Practice Test #4 **STA 2023** **Name ________________**

Section ________________

1. We want to estimate the mean energy consumption level for a home in one region. We want to be 90% confident that our sample mean is within 25 kwh of the true population mean, and past data strongly suggest that the population standard deviation is 137 kwh. How large must our sample be?

______82______

$1-\alpha = 90\%$

$\alpha = 0.10$

$\alpha/2 = 0.05$

$Z_{\alpha/2} = \pm 1.645$

$\sigma = 137$

$E = \pm 25$

$$n = \left[\frac{Z_{\alpha/2}\cdot\sigma}{E}\right]^2 = \left[\frac{1.645 * 137}{25}\right]^2 = 81.26$$

$n = 81.26$ **round up to** $\boxed{82}$

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(1.645*137/25)²
        81.26301316
```

2. A study of shark attacks on humans showed that 15 of 200 attacks occurred in deep water. Construct the 99% confidence interval for the true proportion of shark attacks that occur in deep water.

__0.027 < P < 0.123__

$n = 200$

$x = 15$

$\hat{p} = \frac{x}{n} = \frac{15}{200} = 0.075$

$\hat{q} = 1 - \hat{p} = 1 - 0.075 = 0.925$

______99% C.I. for p______

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p} * \hat{q}}{n}}$$

$$0.075 \pm 2.575\sqrt{\frac{0.075 * 0.925}{200}}$$

$$0.075 \pm .0480$$

$$\boxed{0.027 < p < 0.123}$$

```
1-PropZInt
 x:15
 n:200
 C-Level:99
 Calculate
```

```
1-PropZInt
 (.02703,.12297)
 p̂=.075
 n=200
```

3. A botanist measures the heights of 24 seedlings and obtains a mean and standard deviation of 41.6 cm and 4.8 cm, respectively. Construct the 98% confidence interval for the population mean.

$39.151 < \mu < 44.049$

$n = 24$

$\bar{x} = 41.6$

$s = 4.8$

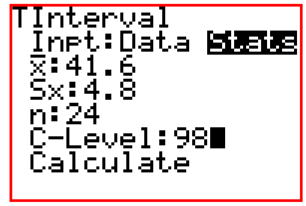
98% C.I. for μ

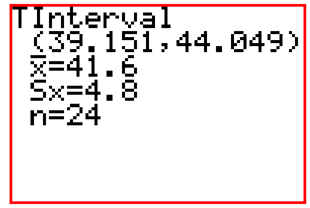
$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$41.6 \pm 2.5 \cdot \frac{4.8}{\sqrt{24}}$$

$$41.6 \pm 2.449$$

$$\boxed{39.151 < \mu < 44.049}$$





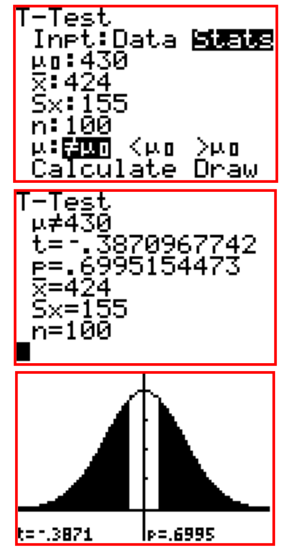
4. (a) Evaluate $t_{\alpha/2}$ for $\alpha = 0.05$ and n = 18. — a. ± 2.110

(b) Evaluate $z_{\alpha/2}$ for $\alpha = 0.06$. — b. ± 1.88

(c) Given the sample data in problem 3, what is the best point estimate of the population mean? — c. $\bar{X} = 41.6$

(d) Given the sample data in problem 2, what is the best point estimate of the population proportion of shark attacks that occur in deep water? — d. $\hat{p} = 0.075$

5. An educational testing company has been using a standard test of verbal ability and the mean has been 430. In analyzing a new version of that test, it is found that a sample of 100 randomly selected subjects produces a mean and a standard deviation of 424 and 155, respectively. At the 0.05 level of significance, test the claim that the new version has a mean equal to that of the past version.



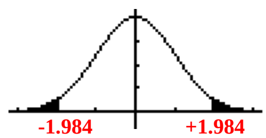
$\alpha = 0.05$
$n = 100$
$\bar{x} = 424$
$s = 155$

(claim) $H_o: \mu = 430$

$H_1: \mu \neq 430$

Test statistic: $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{424 - 430}{155/\sqrt{100}} = -0.387$

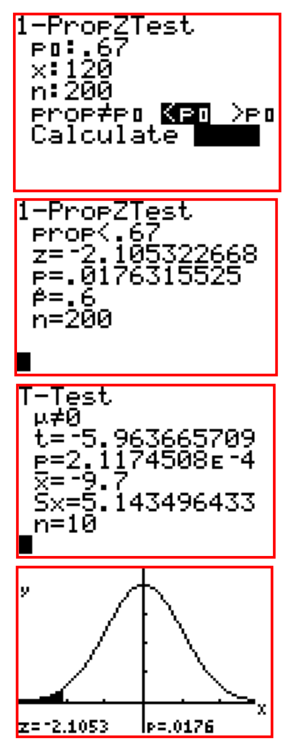
Test:



Fail to Reject H_o

Conclusion - New version has a mean equal to past version.

6. Before endorsing a candidate for political office, a newspaper editor surveys 200 randomly selected readers and finds that 120 favor the candidate in question. At the 0.05 level of significance, test the editor's claim that the candidate is favored by at least 2/3 of the readers.



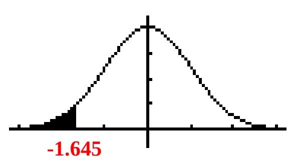
$\alpha = 0.05$
$n = 200$
$x = 120$

(claim) $H_o: p \geq 0.67$

$H_1: p < 0.67$

Test Statistic: $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}} = \dfrac{.60 - .67}{\sqrt{\dfrac{(.67)(.33)}{200}}} = -2.105$

Test:

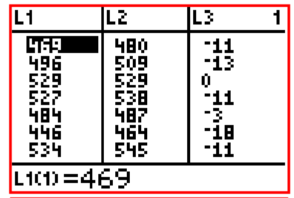


Reject H_o

Conclusion – The candidate is not favored by at least 2/3 of readers

7. A course is designed to improve scores on a college entrance examination. A randomly selected experimental group is used to test the effectiveness of the course and the results are given below.
At the 0.05 significance level, test the claim that course has no effect on the grades.

Subject	A	B	C	D	E	F	G	H	I	J
Before	469	496	529	527	484	446	534	531	539	565
After	480	509	529	538	487	464	545	542	551	572
$d = (x_1 - x_2)$ =	-11	-13	0	-11	-3	-18	-11	-11	-12	-7

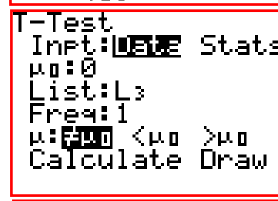


$n = 10$ (claim) H_0: $\mu_d = 0$

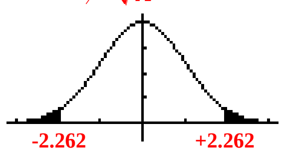
$\bar{d} = -9.7$ H_1: $\mu_d \neq 0$

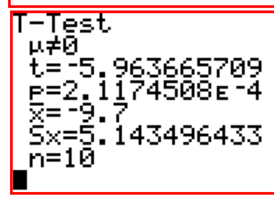
$s_d = 5.14$

$\alpha = 0.05$ Test Statistic: $t = \dfrac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \dfrac{-9.7 - 0}{5.14 / \sqrt{10}} = -5.97$



Test:





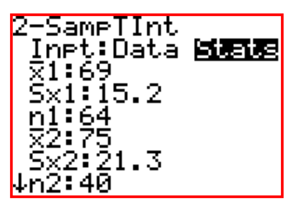
Reject Ho

Conclusion – The course does have an effect.

8. A psychology test was administered to estimate the difference in mean "aptitude for learning" of students in two different school districts, one rural and the other urban. The following data were obtained.

rural district	urban district
n_1 = 64	n_2 = 40
$\bar{x}_1$ = 69	$\bar{x}_2$ = 75
s_1 = 15.2	s_2 = 21.3

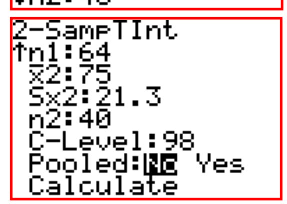
Determine an approximate 98 percent confidence interval for the difference in the population mean aptitudes for the two districts.



98% C.I. for $\mu_1 - \mu_2$

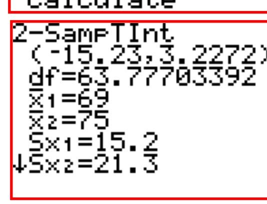
$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(69 - 75) \pm 2.423 \sqrt{\frac{15.2^2}{64} + \frac{21.3^2}{40}}$$



$$-6 \pm 9.37$$

$$\boxed{-15.37 < \mu_1 - \mu_2 < 3.37}$$



9. A marketing survey involves product recognition in New York and California. Of 558 New Yorkers surveyed, 193 knew the product while 196 out of 614 Californians knew the product. At the 0.05 significance level, test the claim that the recognition rates are the same in both states.

$n_1 = 558$ $\quad\quad$ $n_2 = 614$

$x_1 = 193$ $\quad\quad$ $x_2 = 196$

$\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{193}{558} = 0.346$ $\quad$ $\hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{196}{614} = 0.319$ $\quad$ $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{193+196}{558+614} = 0.332$

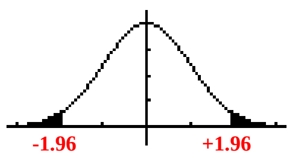
$\hat{q}_1 = 1 - \hat{p}_1 = 0.654$ $\quad$ $\hat{q}_2 = 1 - \hat{p}_2 = 0.0.681$ $\quad$ $\bar{q} = 1 - \bar{p} = 0.668$

(Claim) $p_1 = p_2 \longrightarrow$ $H_o: p_1 - p_2 = 0$

$H_1: p_1 - p_2 \neq 0$

Test Statistic: $$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}} = \frac{(0.346 - 0.319) - 0}{\sqrt{\dfrac{(0.332)(0.668)}{558} + \dfrac{(0.332)(0.668)}{614}}} = 0.980$$

Test:



Fail to Reject H_o

Conclusion – Recognition rates are the same in both states.