations. H_1: Evaluations of female professors have a median less than the median of male professor evaluations.
4. The efficiency rating of 0.95 indicates that with all other factors being the same, the Wilcoxon rank-sum test requires 100 sample observations to achieve the same results as 95 observations with the parametric t test of Section 9-2, assuming that the stricter requirements of the parametric t-test are satisfied.
5. $R_1 = 163$; $R_2 = 188$; $\mu_R = 189$; $\sigma_R = 19.4422$; Test statistic: $z = -1.34$; P-value $= 0.1802$; Critical values: $z = \pm 1.96$; Fail to reject the null hypothesis that the populations have the same median. There is not sufficient evidence to warrant rejection of the claim that evaluation ratings of female professors have the same median as evaluation ratings of male professors.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{14(14+12+1)}{2} = 189$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{14\cdot 12(14+12+1)}{12}} = 19.4422$$

$$z = \frac{Z-\mu_R}{\sigma_R} = \frac{163-189}{19.4422} = -1.34$$

6. $R_1 = 194.5$; $R_2 = 105.5$; $\mu_R = 150$; $\sigma_R = 17.321$; Test statistic: $z = 2.57$; P-value $= 0.0102$; Critical values: $z = \pm 1.96$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to reject the claim that the median amount of strontium-90 from Pennsylvania residents is the same as the median from New York residents.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{12(12+12+1)}{2} = 150$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{12\cdot 12(12+12+1)}{12}} = 17.321$$

$$z = \frac{Z-\mu_R}{\sigma_R} = \frac{194.5-150}{17.321} = 2.57$$

7. $R_1 = 253.5$; $R_2 = 124.5$; $\mu_R = 182$; $\sigma_R = 20.607$; Test statistic: $z = 3.47$; P-value $= 0.0005$; Critical values: $z = \pm 1.96$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to reject the claim that for those treated with 20 mg of Lipitor and those treated with 80 mg of Lipitor, changes in LDL cholesterol have the same median. It appears that the dosage amount does have an effect on the change in LDL cholesterol.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{13(13+14+1)}{2} = 182$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{13\cdot 14(13+14+1)}{12}} = 20.607$$

$$z = \frac{Z-\mu_R}{\sigma_R} = \frac{253.5-182}{20.607} = 3.47$$

8. $R_1 = 437$; $R_2 = 424$; $\mu_R = 525$; $\sigma_R = 37.4166$; Test statistic: $z = -2.35$; P-value $= 0.0188$; Critical values: $z = \pm 2.575$; Fail to reject the null hypothesis of no difference between the two population medians. There is not sufficient evidence to support the claim that the arrangement of the test items has an effect on the score.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{25(25+16+1)}{2} = 525$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{25\cdot 16(25+16+1)}{12}} = 37.4166$$

$$z = \frac{Z-\mu_R}{\sigma_R} = \frac{437-525}{37.4166} = -2.35$$

9. $R_1 = 1615.5$; $R_2 = 2755.5$; $\mu_R = 1880$; $\sigma_R = 128.8669$; Test statistic: $z = -2.05$; P-value $= 0.0404$; Critical values: $z = \pm 1.96$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to warrant rejection of the claim that evaluation ratings of female professors have the same median as evaluation ratings of male professors.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{40(40+53+1)}{2} = 1880$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{40\cdot 53(40+53+1)}{12}} = 128.8669$$

$$z = \frac{Z-\mu_R}{\sigma_R} = \frac{1615.5-1880}{128.8669} = -2.05$$

10. $R_1 = 1434$; $R_2 = 1726$; $\mu_R = 1480$; $\sigma_R = 101.7841$; Test statistic: $z = -0.45$; P-value $= 0.6527$; Critical values: $z = \pm 2.575$; Fail to reject the null hypothesis that the populations have the same median. There is not sufficient evidence to warrant rejection of the claim that the median of the numbers of words spoken by men in a day is the same as the median of the numbers of words spoken by women in a day.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{37(37+42+1)}{2} = 1480$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{37\cdot 42(37+42+1)}{12}} = 101.784$$

$$z = \frac{Z-\mu_R}{\sigma_R} = \frac{1434-1480}{101.7841} = -0.45$$

11. $R_1 = 501$; $R_2 = 445$; $\mu_R = 484$; $\sigma_R = 41.15823$; Test statistic: $z = 0.41$; P-value $= 0.3409$; Critical value: $z = 1.645$; Fail to reject the null hypothesis that the populations have the same median. There is not sufficient evidence to support the claim that subjects with medium lead levels have a higher median of the full IQ scores than subjects with high lead levels. Based on these data, it does not appear that lead level affects full IQ scores.

11. (continued)

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{22(22+21+1)}{2} = 484$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{22\cdot 21(22+21+1)}{12}} = 41.158$$

$$z = \frac{Z-\mu_R}{\sigma_R} = \frac{501-484}{41.15823} = 0.41$$

12. $R_1 = 4178$; $R_2 = 772$; $\mu_R = 3900$; $\sigma_R = 116.833$; Test statistic: $z = 2.38$; P-value $= 0.0087$; Critical value: $z = 1.645$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to support the claim that subjects with low lead levels have performance IQ scores with a higher median than subjects with high lead levels. It appears that exposure to lead does have an adverse effect.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{78(78+21+1)}{2} = 3900$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{78\cdot 21(78+21+1)}{12}} = 116.833$$

$$z = \frac{Z-\mu_R}{\sigma_R} = \frac{4178-3900}{116.833} = 2.38$$

13. The test statistic is the same value with opposite sign.

$$U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R = 12\cdot 15 + \frac{12(12+1)}{2} - 159.5 = 98.5$$

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}}} = \frac{98.5 - \frac{12\cdot 15}{2}}{\sqrt{\frac{12\cdot 15\cdot(12+15+1)}{12}}} = 0.41$$

14. a.

Rank				Rank Sum for Treatment A
1	2	3	4	
A	A	B	B	3
A	B	A	B	4
A	B	B	A	5
B	B	A	A	7
B	A	A	B	5
B	A	B	A	6

b. The R values of 3, 4, 5, 6, 7 have probabilities of $1/6$, $1/6$, $2/6$, $1/6$, and $1/6$, respectively

c. No, none of the probabilities for the values of R would be less than 0.10.

13-5: Kruskal-Wallis Test for Three or More Samples

1. $R_1 = 164.5;\ R_2 = 150;\ R_3 = 150.5$

Age 20-22	Age 23-26	Age 27-29
38.0 (21.0)	39.0 (22.5)	36.0 (17.0)
42.0 (25.5)	31.0 (7.5)	42.0 (25.5)
30.0 (6.0)	36.0 (17.0)	35.5 (14.0)
39.0 (22.5)	35.0 (13.0)	27.0 (4.0)
47.0 (29.5)	41.0 (24.0)	37.0 (20.0)
43.0 (27.0)	45.0 (28.0)	34.0 (12.0)
33.0 (11.0)	36.0 (17.0)	22.0 (2.0)
31.0 (7.5)	23.0 (3.0)	47.0 (29.5)
32.0 (9.5)	36.0 (17.0)	36.0 (17.0)
28.0 (5.0)	20.0 (1.0)	32.0 (9.5)

(Ranks for each value shown in parentheses.)

2. Yes, the samples are independent simple random samples, and each sample has at least five data values.
3. $n_1 = 10,\ n_2 = 10,\ n_3 = 10$, and $N = 10 + 10 + 10 = 30$
4. The efficiency rating of 0.95 indicates that with all other factors being the same, the Kruskal-Wallis test requires 100 sample observations to achieve the same results as 95 observations with the parametric one-way analysis of variance test, assuming that the stricter requirements of the parametric test are satisfied.
5. Test statistic: $H = 0.1748$; Critical value: $\chi^2 = 5.991$; (Tech: P-value $= 0.916$); Fail to reject the null hypothesis of equal medians. There is not sufficient evidence to warrant rejection of the claim that females from the different age brackets give attribute ratings with the same median.
$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right) - 3(N+1) = \frac{12}{30(30+1)}\left(\frac{164.5^2}{10}+\frac{150^2}{10}+\frac{150.5^2}{10}\right) - 3(30+1)$$
$$= 0.1748$$
6. Test statistic: $H = 22.8157$; Critical value: $\chi^2 = 9.210$; (Tech: P-value $= 0.000$); Reject the null hypothesis of equal medians. It appears that the three states have median amounts of arsenic that are not all the same.
$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right) - 3(N+1) = \frac{12}{36(36+1)}\left(\frac{192^2}{12}+\frac{116.5^2}{12}+\frac{357.5^2}{12}\right) - 3(36+1)$$
$$= 22.8157$$
7. Test statistic: $H = 4.9054$; Critical value: $\chi^2 = 5.991$; (Tech: P-value $= 0.086$); Fail to reject the null hypothesis of equal medians. The data do not suggest that larger cars are safer.
$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right) - 3(N+1) = \frac{12}{21(21+1)}\left(\frac{86^2}{7}+\frac{97^2}{7}+\frac{48^2}{7}\right) - 3(21+1)$$
$$= 4.9054$$
8. Test statistic: $H = 11.8349$; Critical value: $\chi^2 = 5.991$; (Tech: P-value $= 0.003$); Reject the null hypothesis of equal medians. The size of a car does appear to affect highway fuel consumption.
$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right) - 3(N+1) = \frac{12}{21(21+1)}\left(\frac{33.5^2}{7}+\frac{85.5^2}{7}+\frac{112^2}{7}\right) - 3(21+1)$$
$$= 11.8349$$

9. Test statistic: $H = 11.4704$; Critical value: $\chi^2 = 5.991$; (Tech: P-value $= 0.003$); Reject the null hypothesis of equal medians. It appears that the three restaurants have dinner drive-through service times with different medians.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right) - 3(N+1) = \frac{12}{150(150+1)}\left(\frac{4580.5^2}{50}+\frac{3606^2}{50}+\frac{3138.5^2}{50}\right) - 3(150+1)$$
$$= 11.4704$$

10. Test statistic: $H = 59.1546$; Critical value: $\chi^2 = 9.210$; (Tech: P-value $= 0.000$); Reject the null hypothesis of equal medians. The data suggest that the amounts of nicotine absorbed by smokers are different from the amounts absorbed by people who don't smoke.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right) - 3(N+1) = \frac{12}{120(120+1)}\left(\frac{3629.5^2}{40}+\frac{2393.5^2}{40}+\frac{1237^2}{40}\right) - 3(120+1)$$
$$= 59.1546$$

11. Test statistic: $H = 2.5999$; Critical value: $\chi^2 = 7.815$; (Tech: P-value $= 0.458$); Fail to reject the null hypothesis of equal medians. It appears that the four hospitals have birth weights with the same median.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}+\frac{R_4^2}{n4}\right) - 3(N+1)$$
$$= \frac{12}{400(400+1)}\left(\frac{20,189^2}{100}+\frac{21,262.5^2}{100}+\frac{20,106.5^2}{100}+\frac{18,642^2}{100}\right) - 3(400+1) = 2.5999$$

12. Test statistic: $H = 39.7561$; Critical value: $\chi^2 = 9.210$; (Tech: P-value $= 0.000$); Reject the null hypothesis of equal medians. There is sufficient evidence to warrant rejection of the claim that the three different types of Chips Ahoy cookies have the same median number of chocolate chips. It appears that the three different types of cookies have different medians for the numbers of chocolate chips.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right) - 3(N+1) = \frac{12}{112(112+1)}\left(\frac{3291.5^2}{40}+\frac{1250.5^2}{32}+\frac{1786^2}{40}\right) - 3(112+1)$$
$$= 39.7561$$

13. The values of t are 2, 2, 2, 2, and 4. (See table below) The values of T are 6, 6, 6, 6, and 60 and $\Sigma T = 84$. Using $\Sigma T = 84$ and $N = 8+6+5 = 19$, the corrected value of H is $0.694\Big/\left(1-\frac{84}{19^3-19}\right) = 0.703$, which is not substantially different from the value of 0.694 found in Example 1. In this case, the large numbers of ties do not appear to have a dramatic effect on the test statistic H.

Lead Level	Rank	t	$t^3 - t$
85	1.5	2	6
90	4.5	2	6
97	6.5	4	60
100	17.0	2	6
107	28.0	2	6
		SUM	84

13-6: Rank Correlation

1. The methods of Section 10-2 should not be used for predictions. The regression equation is based on a linear correlation between the two variables, but the methods of this section do not require a linear relationship. The methods of this section could suggest that there is a correlation with paired data associated by some nonlinear relationship, so the regression equation would not be a suitable model for making predictions.

2. Data at the nominal level of measurement have no ordering that enables them to be converted to ranks, so data at the nominal level of measurement cannot be used with the methods of rank correlation.
3. r represents the linear correlation coefficient computed from sample paired data; r represents the parameter of the linear correlation coefficient computed from a population of paired data; r_s denotes the rank correlation coefficient computed from sample paired data; ρ_s represents the rank correlation coefficient computed from a population of paired data. The subscript s is used so that the rank correlation coefficient can be distinguished from the linear correlation coefficient r. The subscript does not represent the standard deviation s. It is used in recognition of Charles Spearman, who introduced the rank correlation method.
4. The efficiency rating of 0.91 indicates that with all other factors being the same, rank correlation requires 100 pairs of sample observations to achieve the same results as 91 pairs of observations with the parametric test using linear correlation, assuming that the stricter requirements for using linear correlation are met.
5. $r_s = 1.000$; Critical values are –0.886 and 0.886. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support a claim of a correlation between distance and time.
6. $r_s = -1.000$; Critical values are –0.648 and 0.648. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support a claim of a correlation between altitude and time.
7. $r_s = 0.888$; Critical values: –0.618 and 0.618. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support the claim of a correlation between chocolate consumption and the rate of Nobel Laureates. It does not make sense to think that there is a cause/effect relationship, so the correlation could be the result of a coincidence or other factors that affect the variables the same way.
8. $r_s = -0.644$; Critical values: –0.700 and 0.700. Fail to reject the null hypothesis of $\rho_s = 0$. There is not sufficient evidence to support the claim of a correlation between the ages of Best Actresses and Best Actors.
9. $r_s = 1.000$; Critical values: –0.700 and 0.700. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support the claim of a correlation between the cost of a slice of pizza and the subway fare.
10. $r_s = 1.000$; Critical values: –0.700 and 0.700. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support the claim of a correlation between the CPI (Consumer Price Index) and the subway fare.
11. $r_s = 1.000$; Critical values: -0.886, 0.886. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to conclude that there is a correlation between overhead widths of seals from photographs and the weights of the seals.
12. $r_s = 0.857$; Critical values: –0.738 and 0.738. Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to conclude that there is a correlation between the number of chirps in 1 min and the temperature.
13. $r_s = 0.902$; Critical values: $r_s = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.96}{\sqrt{23-1}} = \pm 0.418$; Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to support the claim of a correlation between chocolate consumption and the rate of Nobel Laureates. It does not make sense to think that there is a cause/effect relationship, so the correlation could be the result of a coincidence or other factors that affect the variables the same way.
14. $r_s = 0.006$; Critical values: $r_s = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.96}{\sqrt{87-1}} = \pm 0.211$; Fail to reject the null hypothesis of $\rho_s = 0$. There is not sufficient evidence to support the claim of a correlation between the ages of Best Actresses and Best Actors.
15. $r_s = 0.360$; Critical values: $r_s = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.96}{\sqrt{153-1}} = \pm 0.159$; Reject the null hypothesis of $\rho_s = 0$. There is sufficient evidence to conclude that there is a correlation between the systolic and diastolic blood pressure levels in males.

16. $r_s = 0.106$; Critical values: $r_s = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.96}{\sqrt{20-1}} = \pm 0.450$; Fail to reject the null hypothesis of $\rho_s = 0$. There is not sufficient evidence to support the claim of a correlation between brain volumes and IQ scores.

17. $r_s = \pm\sqrt{\frac{1.975799^2}{1.975799^2 + 153 - 2}} = \pm\sqrt{\frac{1.978^2}{1.978^2 + 153 - 2}} = \pm 0.159$; (Use either $t = 1.975799$ from technology or use interpolation in Table A-3 with 151 degrees of freedom, so the critical value of t is approximately halfway between 1.984 and 1.972, which is 1.978.) The critical values are the same as those found by using Formula 13-1.

13-7: Runs Test for Randomness

1. No, the runs test can be used to determine whether the sequence of political parties is not random, but the runs test does not show whether the proportion of Republicans is significantly greater than the proportion of Democrats.
2. $n_1 = 17$; $n_2 = 10$; $G = 16$
3. The critical values are 8 and 19. Because $G = 16$ is not less than or equal to 8 nor is $G = 16$ greater than or equal to 18, fail to reject randomness. It appears that the sequence of political parties is random.
4. No, it is very possible that the sequence of data appears to be random, yet the sampling method (such as voluntary response sampling) might be very unsuitable for statistical methods.
5. $\bar{x} = 157.7$ fatalities; $n_1 = 11$; $n_2 = 9$; $G = 12$; critical values: 6, 16; Fail to reject randomness. There is not sufficient evidence to warrant rejection of the claim that there is randomness above and below the mean. There does not appear to be a trend.
6. $n_1 = 14$; $n_2 = 11$; $G = 11$; critical values: 8, 19; Fail to reject randomness. There is not sufficient evidence to warrant rejection of the claim that odd and even digits occur in random order. The statement from the New York Times appears to be accurate.
7. $n_1 = 20$; $n_2 = 10$; $G = 16$; critical values: 9, 20; Fail to reject randomness. There is not sufficient evidence to reject the claim that the dates before and after July 1 are randomly selected.
8. The median is 1480. Discard the sample value of 1480 (which is the median) to get $n_1 = 10$; $n_2 = 10$; $G = 2$; and critical values: 6, 16; Reject randomness. The numbers of daily newspapers do not appear to be in a random sequence. Because all of the values above the median occur in the beginning and all of the values below the median occur at the end, there appears to be a downward trend in the numbers of daily newspapers.
9. $n_1 = 26$; $n_2 = 23$; $G = 20$; $\mu_G = 25.40816$; $\sigma_G = 3.450091$; Test statistic: $z = -1.57$; P-value $= 0.1164$; Critical values: $z = \pm 1.96$; Fail to reject randomness. There is not sufficient evidence to reject randomness. The runs test does not test for disproportionately more occurrences of one of the two categories, so the runs test does not suggest that either conference is superior.

$$\mu_G = \frac{2n_1n_2}{n_1 + n_2} + 1 = \frac{2 \cdot 26 \cdot 23}{26 + 23} + 1 = 25.40816$$

$$\sigma_G = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}} = \sqrt{\frac{2 \cdot 26 \cdot 23(2 \cdot 26 \cdot 23 - 26 - 23)}{(26 + 23)^2(26 + 23 - 1)}} = 3.450091$$

$$z = \frac{G - \mu_G}{\sigma_G} = \frac{20 - 25.40816}{3.450091} = -1.57$$

10. $n_1 = 63$; $n_2 = 47$; $G = 64$, $\mu_G = 54.83636$; $\sigma_G = 5.108473$; Test statistic: $z = 1.79$; P-value = 0.0735; Critical values: $z = \pm 1.96$; Fail to reject randomness. There is not sufficient evidence to reject randomness. American League teams and National League teams appear to win World Series in a random sequence.

$$\mu_G = \frac{2n_1n_2}{n_1+n_2}+1 = \frac{2\cdot 63\cdot 47}{63+47}+1 = 54.83636$$

$$\sigma_G = \sqrt{\frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}} = \sqrt{\frac{2\cdot 63\cdot 47(2\cdot 63\cdot 23-63-47)}{(63+47)^2(63+47-1)}} = 5.108473$$

$$z = \frac{G-\mu_G}{\sigma_G} = \frac{64-54.83636}{5.108473} = 1.79$$

11. The median is 2895.5; $n_1 = 25$; $n_2 = 25$; $G = 2$; $\mu_G = 26$; $\sigma_G = 3.49927$; Test statistic: $z = -6.86$; P-value = 0.0000; Critical values: $z = \pm 1.96$; Reject randomness. The sequence does not appear to be random when considering values above and below the median. There appears to be an upward trend, so the stock market appears to be a profitable investment for the long term.

$$\mu_G = \frac{2n_1n_2}{n_1+n_2}+1 = \frac{2\cdot 25\cdot 25}{25+25}+1 = 26$$

$$\sigma_G = \sqrt{\frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}} = \sqrt{\frac{2\cdot 25\cdot 25(2\cdot 25\cdot 25-25-25)}{(25+25)^2(25+25-1)}} = 3.49927$$

$$z = \frac{G-\mu_G}{\sigma_G} = \frac{2-26}{3.49927} = -6.86$$

12. $n_1 = 56$; $n_2 = 44$; $G = 39$; $\mu_G = 50.28$; $\sigma_G = 4.902317$; Test statistic: $z = -2.30$; P-value = 0.0214; Critical values: $z = \pm 1.96$; Reject randomness. The sequence of genders of newborn babies does not appear to be random.

$$\mu_G = \frac{2n_1n_2}{n_1+n_2}+1 = \frac{2\cdot 56\cdot 44}{56+44}+1 = 50.28$$

$$\sigma_G = \sqrt{\frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}} = \sqrt{\frac{2\cdot 56\cdot 44(2\cdot 56\cdot 44-56-44)}{(56+44)^2(56+44-1)}} = 4.902317$$

$$z = \frac{G-\mu_G}{\sigma_G} = \frac{39-50.28}{4.902317} = -2.30$$

13. a. List of sequences not provided.
    b. The 84 sequences yield these results: 2 sequences have 2 runs, 7 sequences have 3 runs, 20 sequences have 4 runs, 25 sequences have 5 runs, 20 sequences have 6 runs, and 10 sequences have 7 runs.
    c. With $P(2\text{ runs}) = 2/84$, $P(3\text{ runs}) = 7/84$, $P(4\text{ runs}) = 20/84$, $P(5\text{ runs}) = 25/84$, $P(6\text{ runs}) = 20/84$, and $P(7\text{ runs}) = 10/84$, each of the G values of 3, 4, 5, 6, 7 can easily occur by chance, whereas $G = 2$ is unlikely because $P(2\text{ runs})$ is less than 0.025. The lower critical value of G is therefore 2, and there is no upper critical value that can be equaled or exceeded.
    d. The critical value of $G = 2$ agrees with Table A-10. The table lists 8 as the upper critical value, but it is impossible to get 8 runs using the given elements.

Quick Quiz

1. The ranks are 1, 3, 3, 5, 3. The rank for 7 is found using $\frac{2+3+4}{3} = 3$.

2. The efficiency rating of 0.91 indicates that with all other factors being the same, rank correlation requires 100 pairs of sample observations to achieve the same results as 91 pairs of observations with the parametric test for linear correlation, assuming that the stricter requirements for using linear correlation are met.

3. a. distribution-free test
   b. The term "distribution-free test" suggests correctly that the test does not require that a population must have a particular distribution, such as a normal distribution. The term "nonparametric test" incorrectly suggests that the test is not based on a parameter, but some nonparametric tests are based on the median, which is a parameter; the term "distribution-free test" is better because it does not make that incorrect suggestion.
4. Rank correlation should be used. The rank correlation test is used to investigate whether there is a correlation between foot length and height.
5. No, the P-values are almost always different, and the conclusions may or may not be the same.
6. Rank correlation can be used in a wider variety of circumstances than linear correlation. Rank correlation does not require a normal distribution for any population. Rank correlation can be used to detect some (not all) relationships that are not linear. 7. Because there are only two runs, all of the values below the mean occur at the beginning and all of the values above the mean occur at the end, or vice versa. This indicates the presence of an upward (or downward) trend.
7. Because there are only two runs, all of the values below the mean occur at the beginning and all of the values above the mean occur at the end, or vice versa. This indicates the presence of an upward (or downward) trend.
8. a. false
   b. false
9. Because the sign test uses only signs of differences while the Wilcoxon signed-ranks test uses ranks of the differences, the Wilcoxon signed-ranks test uses more information about the data and tends to yield conclusions that better reflect the true nature of the data.
10. Kruskal-Wallis test

Review Exercises

1. $r_s = 0.400$; The critical values are -0.700 and 0.700. Fail to reject the null hypothesis of $\rho_s = 0$. There is not sufficient evidence to support the claim of a correlation between job stress and annual income. Based on the given data, it does not appear that jobs with more stress have higher salaries.
2. Test statistic: $H = 2.5288$; (Tech: P-value $= 0.2824$); Critical value: $\chi^2 = 5.991$; Fail to reject the null hypothesis of equal medians. It appears that times of longevity after inauguration for presidents, popes, and British monarchs have the same median.
$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right)-3(N+1) = \frac{12}{76(76+1)}\left(\frac{1485.5^2}{38}+\frac{806.5^2}{24}+\frac{635^2}{14}\right)-3(76+1)$$
$$= 2.5289$$
3. The test statistic of $z = \dfrac{(47+0.5)-\frac{110}{2}}{\sqrt{110}/2} = -1.43$ results in a P-value of 0.1527 and it is not less than or equal to the critical value of $z = -1.96$. Fail to reject the null hypothesis of $p = 0.5$. There is not sufficient evidence to warrant rejection of the claim that in each World Series, the American League team has a 0.5 probability of winning.
4. $n_1 = 16$; $n_2 = 14$; $G = 11$; critical values: 10, 22; Fail to reject randomness. There is not sufficient evidence to warrant rejection of the claim that odd and even digits occur in random order. The lottery appears to be working as it should.
5. The test statistic of $x = 3$ is less than or equal to the critical value of 5 (from Table A-7). There is sufficient evidence to warrant rejection of the claim that the sample is from a population with a median equal to 5.
6. The test statistic $T = 21$ is less than or equal to the critical value of 59. There is sufficient evidence to warrant rejection of the claim that the sample is from a population with a median equal to 5.

7. $R_1 = 204.5$; $R_2 = 230.5$; $\mu_R = 255$; $\sigma_R = 22.58318$; Test statistic: $z = -2.24$; (Tech: P-value = 0.025); Critical values: $z = \pm 1.96$; Reject the null hypothesis that the populations have the same median. There is sufficient evidence to warrant rejection of the claim that the recent eruptions and past eruptions have the same median time interval between eruptions. The conclusion does change with a 0.01 significance level.

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{17(17+12+1)}{2} = 255$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{17\cdot 12(17+12+1)}{12}} = 22.58318$$

$$z = \frac{Z-\mu_R}{\sigma_R} = \frac{204.5-255}{22.58318} = -2.24$$

8. The test statistic of $x = 0$ is less than or equal to the critical value of 0. There is sufficient evidence to reject the claim of no difference. It appears that there is a difference in cost between flights scheduled 1 day in advance and those scheduled 30 days in advance. Because all of the flights scheduled 30 days in advance cost less than those scheduled 1 day in advance, it appears to be wise to schedule flights 30 days in advance.
9. The test statistic of $T = 0$ is less than or equal to the critical value of 4. There is sufficient evidence to reject the claim that differences between fares for flights scheduled 1 day in advance and those scheduled 30 days in advance have a median equal to 0. Because all of the flights scheduled 30 days in advance cost less than those scheduled 1 day in advance, it appears to be wise to schedule flights 30 days in advance.
10. $r_s = 0.714$. Critical values: $r_s = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.96}{\sqrt{8-1}} = \pm 0.741$; Fail to reject the null hypothesis of $\rho_s = 0$.

    There is not sufficient evidence to support the claim that there is a correlation between the student ranks and the magazine ranks. When ranking colleges, students and the magazine do not appear to agree.

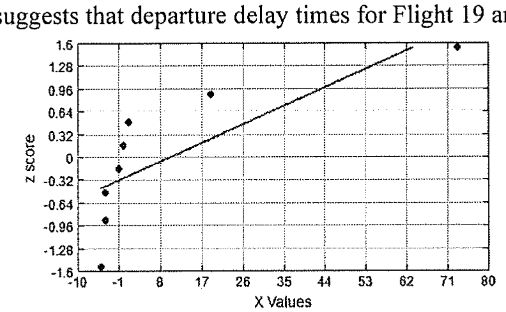
Cumulative Review Exercises

1. Flight 1: $\bar{x} = -1.3$ min; $Q_2 = -2.0$ min; $s = 1.6$ min

   Flight 19: $\bar{x} = 9.9$ min; $Q_2 = -0.5$ min; $s = 26.6$ min

   Flight 21: $\bar{x} = 33.4$ min; $Q_2 = 15.5$ min; $s = 50.3$ min

   The means appear to be very different, with Flight 21 having the longest departure delay times. The medians appear to be very different, with Flight 21 having the longest departure delay times. The standard deviations appear to be very different, with Flight 21 having the greatest amount of variation. Flight 21 appears to be the least predictable flight because it has the highest variation, and it appears to have the longest departure delay times.
2. The normal quantile plot suggests that departure delay times for Flight 19 are not normally distributed.



3. Kruskal-Wallis test statistic: $H = 3.2600$; Tech: P-value = 0.1959; Critical value ($\alpha = 0.05$): $\chi^2 = 5.991$; Fail to reject the null hypothesis of equal medians. There is not sufficient evidence to warrant rejection of the claim that the three samples are from populations with the same median departure delay time.

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}\right) - 3(N+1) = \frac{12}{24(24+1)}\left(\frac{78^2}{8}+\frac{94^2}{8}+\frac{128^2}{8}\right) - 3(24+1)$$
$$= 3.2600$$

4. 95% CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.039 \pm 1.96\sqrt{\frac{(0.039)(0.961)}{2000}} \Rightarrow 0.031 < p < 0.047$, or $3.1\% < p < 4.7\%$; We have 95% confidence that the limits of 3.1% and 4.7% actually contain the true percentage of the population of workers who test positive for drugs.

5. $H_0\colon p = 0.03$; $H_1\colon p > 0.03$; Test statistic: $z = \frac{0.039 - 0.03}{\sqrt{\frac{(0.03)(0.97)}{2000}}} = 2.36$;

   P-value $= P(z > 2.36) = 0.0091$ (Tech: 0.0092); Critical value: $z = 1.645$;

   Reject H_0. There is sufficient evidence to support the claim that the rate of positive drug test results among workers in the United States is greater than 3.0%.

6. The sample mean is 54.8 years. $n_1 = 19$; $n_2 = 19$; The number of runs is $G = 18$. The critical values are 13 and 27. Fail to reject the null hypothesis of randomness. There is not sufficient evidence to warrant rejection of the claim that the sequence of ages is random relative to values above and below the mean. The results do not suggest that there is an upward trend or a downward trend.

7. $n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2} = \frac{[1.96]^2 \cdot 0.25}{0.02^2} = 2401$

8. There is a relatively small number of players with salaries that are substantially large, so the mean is strongly affected by those values, resulting in a large value of the mean, but the median is not affected by the small number of very large salaries.

9. $H_0\colon p = 0.5$; $H_1\colon p > 0.5$; Test statistic: $z = \frac{0.54 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{285}}} = 1.36$;

   P-value $= P(z > 1.36) = 0.0869$ (Tech: 0.0865); Critical value: $z = 1.645$;

   Fail to reject H_0. There is not sufficient evidence to support the claim that the majority of the population is not afraid of heights in tall buildings. Because respondents themselves chose to reply, the sample is a voluntary response sample, not a random sample, so the results might not be valid.

10. There must be an error, because the rates of 13.7% and 10.6% are not possible with samples of size 100.

Chapter 14: Statistical Process Control

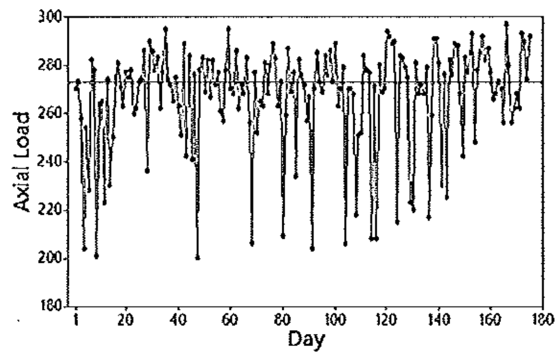
Section 14-1: Control Charts for Variation and Mean

1. No, if we know that the process is within statistical control, we know that none of the three out-of-control criteria are satisfied, but we know nothing about whether any specifications or requirements are satisfied. It is possible to be within statistical control while manufacturing altimeters with errors that are too large to satisfy the FAA requirements.
2. An $\bar{x}$ control chart is a plot of sample means and it includes a centerline as well as a line representing an upper control limit and a line representing a lower control limit. An R control chart is a plot of sample ranges and it also includes a centerline as well as a line representing an upper control limit and a line representing a lower control limit. $\bar{\bar{x}}$ denotes the mean of all of the 20 sample means, $\bar{R}$ denotes the mean of the 20 ranges, UCL denotes the value used to locate the upper control limit in a control chart, and LCL denotes the value used to locate the lower control limit in a control chart.
3. The mean is out of statistical control. The elevations have decreased substantially in recent years, so Lake Mead is becoming shallower. The decreases have been significant (and they are having a dramatic impact on the affected populations).
4. The variations in Lake Mead elevations have become more stable in recent years, so the elevations do not vary as much as they once did, but the variation is out of statistical control.
5. $\bar{\bar{x}} = 267.11$ lb; $\bar{R} = 54.96$ lb, $n = 7$

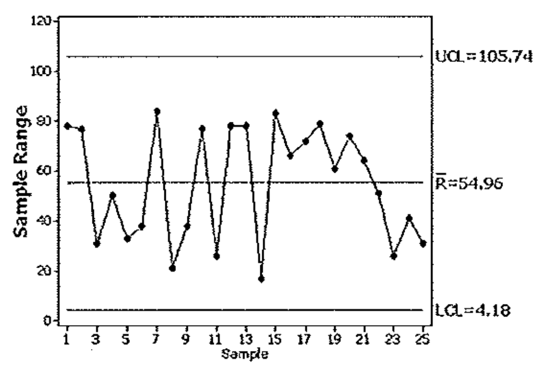
   For the R chart: $\text{LCL} = D_3\bar{R} = 0.076 \cdot 54.96 = 4.18$ lb and $\text{UCL} = D_4\bar{R} = 1.924 \cdot 54.96 = 105.74$ lb.

   For the $\bar{x}$ chart: $\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 267.11 - 0.419 \cdot 54.96 = 244.08$ lb and

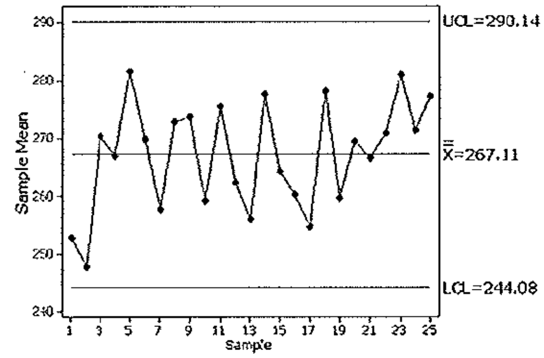
   $\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 267.11 + 0.419 \cdot 54.96 = 290.14$ lb.
6. The run chart does not reveal any patterns that would indicate manufacturing problems needing correction.



7. The R chart does not meet any of the three out-of-control criteria, so the variation of the process appears to be within statistical control.



8. The $\bar{x}$ chart does not meet any of the out-of-control criteria, so the mean of the process appears to be within statistical control.



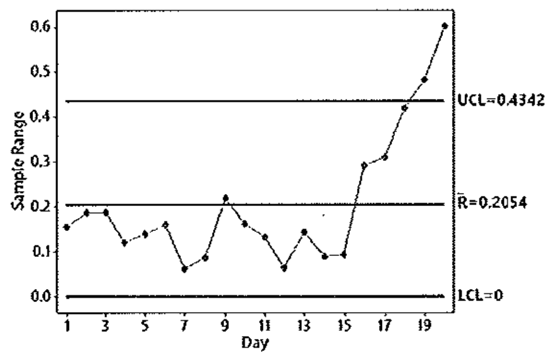
9. $\bar{\bar{x}} = 5.6955$ g; $\bar{R} = 0.2054$ g; $n = 5$

   For R chart: $\text{LCL} = D_3\bar{R} = 0.000 \cdot 0.2054 = 0.0000$ g and $\text{UCL} = D_4\bar{R} = 2.114 \cdot 0.2054 = 0.4342$ g

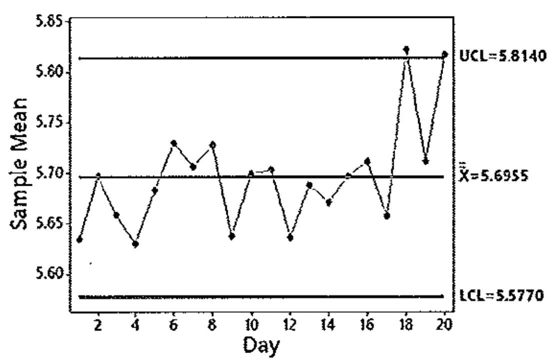
   For $\bar{x}$ chart: $\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 5.6955 - 0.577 \cdot 0.2054 = 5.5770$ g and

   $\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 5.6955 + 0.577 \cdot 0.2054 = 5.8140$ g

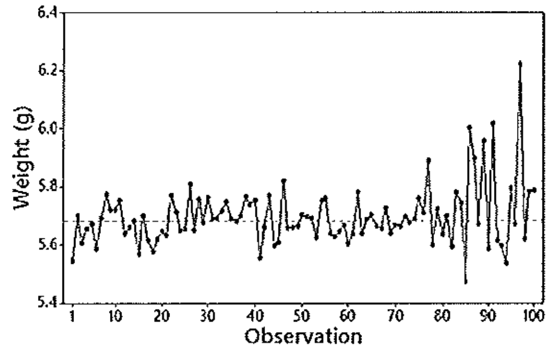
10. The R chart has eight consecutive points below the centerline, there appears to be an upward trend, and there are points that lie beyond the upper control limit, so the variation of the process appears to be out of statistical control.



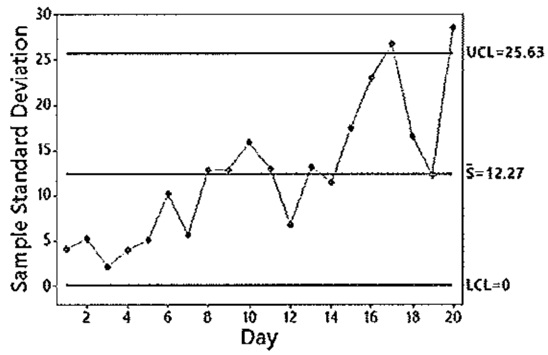
11. There are points lying beyond the upper control limit, so the process mean appears to be out of statistical control.



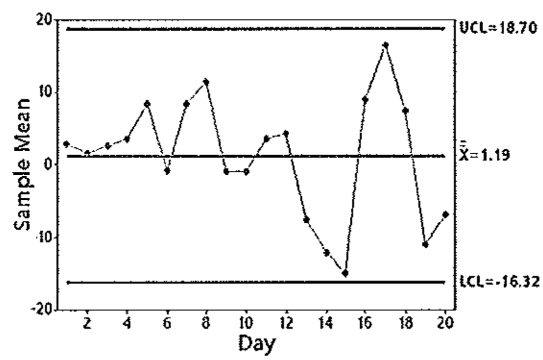
12. There is a pattern of increasing variation, which is quite bad. The process should be fixed.



13. $\bar{s}=12.27$ g; $n=5$; LCL $=B_3\bar{s}=0\cdot 12.27=0$ g; UCL $=B_4\bar{s}=2.089\cdot 12.27=25.63$ g; Except for the values on the vertical scale, the s chart is nearly identical to the R chart shown in Example 3.



14. Except for the values used for the vertical scale, the two $\bar{x}$ charts are nearly identical.
LCL $=\bar{\bar{x}}-A_3\bar{s}=5.6955-1.427\cdot 12.27=-16.32$ g and UCL $=\bar{\bar{x}}+A_3\bar{s}=1.19+1.427\cdot 12.27=18.70$ g



Section 14-2: Control Charts for Attributes

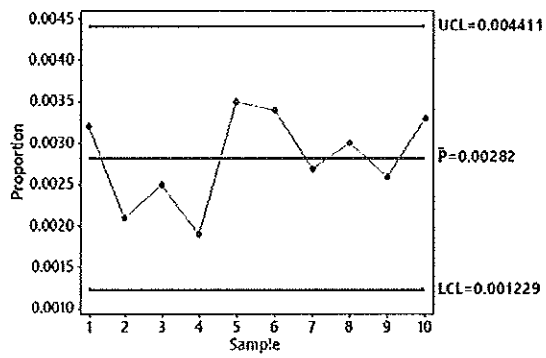
1. No, the process appears to be out of statistical control. There is a downward trend and there are at least eight consecutive points all lying above the centerline. Because the proportions of defects are decreasing, the manufacturing process is not deteriorating; it is improving.
2. $\bar{p}$ is the pooled estimate of the proportion of defective items. It is obtained by finding the total number of defects in all samples combined and dividing that total by the total number of items sampled. UCL is the upper control limit, and LCL is the lower control limit.
3. Because the value of -0.00325 is negative and the actual proportion of defects cannot be less than 0, we should replace that value with 0.

4. No, it is very possible that the proportions of defects in repeated samplings behave in a way that makes the process appear to be within statistical control, but the actual proportions of defects could be very high (such as 90%), so that almost all of the euro coins fail to meet the manufacturing specifications. Upper and lower control limits of a control chart for the proportion of defects are based on the actual behavior of the process, not the desired behavior according to specifications.
5. The process appears to be within statistical control. (Considering a shift up, note that the first and last points are about the same.)

$$\bar{p} = \frac{32+21+25+19+35+34+27+30+26+33}{10{,}000 \cdot 10} = 0.00282; \ \bar{q} = 1 - 0.00282 = 0.99718$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.00282 - 3\sqrt{\frac{(0.00282)(0.99718)}{10{,}000}} = 0.001229$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.00282 + 3\sqrt{\frac{(0.00282)(0.99718)}{10{,}000}} = 0.004411$$



6. The process appears to be within statistical control. The control chart is the same as the control chart from Exercise 5, except for the values identifying the centerline and control limits ($\text{UCL} = 0.4170$, centerline is at $\bar{p} = 0.282$, and $\text{LCL} = 0.1470$). In this exercise, the proportions of defects are very high. Even though the process is within statistical control, this manufacturing process is yielding far too many defective euro coins, so corrective action should be taken to lower the defect rate.

$$\bar{p} = \frac{32+21+25+19+35+34+27+30+26+33}{100 \cdot 10} = 0.282; \ \bar{q} = 1 - 0.282 = 0.718$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.282 - 3\sqrt{\frac{(0.282)(0.718)}{100}} = 0.1470$$

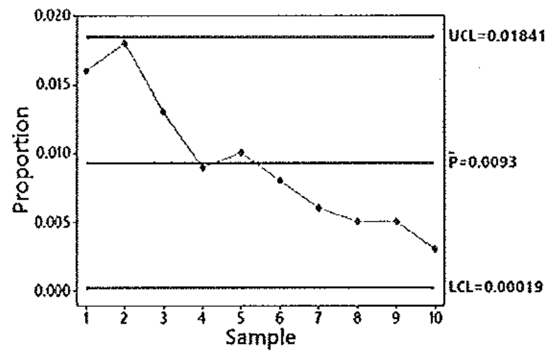
$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.282 + 3\sqrt{\frac{(0.282)(0.718)}{100}} = 0.4170$$

7. The process appears to be out of statistical control because of a downward trend, but the number of defects appears to be decreasing, so the process is improving. Causes for the declining number of defects should be identified so that they can be continued.

$$\overline{p}=\frac{16+18+13+9+10+8+6+5+5+3}{1000\cdot 10}=0.0093;\ \overline{q}=1-0.0093=0.9907$$

$$\text{LCL}=\overline{p}-3\sqrt{\frac{\overline{pq}}{n}}=0.0093-3\sqrt{\frac{(0.0093)(0.9907)}{1000}}=0.00019$$

$$\text{UCL}=\overline{p}+3\sqrt{\frac{\overline{pq}}{n}}=0.0093+3\sqrt{\frac{(0.0093)(0.9907)}{1000}}=0.01841$$



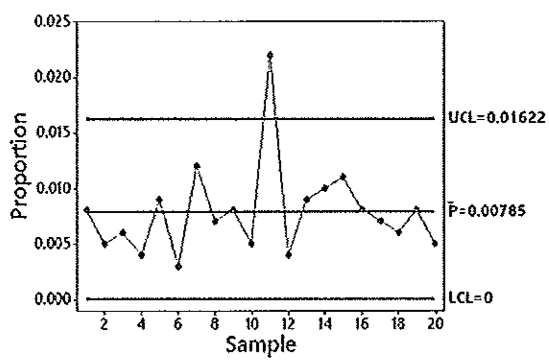
8. There is a point that lies beyond the upper control limit, so the process is out of statistical control. The cause of the high number of defects in the 11th sample should be identified and corrected.

$$\overline{p}=\frac{8+5+6+4+9+3+12+7+8+5+22+4+9+10+11+8+7+6+8+5}{1000\cdot 20}=0.00785$$

$\overline{q}=1-0.00785=0.99215$

$$\text{LCL}=\overline{p}-3\sqrt{\frac{\overline{pq}}{n}}=0.00785-3\sqrt{\frac{(0.00785)(0.99215)}{1000}}=-0.000522,\text{ so LCL}=0$$

$$\text{UCL}=\overline{p}+3\sqrt{\frac{\overline{pq}}{n}}=0.00785+3\sqrt{\frac{(0.00785)(0.99215)}{1000}}=0.01622$$



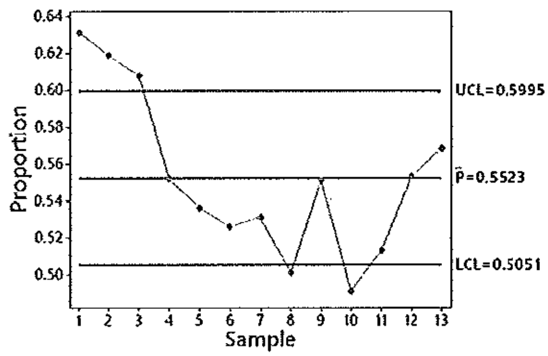
9. The process is out of statistical control because there are points lying beyond the upper control limit and there are points lying beyond the lower control limit. Also, there are eight consecutive points all lying below the centerline. The percentage of voters is increasing in recent presidential elections, and it should be much higher than any of the rates shown.

$$\bar{p} = \frac{631+619+608+552+536+526+531+501+551+491+513+553+568}{1000 \cdot 13} = 0.5523$$

$$\bar{q} = 1 - 0.5523 = 0.4477$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.5523 - 3\sqrt{\frac{(0.5523)(0.4477)}{1000}} = 0.5051$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.5523 + 3\sqrt{\frac{(0.5523)(0.4477)}{1000}} = 0.5995$$

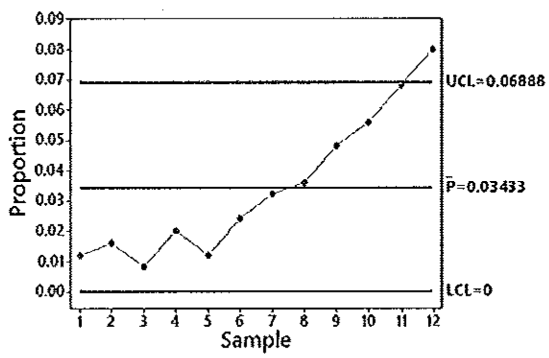


10. The process appears to be out of statistical control. There is a point lying beyond the upper control limit, and there is a pattern of increasing proportions of defects. The manufacturing process requires corrective action.

$$\bar{p} = \frac{3+4+2+5+3+6+8+9+12+14+17+20}{250 \cdot 12} = 0.03433; \bar{q} = 1 - 0.03433 = 0.96557$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.03433 - 3\sqrt{\frac{(0.03433)(0.96557)}{250}} = -0.000216, \text{ so LCL} = 0$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.03433 + 3\sqrt{\frac{(0.03433)(0.96557)}{250}} = 0.06888$$



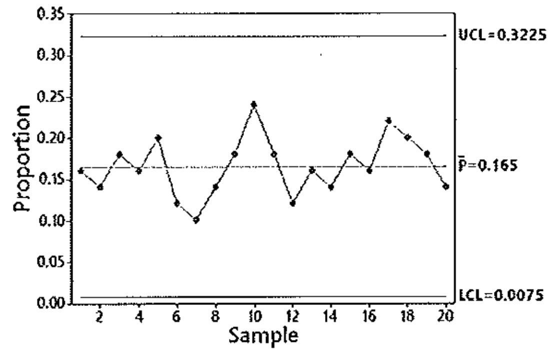
11. Although the process is within statistical control, the proportions of defects are substantially high, so immediate corrective action should be taken to substantially lower the proportions of defects.

$$\bar{p} = \frac{8+7+9+8+10+6+5+7+9+12+9+6+8+7+9+8+11+10+9+7}{50 \cdot 20} = 0.165$$

$$\bar{q} = 1 - 0.165 = 0.835$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.165 - 3\sqrt{\frac{(0.165)(0.835)}{50}} = 0.0075$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.165 + 3\sqrt{\frac{(0.165)(0.835)}{50}} = 0.3225$$

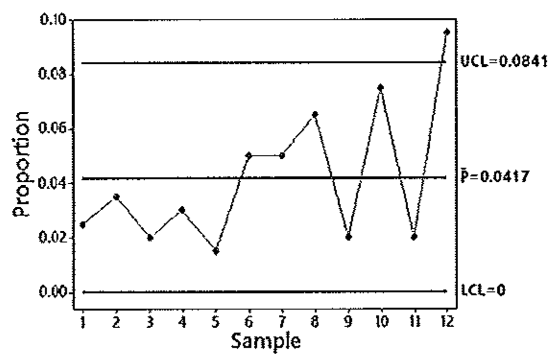


12. There appears to be a pattern of increasing variation and there is a point lying beyond the upper control limit, so the process is out of statistical control. The cause of the increasing variation should be identified and corrected.

$$\bar{p} = \frac{5+7+4+6+3+10+10+13+4+15+4+19}{200 \cdot 12} = 0.0417;\ \bar{q} = 1 - 0.0417 = 0.9583$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.0417 - 3\sqrt{\frac{(0.0417)(0.9583)}{200}} = -0.000705, \text{ so LCL} = 0$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.0417 + 3\sqrt{\frac{(0.0417)(0.9583)}{200}} = 0.0841$$

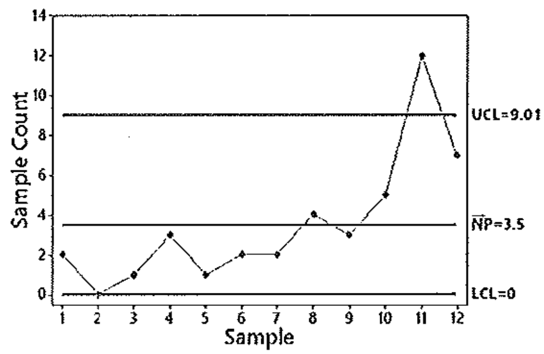


13. Except for a different vertical scale, the basic control chart is identical to the one given for Example 1.

$$\bar{p} = \frac{2+0+1+3+1+2+2+4+3+5+12+7}{100 \cdot 12} = 0.035; \bar{q} = 1 - 0.035 = 0.965; n\bar{p} = 100(0.035) = 3.5$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}\bar{q}} = 3.5 - 3\sqrt{100(0.035)(0.965)} = -2.013, \text{ so LCL} = 0$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}\bar{q}} = 3.5 - 3\sqrt{100(0.035)(0.965)} = 9.01$$



Quick Quiz

1. Process data are data arranged according to some time sequence. They are measurements of a characteristic of goods or services that result from some combination of equipment, people, materials, methods, and conditions.
2. Random variation is due to chance, but assignable variation results from causes that can be identified, such as defective machinery or untrained employees.
3. There is a pattern, trend, or cycle that is obviously not random. There is a point lying outside the region between the upper and lower control limits. There are at least eight consecutive points all above or all below the centerline.
4. An R chart uses ranges to monitor variation, but an $\bar{x}$ chart uses sample means to monitor the center (mean) of a process.
5. No, the R chart has at least eight consecutive points all lying below the centerline, there are at least eight consecutive points all lying above the centerline, there are points lying beyond the upper and lower control limits, and there is a pattern showing that the ranges have jumped in value for the more recent samples. What a mess!
6. $\bar{R} = 67.0$ ft; In general, a value of $\bar{R}$ is found by first finding the range for the values within each individual subgroup; the mean of those ranges is the value of $\bar{R}$.
7. No, the $\bar{x}$ chart has a point lying beyond the upper control limit, and there are at least eight consecutive points lying below the centerline.
8. $\bar{\bar{x}} = -2.24$ ft; In general, a value of $\bar{\bar{x}}$ is found by first finding the mean of the values within each individual subgroup; the mean of those subgroup means is the value of $\bar{\bar{x}}$.
9. No, the control charts can be used to determine whether the mean and variation are within statistical control, but they do not reveal anything about specifications or requirements.
10. Because there is a downward trend, the process is out of statistical control, but the rate of defects is decreasing, so we should investigate and identify the cause of that trend so that it can be continued.

Review Exercises

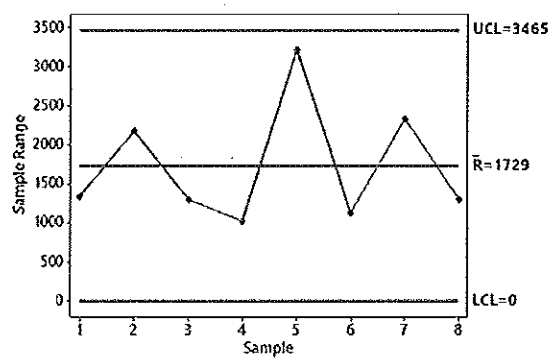
1. $\bar{\bar{x}} = 3157$ kWh; $\bar{R} = 1729$ kWh; $n = 6$

   For R chart: $\text{LCL} = D_3\bar{R} = 0.000 \cdot 1729 = 0$ kWh and $\text{UCL} = D_4\bar{R} = 2.004 \cdot 1729 = 3465$ kWh

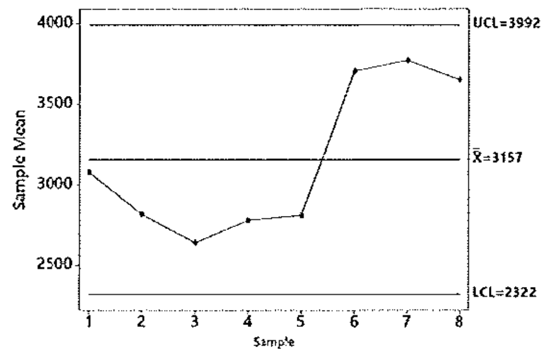
   For $\bar{x}$ chart: $\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 3157 - 0.483 \cdot 1729 = 2322$ kWh and

   $\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 3157 + 0.483 \cdot 1729 = 3992$ kWh

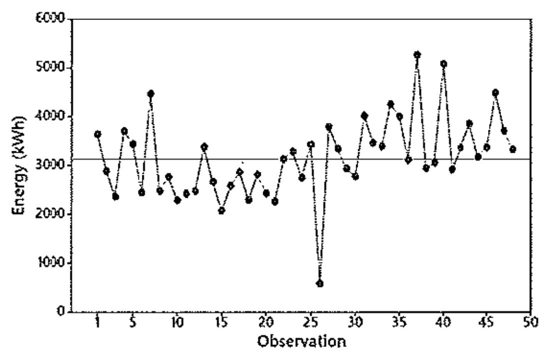
2. The process variation is within statistical control.



3. There appears to be a shift up in the mean values, so the process mean is out of statistical control.



4. There appears to be a slight upward trend. There is 1 point that appears to be exceptionally low. (The author's power company made an error in recording and reporting the energy consumption for that time period.)

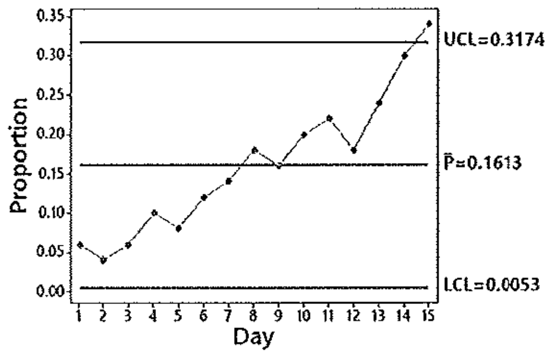


5. Because there is a distinct upward trend and there is a point beyond the upper control limit, the process is out of statistical control. Because the order times are clearly increasing, immediate corrective action should be taken.

$$\bar{p}=\frac{3+2+3+5+4+6+7+9+8+10+11+9+12+15+17}{50\cdot 15}=0.16133; \bar{q}=1-0.16133=0.83867$$

$$\text{LCL}=\bar{p}-3\sqrt{\frac{\bar{p}\bar{q}}{n}}=0.16133-3\sqrt{\frac{(0.16133)(0.83867)}{50}}=0.0053$$

$$\text{UCL}=\bar{p}+3\sqrt{\frac{\bar{p}\bar{q}}{n}}=0.16133+3\sqrt{\frac{(0.16133)(0.83867)}{50}}=0.3174$$



Cumulative Review Exercises

1. 95% CI: $\hat{p}\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}=\frac{1108}{2015}\pm 1.96\sqrt{\frac{\left(\frac{1108}{2015}\right)\left(\frac{907}{2015}\right)}{2015}}\Rightarrow 0.528<p<0.573$; Because the entire range of values in the confidence interval includes values that are all greater than 0.5, it does appear that the majority of adults learn about medical symptoms more often from the Internet than from their doctor.

2. $H_0: p=0.5$; $H_1: p>0.5$; Test statistic: $z=\frac{\frac{1108}{2015}-0.5}{\sqrt{\frac{(0.5)(0.5)}{2015}}}=4.48$;

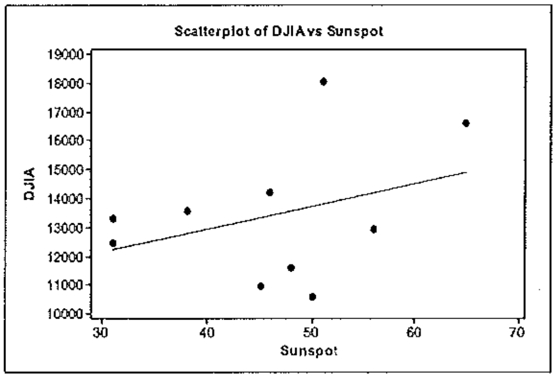
   P-value $=P(z>4.48)=0.0001$ (Tech: 0.0000); Critical value: $z=1.645$; Reject H_0. There is sufficient evidence to support the claim that the majority of adults learn about medical symptoms more often from the Internet than from their doctor.

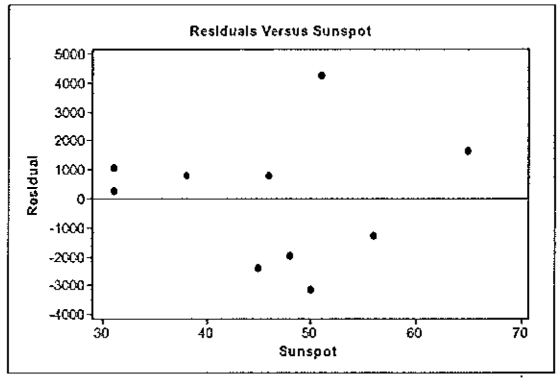
3. The graph is misleading. The vertical scale begins with a frequency of 800 instead of 0, so the difference between the "yes" and "no" responses is greatly exaggerated.

4. a. $(0.55)^3=0.166$ b. $1-(0.45)^3=0.909$

5. $H_0: \rho=0$; $H_1: \rho\neq 0$; $r=0.0.356$; P-value $=0.313$ (Table: P-value >0.05); Critical values: $r\approx\pm 0.632$; Fail to reject H_0. There is not sufficient evidence to support a claim of a linear correlation between the DJIA and sunspot numbers. Because we do not expect any relationship between sunspot numbers and the behavior of stocks, this result is not surprising.

6. $\hat{y} = 9772 + 79.2x$; With no significant linear correlation, the best predicted value of the DJIA in the year 2004 is $\bar{y} = 13,423.6$, and that value is not close to the actual 2004 value of 10,855.





7. a. $z_{x=60} = \dfrac{60.0-68.6}{2.8} = -3.07$ and $z_{x=80} = \dfrac{80.0-68.6}{2.8} = 4.07$, which have a probability of $0.9999 - 0.0001 = 0.9998$, or 99.98% (Tech: 99.89%) between them.

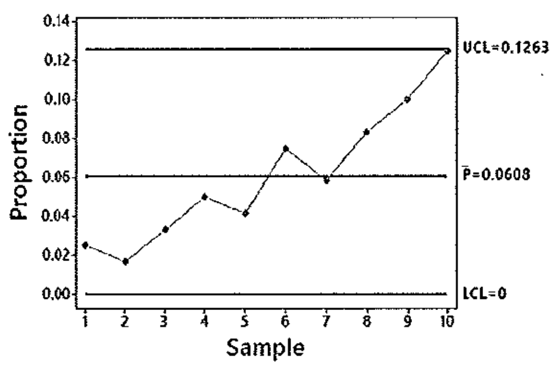
   b. $z_{x=70} = \dfrac{70.0-68.6}{2.8/\sqrt{4}} = 1.00$; which has a probability of $1 - 0.8413 = 0.1587$ to the right.

8. There is a pattern of an upward trend, so the process is out of statistical control.

$$\bar{p} = \frac{3+2+4+6+5+9+7+10+12+15}{1200} = 0.060833; \bar{q} = 1 - 0.0608 = 0.93917$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.06083 - 3\sqrt{\frac{(0.06083)(0.93917)}{120}} = -0.0046, \text{ so LCL} = 0$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}} = 0.06083 + 3\sqrt{\frac{(0.06083)(0.93917)}{120}} = 0.1263$$



9. $\bar{x} = 7.3$; $Q_2 = 6.5$; $s = 4.2$; These statistics do not convey information about the changing pattern of the data over time.

10. H_0: Whether sentence is independent of plea.

    H_1: Whether sentence depends on plea.

    Test statistic: $\chi^2 = 42.557$; P-value $= 0.000$ (Table: P-value < 0.005); Critical value: $\chi^2 = 3.841$; $\text{df} = (2-1)(2-1) = 1$; Reject H_0. There is sufficient evidence to warrant rejection of the claim that the sentence is independent of the plea. The results encourage pleas for guilty defendants.

$$\chi^2 = \frac{(392-418.48)^2}{418.48} + \frac{(58-31.52)^2}{31.52} + \frac{(564-537.52)^2}{537.52} + \frac{(14-40.48)^2}{40.48} = 42.557$$